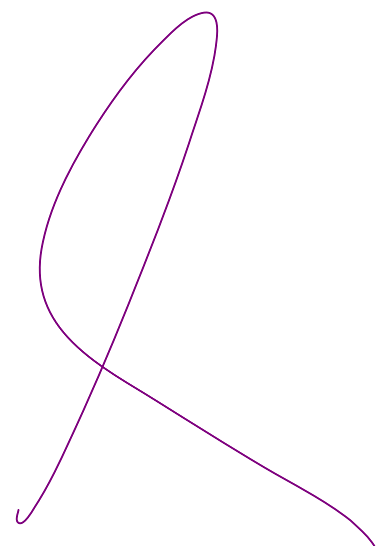
Note Title 3/23/2015



Spinodal Effects During Inflation

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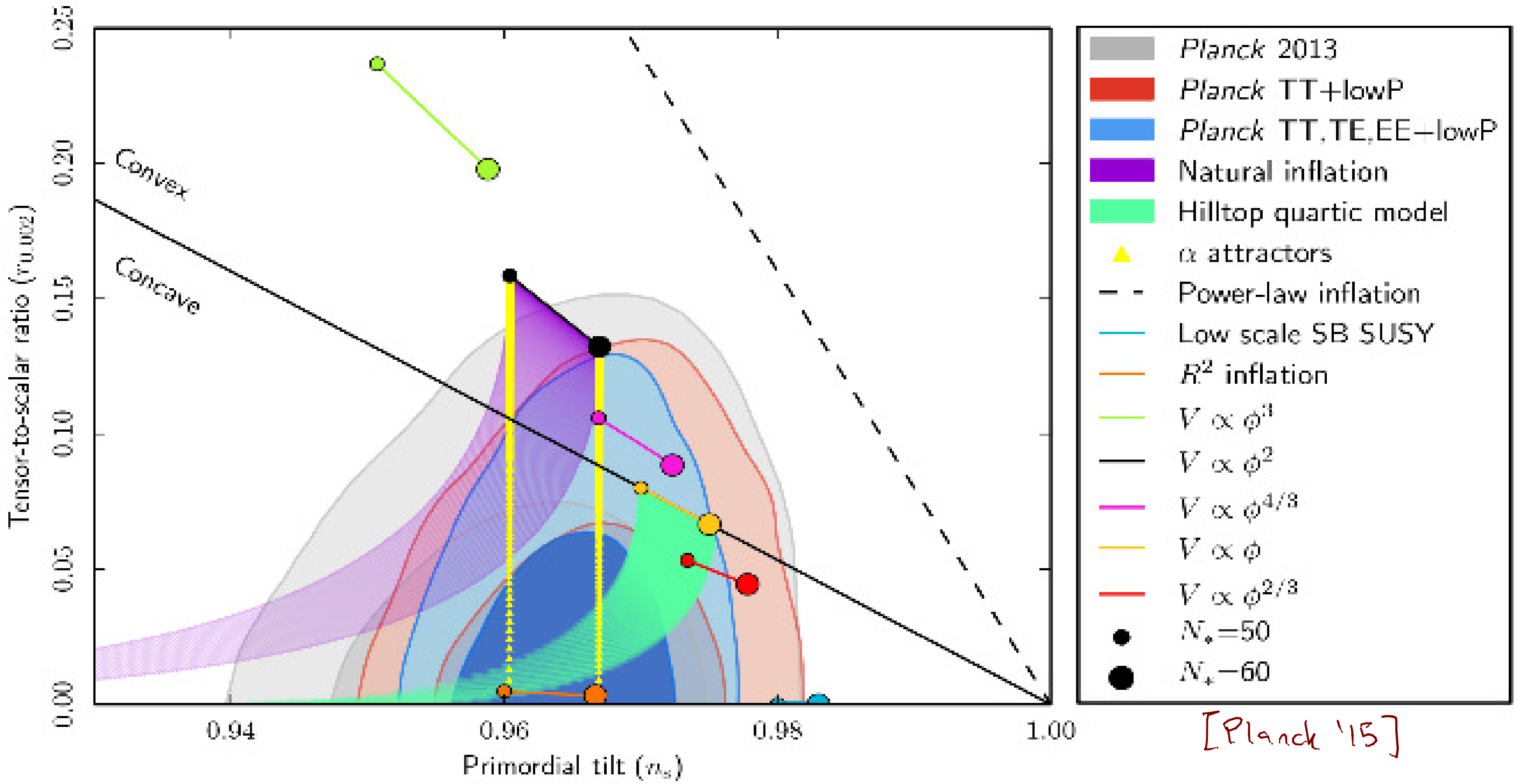
[arxiv: 1504.03332
w/ B. Richard]



[Planck '15]

$$r = 8\frac{V'^2}{V^2} \qquad n_s - 1 = -3\frac{V'^2}{V^2} + 2\frac{V''}{V} \qquad r < .08 \Rightarrow V'' < 0$$

Outline

↳ Tachyonic Inflaton ⇒ IR instability

↳ Power can grow and influence dynamics

↳ Can encapsulate in a two field model [Cormier, Holman '99]

↳ Diakyon field σ ⟷ long wavelength modes

$$\boxed{V^H(\bar{\Phi},\sigma) = \int_{\mathbb{R}} d\psi \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\psi^2}{2\sigma^2}} V(\bar{\Phi}+\psi)}$$

↳ Many implications to explore

Setup

Action $S(\phi)$, $\phi = \Phi(t) + \psi$

Expansion history $\quad$ perturbation spectrum

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$
$$(-\Box + V''(\Phi))\psi = 0$$

lowest order

However:

$V'' < 0 \Rightarrow$ tachyon

Long wavelength modes can become large can backreact on <u>BOTH</u> equations!

Path Integral

$$Z = \int \mathcal{D}\psi \, e^{iS(\Phi+\psi) + i\int dx \, j_x \psi_x - \frac{i}{2}\int dx\,dy\, K_{xy}\psi_x\psi_y}$$

$$\boxed{\begin{aligned} &\text{Tadpole:} \quad \frac{\delta}{\delta j_x} Z \Big|_{j=K=0} = 0 \\ &\text{Gap:} \quad \frac{\delta}{\delta K_{xy}} Z \Big|_{j=K=0} = 0 \end{aligned}}$$

Involve $\langle \psi^n \rangle$,

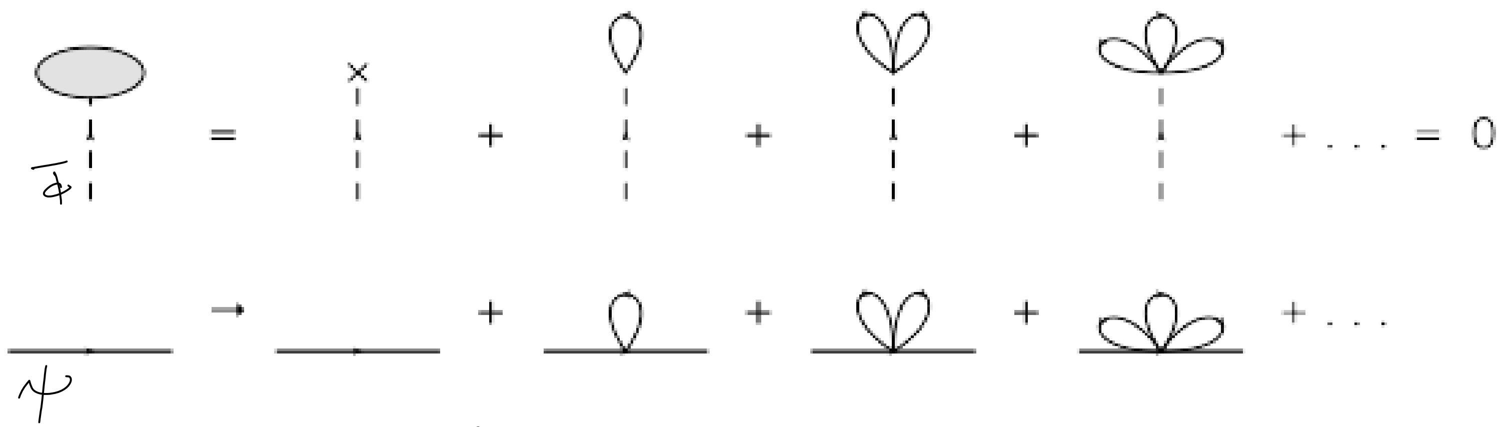
n arbitrary:

impossible to solve

Hartree approximation:

$\langle \psi^{2n} \rangle = \# \langle \psi^2 \rangle^n \quad , \quad \langle \psi^{2n+1} \rangle = \# \langle \psi^2 \rangle^n \psi$

truncates system to just Φ and $\langle \psi^2 \rangle$.



(& iterate)

Derivation

$$S(\phi) = \bar{S}_0 + \bar{S}_1 \phi + \bar{S}_2 \phi^2 + S_{int}$$

$$\langle F \rangle_0 = \int \mathcal{D}\phi \, F \, e^{i \bar{S}_2 \phi^2}$$

$$\left.\frac{\delta Z}{\delta j_x}\right|_{j=0} = \langle \phi_x \rangle = e^{i\bar{S}_0} \left\langle \phi_x \, e^{i \frac{\delta L}{\delta \phi} \phi + i \bar{S}_{int}} \right\rangle_0$$

$$\approx i e^{i\bar{S}_0} \int d^4y \left[\frac{\delta L}{\delta \phi} \langle \phi_x \phi_y \rangle_0 + \langle \phi_x V_{int}(\phi_y) \rangle_0 \right]$$

$$= i e^{i\bar{S}_0} \int d^4y \, G_{xy} \left[\frac{\delta L}{\delta \phi_y} + \langle V_{int}'(\phi_y) \rangle_0 \right] = 0$$

The diakyon

$$\boxed{\begin{array}{l}\text{Long wavelength modes important} \\ \langle \psi_x \psi_y \rangle_0 \longrightarrow \sigma(t)^2 \quad \text{[Cormier, Holman]}\end{array}}$$

$$\boxed{V^H(\Phi, \sigma) \equiv \langle V(\Phi + \psi) \rangle_0 = \int_{\mathbb{R}} d\psi \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\psi^2}{2\sigma^2}} V(\Phi + \psi)}$$

$$\left.\begin{array}{ll}\text{Tadpole:} & (\partial_t^2 + 3H\partial_t)\Phi + V_\Phi^H = 0 \\ \text{Gap:} & (\partial_t^2 + 3H\partial_t)\sigma + V_\sigma^H = 0\end{array}\right\} \longrightarrow \begin{array}{c}\text{EOMs for} \\ \Phi \;\&\; \sigma\end{array}$$

Properties of $V^H(\bar{\Phi},\sigma) = \int_{\mathbb{R}} d\psi \, \frac{1}{\sqrt{2\pi}\,\sigma} \, e^{-\frac{\psi^2}{2\sigma^2}} \, V(\bar{\Phi}+\psi)$

$$V^H(\phi, 0) = V(\phi)$$

$$\boxed{\sigma \, \partial_\phi^2 V^H = \partial_\sigma V^H}$$

$\Rightarrow m_\sigma^2 = m_\phi^2 + \sigma^2 \partial_\phi^4 V^H$ $\quad$ σ also slow rolls!

$\sigma \ll M_p \Rightarrow \sigma$ eqn integrable $\quad$ $\sigma(N) = \sigma_0 \dfrac{V_\phi(N)}{V_\phi(0)}$

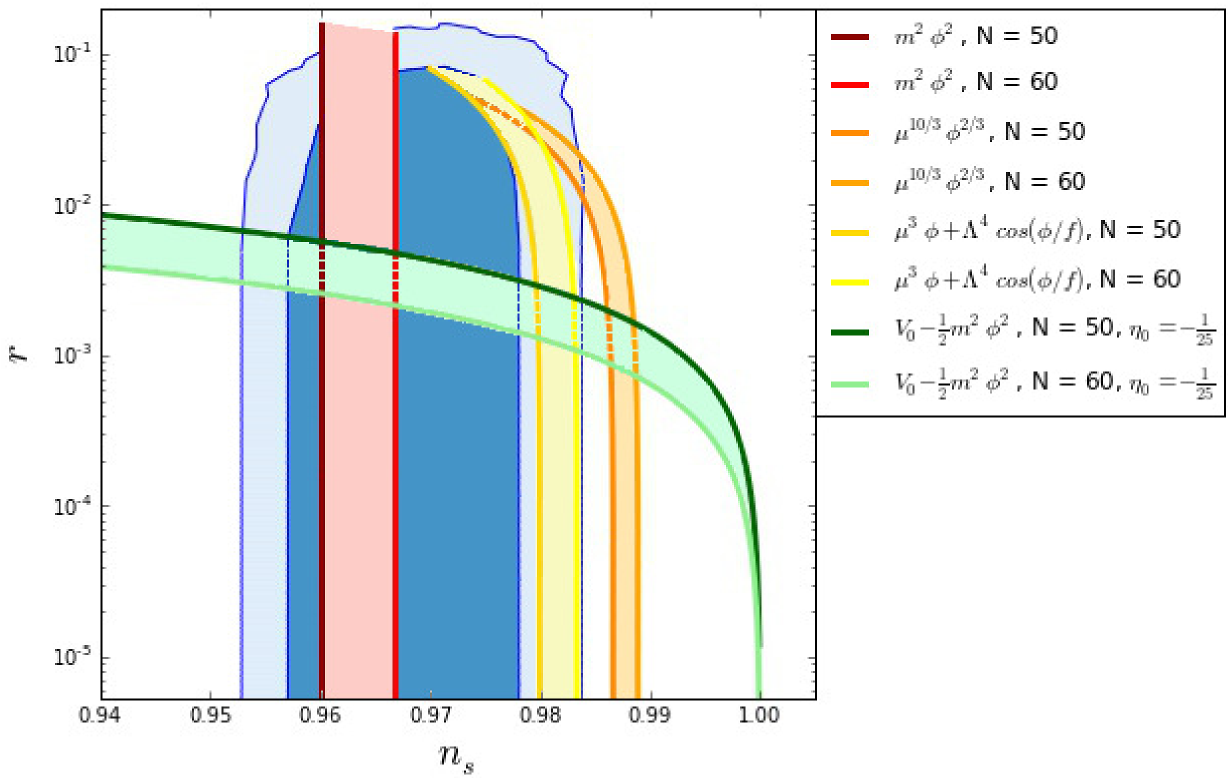
Perturbations

$$P_k = \frac{\langle (V(\phi+\varphi) - V(\phi))^2 \rangle_0}{\dot{H}} \approx \left(\frac{V^H}{V_\phi^{H2} + V_\sigma^{H2}} \right)^2 \langle V_\phi^2 \varphi^2 \rangle_0$$

$\sigma \ll M_p$: $$P_k \approx \frac{V^2}{V_\phi^2} \langle \varphi^2 \rangle_0 = \frac{V^2}{V_\phi^2} \sigma^2$$

$$\boxed{\begin{aligned} n_s - 1 &= -\epsilon \left(4 + \frac{2}{1+\sigma^2/H^2} \right) + 2\eta \frac{1}{1+\sigma^2/H^2} \longrightarrow \begin{cases} -6\epsilon + 2\eta & \sigma \to H \\ -4\epsilon & \sigma \gg H \end{cases} \\ r &= \frac{16\epsilon}{1+\sigma^2/H^2} \longrightarrow \begin{cases} 16\epsilon & \sigma \to H \\ 0 & \sigma \gg H \end{cases} \end{aligned}}$$

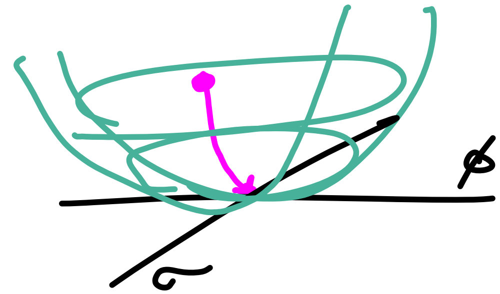
Specific Models



$m^2\phi^2$, hilltop , flattened , monodromy

$$\underline{V = m^2\phi^2}$$

$$V^H = \frac{1}{2}m^2(\phi^2+\sigma^2) \qquad N_{CMB} = \frac{\phi^2+\sigma^2}{4}$$



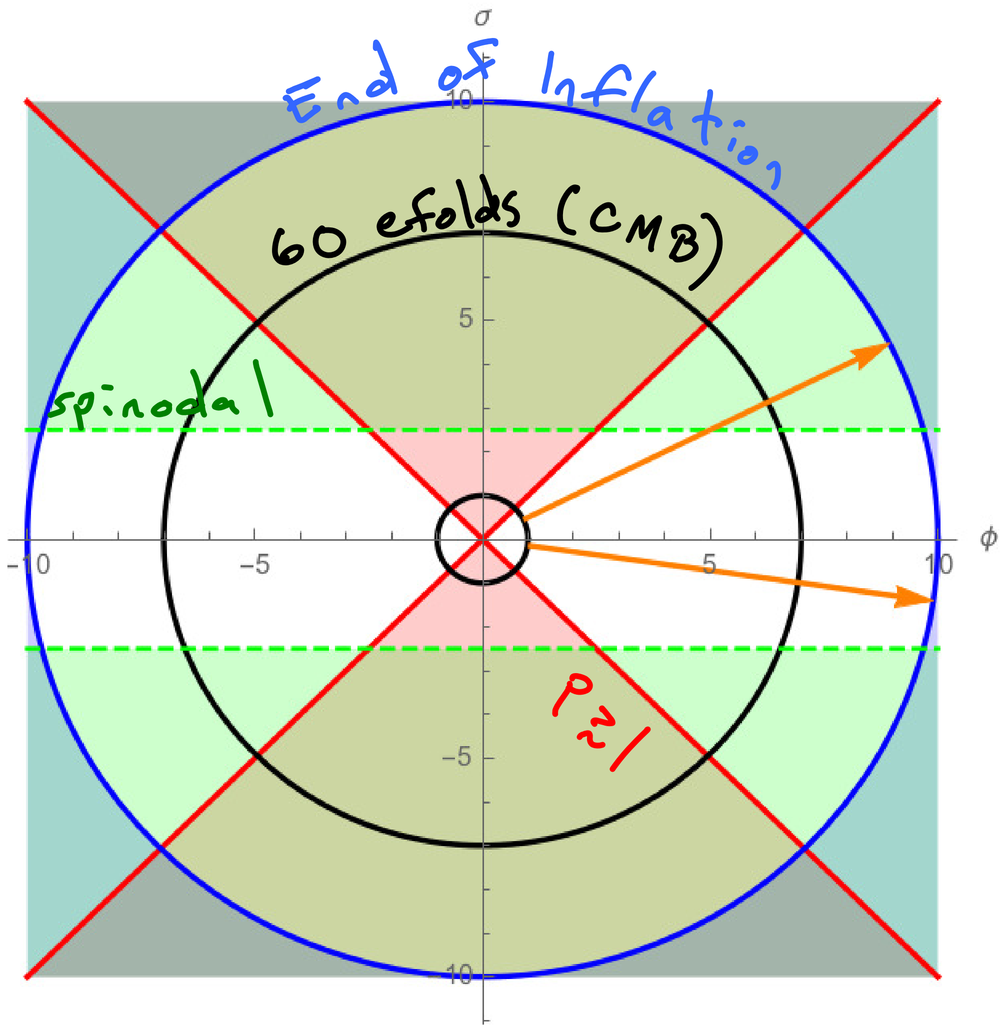
$P_k = N_{CMB}\,\sigma^2$ * $\left\{ \begin{array}{l} \text{Amplitude set by } \sigma, \\ \underline{\text{NOT}} \text{ potential.} \\ P_k \sim 10^{-9} \Rightarrow \sigma_0 \sim 10^{-5} M_p \end{array} \right.$

$n_s - 1 = \frac{-2}{N_{CMB}}$ * tilt unchanged

$r \propto \frac{m^2}{\sigma^2}$ * small m $\Rightarrow$ small r

Hilltop

$$V^H = V_0 - \frac{1}{2} m^2 (\phi^2 + \sigma^2)$$



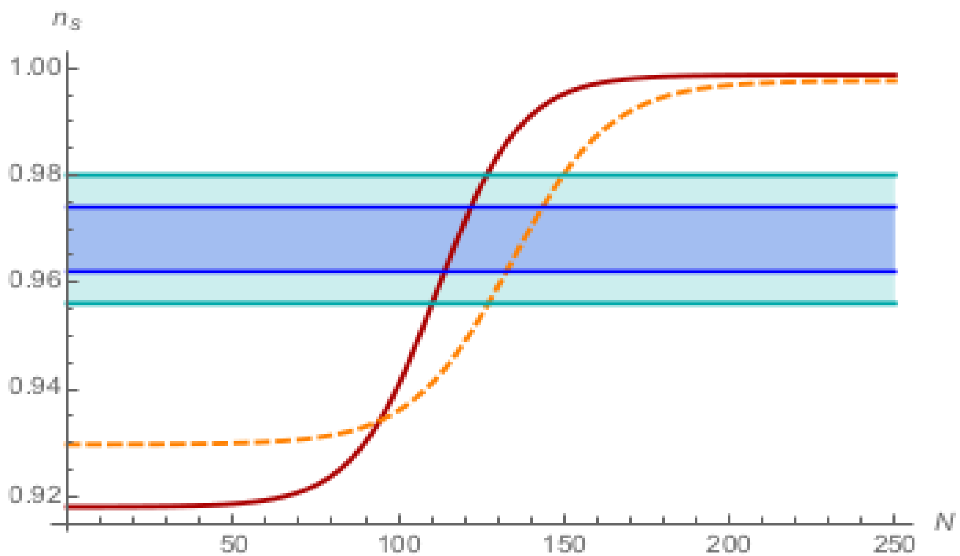
$$P_k \approx \frac{V_0^2}{M^4} \frac{\sigma^2 + H^2}{\phi^2}$$

$$\phi(N) \sim \phi_0 \, e^{|\eta_0| N}$$

$$\sigma(N) \sim \sigma_0 \, e^{|\eta_0| N}$$

$$\phi/\sigma = \text{const}$$

Hilltop (Continued)



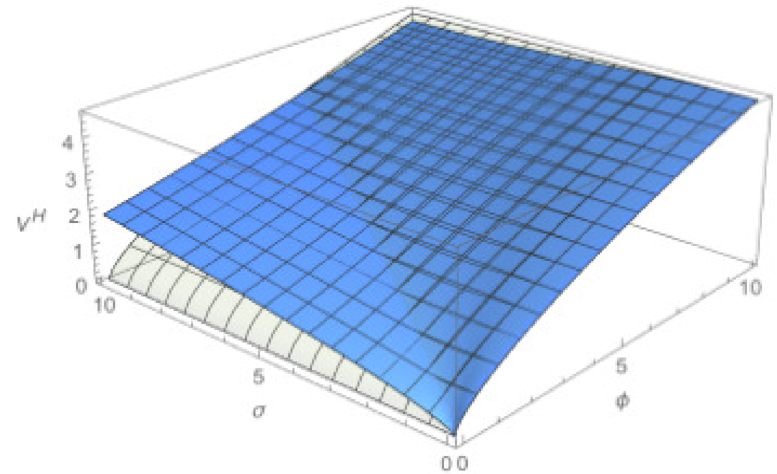
~30 efolds

- Usual hilltop models are excluded - diakyon can help!
- Tends towards scale invariance.
- Fairly slow climb

Quartic: $V^H = V_0 - \lambda\left(\phi^4 + 6\sigma^2\phi^2 + 3\sigma^4\right)$

$$V = \mu^{10/3}\,\phi^{2/3}$$

$$V^{\rm H}_{\rm flat}(\phi,\sigma) = \mu^{10/3}\left[\frac{\Gamma(5/6)\,\sigma^{2/3}}{2^{2/3}\pi^{1/2}}\,{}_1F_1\left(-\frac{1}{3};\frac{1}{2};-\frac{\phi^2}{2\sigma^2}\right) + \frac{\Gamma(4/3)\,\phi}{2^{1/6}\pi^{1/2}\sigma^{1/3}}\,{}_1F_1\left(\frac{1}{6};\frac{3}{2};-\frac{\phi^2}{2\sigma^2}\right)\right]$$



$\boxed{\text{IF}}$ Starts from the eternal regime:

$P_k \sim 1 \qquad \phi_0 \sim \dfrac{M_P^{9/4}}{\mu^{5/4}} \qquad \sigma_0 \sim H_0$

$\boxed{\text{Then}}$ From $\sigma(N) = \sigma_0 \dfrac{V_\phi(N)}{V_\phi(0)}$

$$\sigma_{CMB} \sim \left(\frac{M_P}{\mu}\right)^{5/6} H_{CMB} \gg H_{CMB}$$

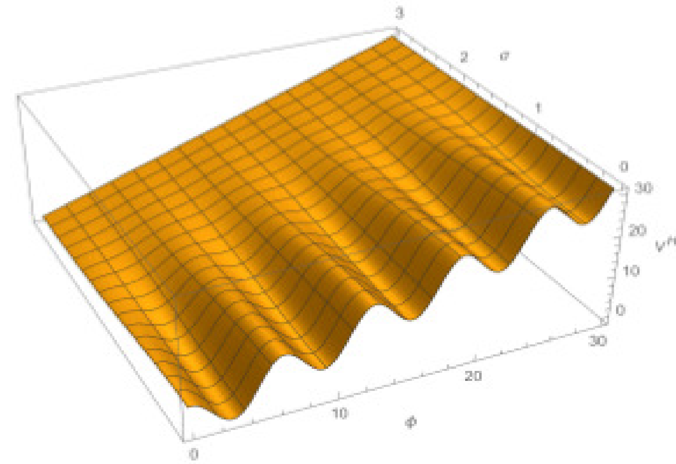
$n_s = .989$

$r = 10^{-8}$

This effect HAS to be taken into account.

Monodromy

$$V^H(\phi,\sigma) = \mu^3\phi + \Lambda^4 e^{-\frac{\sigma^2}{2f^2}} \cos \phi/f$$



Large σ_0 $\Longrightarrow$ decreased Λ^4_{eff}

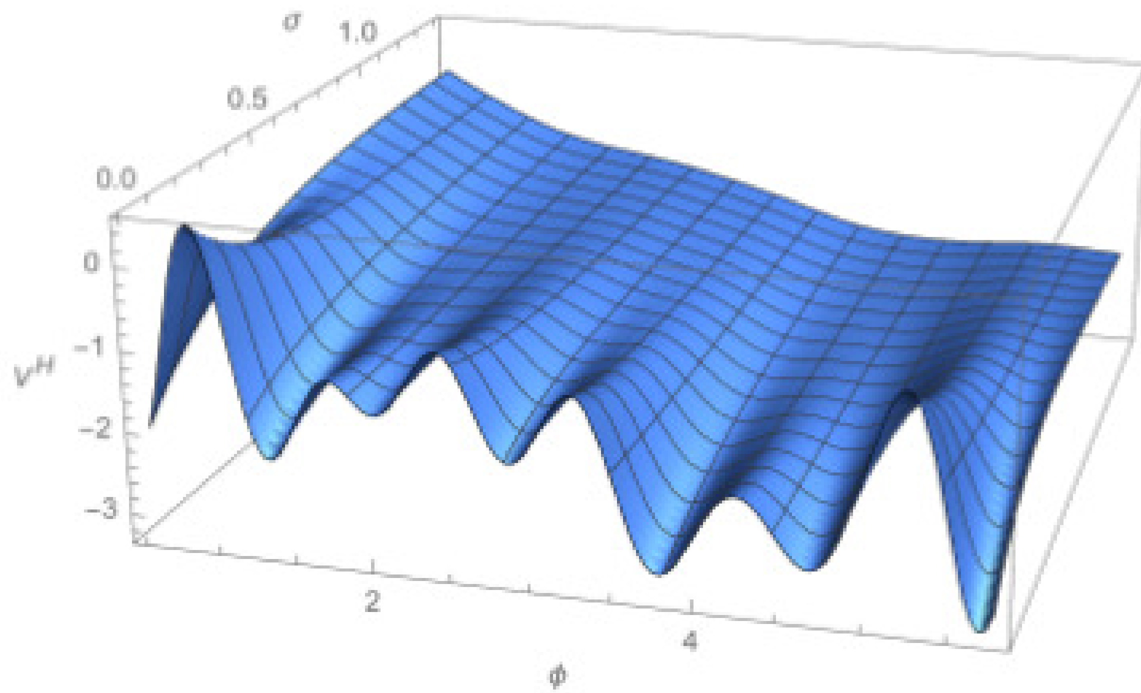
would <u>not</u> expect wiggles in CMB

Disclaimer: $P_k \sim 10^{-9} \Rightarrow \sigma \sim 10^{-5} M_p$

Screening efficient for $\sigma \gtrsim 4f$

Irons out wiggles only for $f < 10^{-6} M_p$.

Generic Potential



can "iron out" false vacua and produce a slow roll potential

Conclusions

↳ Spinodal effects have important consequences

Amplitude can depend on initial conditions

LARGE FIELD	SMALL FIELD
decreases tensor to scalar ratio	Spectral tilt tends toward scale invariance
Effect has to be taken into account	Can dampen features in the potential

Future Work

↳ Need to check validity of Hartree approx.

↳ Incorporate into gauge invariant perturbation theory

↳ Check effects on 3 point function

↳ etc...