A relation between deformed superspace and Lee-Wick higher-derivative theories

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Summary

- Motivations: Quantum Gravity, and the nature of spacetime at the Planck scale
- Non anti commutative (NAC) SUSY: $\mathcal{N} = 1/2$ SUSY
- (Interlude) NAC in (2+1)D SUSY
- A different proposal for (3+1)D NAC Supersymmetry
- Higher derivative theories, Lee-Wick models
- Conclusions

Motivations

- Already in the 40's, non commutativity (NC) of spacetime was proposed as a way to tame the divergences of QFT (Heisenberg, Snyder).
- Renormalization theory was a more conservative approach, and achieved enormous success, peaking at the formulation of the Standard Model.
- In the last decades, NC spacetime has regained attention as a proposal for a modification of spacetime structure at the Planck scale, related to quantum gravity.

Motivations

- The **Planck length** is the length scale where we expect gravitational quantum effects to become relevant.

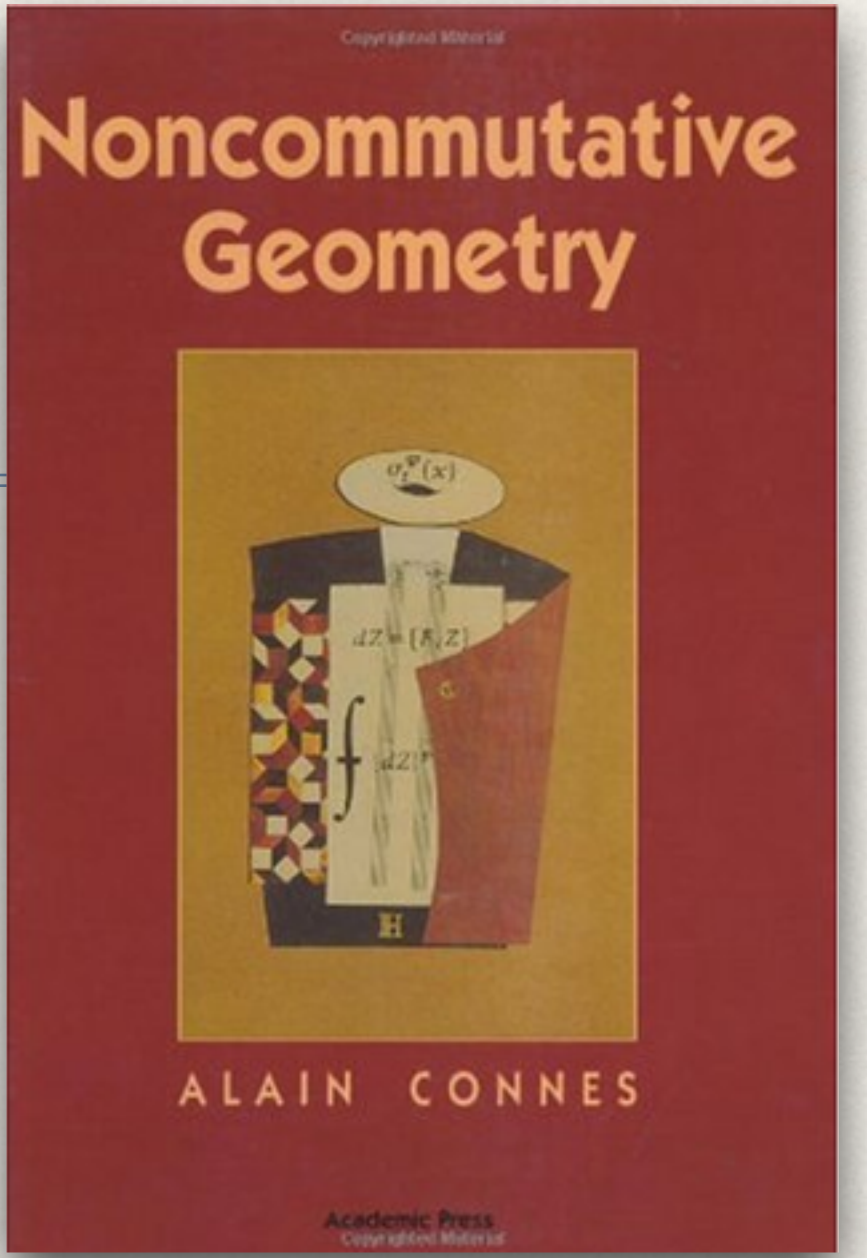
$$\ell_P \sim \sqrt{G\hbar/c^3} \sim 10^{-35}\mathrm{m}$$

- Gravity is closely related to the geometry of spacetime, so it would not be surprising that **quantum** gravity would require some fundamental changes in the way we describe spacetime at the Planck length.

- Perturbative quantum gravity lives in an usual Minkowski spacetime, but is non renormalizable, therefore, it works at the most as an effective field theory, and cannot be a **complete** description of quantum gravitational effects.

Motivations

- Since the 1990's, Alain Connes has developed NC generalisations of geometric concepts, and the idea of "quantization of spacetime" is one of the motivations.



- Deformations and generalisations of spacetime, involving some kind of non commutativity, Lorentz violations etc… have been studied in the last decades, either via a more mathematical standpoint, or by studying models with a more obvious physical application.

Motivations

- Doplicher et.al. (1995): semiclassical arguments suggest the existence of a fundamental limit in the localisation of spacetime events. **Spacetime uncertainty relations**.

$$\Delta x_0(\Delta x_1 + \Delta x_2 + \Delta x_3) \gtrsim \lambda_P^2 ,$$

$$\Delta x_1 \Delta x_2 + \Delta x_2 \Delta x_3 + \Delta x_3 \Delta x_1 \gtrsim \lambda_P^2 ,$$

- This could be explained by assuming non commutativity of spacetime coordinates.

$$[q_\mu, q_\nu] = \mathrm{i} Q_{\mu\nu}$$

Motivations

- κ-Poincaré: deformation of the Poincaré algebra, including a parameter κ with dimension of Energy, such that the limit $\kappa \to \infty$ goes back to the usual Poincaré algebra (Kosinski, Lukierski et al).
- It is related to a non commutative spacetime: κ-Minkowski.

$$\left[x^0, x^i\right] = \frac{i}{\kappa} x^i, \quad \left[x^i, x^j\right] = 0$$

- This kind of deformation arises from quantum gravity in(2+1)D (Freidel, Kowalski-Glikman, Smolin), and might be related to the existence of an invariant length scale (Doubly Special Relativity).

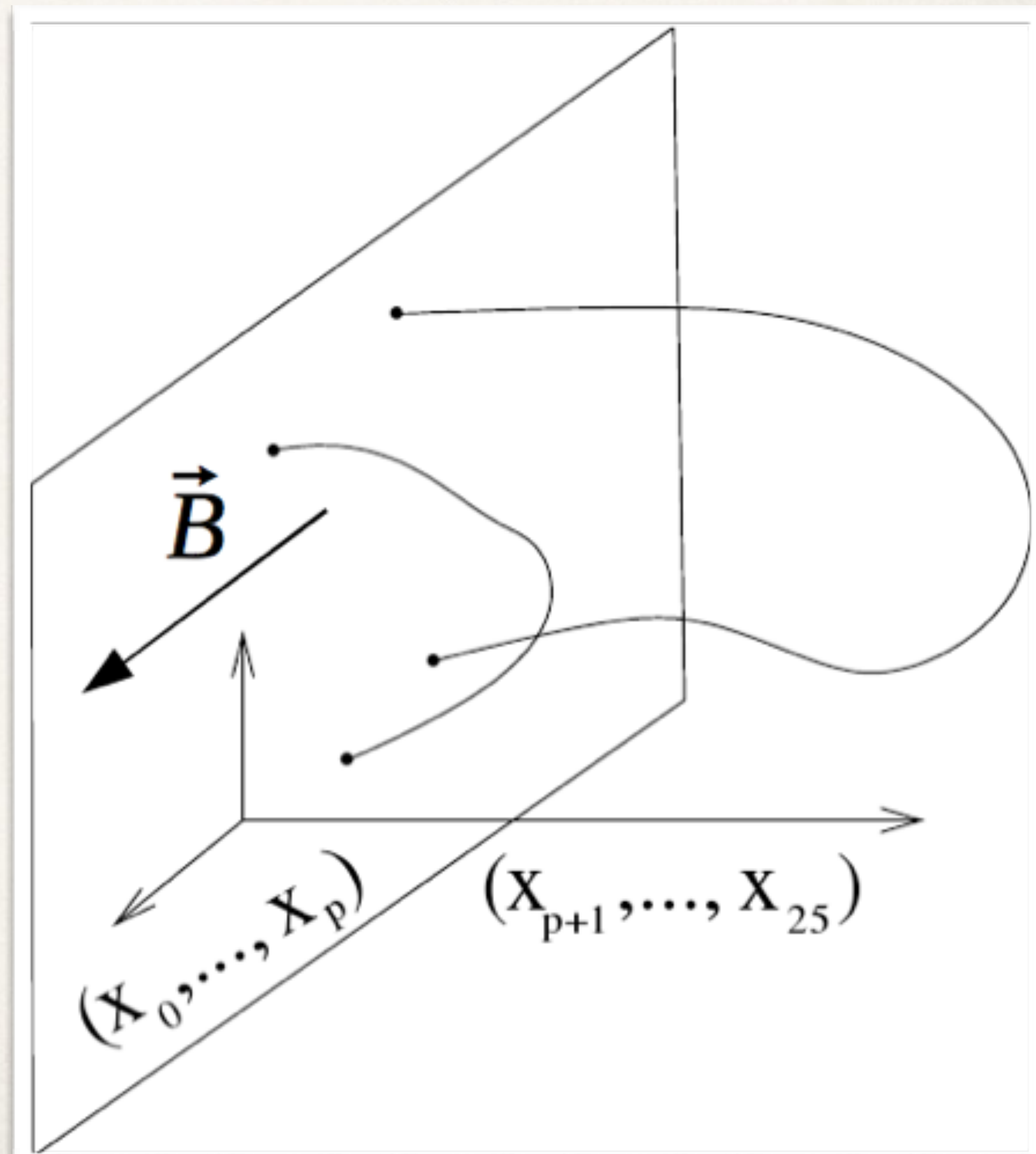
Motivations

- Seiberg, Witten, Douglas: non commutativity from string theory. **Canonical non commutativity.**

$$[\mathbf{x}^{\mu}\,,\,\mathbf{x}^{\nu}] = i\theta^{\mu\nu}$$

- Easily implemented via a *star product*, and many NC models could be studied.

$$f_1(x) * f_2(x) = f_1(x)\,\exp\left(\frac{i}{2}\,\frac{\overleftarrow{\partial}}{\partial x^{\mu}}\,\theta^{\mu\nu}\,\frac{\overrightarrow{\partial}}{\partial x^{\nu}}\right) f_2(x)$$



Motivations

- Canonical non commutativity was a hot topic during the years 2.000, since it was simple enough that many interesting models could be extended to a NC spacetime, and explicit calculations could be done, yet rich enough to provide several new effects:
  - UV/IR mixing
  - self-interacting Abelian gauge theories
  - restriction on gauge groups and representations for non Abelian theories
  - SUSY presented itself as a nice way to avoid UV/IR problems (*Girotti et al NPB 587(2000)299, AFF et al PRD69(2004)025008, PRD70(2004)085012, …*)

NAC Superspace

- Supersymmetry is itself, at a fundamental level, a proposal for a modification of spacetime: Minkowski space is replaced by a **superspace**, and Poincaré symmetry is generalised (in an essentially unique way) to **supersymmetry**.

$$x^\mu \to z^A = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$$

$$\{\theta^\alpha, \theta^\beta\} = \left\{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\} = 0$$

The set of spacetime coordinates is enlarged by a set of **anti commuting** coordinates θ

$$Q_\alpha = \partial_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \ , \ \bar{Q}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

Poincaré symmetry is enlarged by the inclusion of SUSY generators

NAC Superspace

- *Seiberg JHEP 06(2003)010*: **non anticommutative** deformation of superspace.

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} \qquad \left\{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\right\} = 0$$

- As the canonical non commutativity, this can be motivated by a particular limit of String Theory.

- *Klemm et al CQG 20(2003)2905*: general case of NAC SUSY. In general, **associativity** of the star product and SUSY is lost, except for very specific cases (including the one studied by Seiberg).

$$\begin{aligned}
\left\{\theta^\alpha, \theta^\beta\right\} &= \mathcal{A}^{\alpha\beta}(x,\theta,\bar\theta) \quad , \quad & \left\{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\right\} &= \bar{\mathcal{A}}^{\dot\alpha\dot\beta}(x,\theta,\bar\theta)\\
\left\{\theta^\alpha, \bar\theta^{\dot\alpha}\right\} &= \mathcal{B}^{\alpha\dot\alpha}(x,\theta,\bar\theta) & &\\
\left[x^{\underline{a}}, \theta^\beta\right] &= i\mathcal{C}^{\underline{a}\beta}(x,\theta,\bar\theta) \quad , \quad & \left[x^{\underline{a}}, \bar\theta^{\dot\beta}\right] &= i\bar{\mathcal{C}}^{\underline{a}\dot\beta}(x,\theta,\bar\theta)\\
\left[x^{\underline{a}}, x^{\underline{b}}\right] &= i\mathcal{D}^{\underline{ab}}(x,\theta,\bar\theta) & &
\end{aligned}$$

NAC Superspace

SUSY algebra in terms of chiral coordinates:

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \bar{\theta}^{\dot{\alpha}}] = 0$$

Chiral superfields Φ(y,θ) can be easily defined.

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^\mu},$$

$$\{Q_\alpha, Q_\beta\} = 0,$$

Half of the SUSY algebra is undeformed.

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = -4C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\frac{\partial^2}{\partial y^\mu \partial y^\nu},$$

$\mathcal{N}$ = 1/2 supersymetry.

NAC Superspace

- Since the θ do not anti commute, when building functions of θ we need to define a proper ordering. One way is using a *star product*

$$f(\theta) \star g(\theta) = f(\theta)\exp\left(-\frac{c^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\alpha}\right)g(\theta)$$

that enforces the *symmetric* ordering.

- The star product is associative and *finite*. When defining a scalar model, the only modification is the inclusion of an additional term in the Lagrangian.

$$\mathcal{L} = \int d^4\theta\,\overline{\Phi}\Phi + \int d^2\theta\,(\frac{1}{2}m\Phi * \Phi + \frac{1}{3}g\Phi * \Phi * \Phi) + \int d^2\overline{\theta}\,(\frac{1}{2}\overline{m}\overline{\Phi} * \overline{\Phi} + \frac{1}{3}\overline{g}\overline{\Phi} * \overline{\Phi} * \overline{\Phi})$$
$$= \mathcal{L}(C = 0) - \frac{1}{3}g\det CF^3 + \text{total derivative.}$$

(Interlude) NAC in 3D Superspace

- In 3D, the Grassmanian coordinates are *real*. Because of this, we do not have the option, in principle, to break half of SUSY, as in 4D.

$$x^\mu \to z^A = (x^\mu, \theta^\alpha)$$

- What we can do is start with $\mathcal{N} = 2$, then break half of SUSY to end up with an $\mathcal{N} = 1$ SUSY model.

$$\{\theta^{1\,\alpha}, \theta^{1\,\beta}\} = \{\theta^{1\,\alpha}, \theta^{2\,\beta}\} = \{\theta^{2\,\alpha}, \theta^{2\,\beta}\} = 0 \quad \Rightarrow \quad \{\theta^{i\alpha}, \theta^{j\beta}\} = \Sigma^{ij\alpha\beta}$$

- Since $\mathcal{N} = 2$ 3D SUSY can be rewritten as a kind of $\mathcal{N} = 1$ 4D SUSY model, this is essentially adapting Seiberg's procedure to the 3D case.

$$\theta^{1\,\alpha}, \theta^{2\,\beta} \to \theta^\alpha, \bar{\theta}^\beta$$

(Interlude) NAC in 3D Superspace

- We can choose for example

$$\{\theta^{1\,\alpha}, \theta^{1\,\beta}\} = \{\theta^{1\,\alpha}, \theta^{2\,\beta}\} = 0\,;\ \{\theta^{2\,\alpha}, \theta^{2\,\beta}\} = \Sigma^{\alpha\beta}$$

- Half of the $\mathcal{N} = 2$ SUSY algebra is broken.

$$\{Q^{1\,\alpha}, Q^{1\,\beta}\} = 2i\partial^{\alpha\beta}\ ;\ \{Q^{1\,\alpha}, Q^{2\,\beta}\} = 0$$
$$\{Q^{2\,\alpha}, Q^{2\,\beta}\} = 2i\partial^{\alpha\beta} - \Sigma^{\mu\nu}\partial_{\mu\alpha}\partial_{\nu\beta}$$

- Compatible star product. The key here is to use the covariant derivative D, instead of the Grassmanian derivative ∂

$$\Phi_1(z) * \Phi_2(z) = \exp\Big[-\frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2)\Big] \times \Phi_1(z_1)\Phi_2(z_2)\Big|_{z_1=z_2=z}.$$

(Interlude) NAC in 3D Superspace

- Quadratic action for a scalar model, where Δ is a kinetic operator, independent of θ^2.

$$S = -\frac{1}{4}\int d^7 z \Phi * \Delta\Phi,$$

- After some algebraic work, we can show the star product can be explicitly worked out, reducing to a simple exponential factor, depending on the momenta.

$$S = \frac{1}{2}\int d^7 z \Phi(z) e^{R(P)/2} \Delta\Phi(z).$$

$$\begin{aligned} R(P) &\equiv \Sigma^{\alpha\beta}\left(P_{\alpha\beta} + \frac{1}{2}\Sigma^{\gamma\delta} P_{\gamma\alpha} P_{\delta\beta}\right) \\ &= 2\Sigma \cdot P + 2(\Sigma \cdot P)^2 - \Sigma^2 P^2, \end{aligned}$$

- Propagator:

$$\langle \Phi(-p, \theta_1)\Phi(p, \theta_2)\rangle = i e^{-R(P)/2} \Delta^{-1}(p) \delta^4_{12}.$$

(Interlude) NAC in 3D Superspace

- For a clever choice of the vector Σ: $\Sigma = (\Sigma^0,0,0)$, we end up with

$$\langle\Phi(-p,\theta_1)\Phi(p,\theta_2)\rangle = ie^{\Sigma_0 p_0 - \Sigma_0^2(p_0^2+\vec{p}^2)}\Delta^{-1}(p)\delta_{12}^4$$

- This propagator has exponential UV suppression: UV finiteness!
- So it is possible to build up things in NAC 3D SUSY to obtain interesting models, such as UV finite ones.
- In our paper *Ferrari et al, PRD 74(2006)125016*, we also give another prescription, that ends up being quite similar to 4D $\mathcal{N} = 1/2$ SUSY.

Alternative NAC Superspace

- Now we go back to 4D…
- We want to show another alternative formulation of NAC SUSY in 4D that has interesting properties.
- Based on *Dias, AFF, et al, JPA 48(2015)275403*.
- We start by proposing the star product, also involving a particular combination of covariant derivatives:

$$f(z) \star g(z) = f(z) \exp\left[\frac{\xi}{2} C^{\alpha\dot{\alpha}} \left(\overleftarrow{D}_{\alpha} \overrightarrow{\bar{D}}_{\dot{\alpha}} + \overleftarrow{\bar{D}}_{\dot{\alpha}} \overrightarrow{D}_{\alpha} \right)\right] g(z)$$

$$\xi = \frac{1}{M^2},$$

Alternative NAC Superspace

$$f(z) \star g(z) = f(z) \exp\left[\frac{\xi}{2} C^{\alpha\dot{\alpha}} \left(\overleftarrow{D}_\alpha \overrightarrow{\bar{D}}_{\dot{\alpha}} + \overleftarrow{\bar{D}}_{\dot{\alpha}} \overrightarrow{D}_\alpha \right)\right] g(z)$$

- This implies in

$$\left\{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right\}_\star = \xi C^{\alpha\dot{\alpha}},$$

$$\left\{\theta^\alpha, \theta^\beta\right\}_\star = \left\{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\}_\star = 0,$$

$$[y^\mu, y^\nu]_\star = [\bar{y}^\mu, \bar{y}^\nu]_\star = 0,$$

$$[y^\mu, \theta^\alpha]_\star = 0, \quad \left[\bar{y}^\mu, \bar{\theta}^{\dot{\alpha}}\right]_\star = 0.$$

all remaining (anti)commutations relations are deformed.

Alternative NAC Superspace

- SUSY algebra is broken, in general.

$$\left\{ Q_\alpha, \bar{Q}_{\dot\alpha} \right\}_\star = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu + \xi C^{\beta\dot\beta}\sigma^\mu_{\alpha\dot\beta}\sigma^\nu_{\beta\dot\alpha}\partial_\mu\partial_\nu,$$

- So our star product breaks SUSY. It is also *not finite* as an expansion in power of C, and it is *not associative*.
- In principle, we seem to loose too much. Can this deformation be still interesting?
- It happens that this product can still lead, in specific examples, to interesting models.

NAC Wess-Zumino Model

- Chiral and anti chiral superfields can be easily defined, due to ☼.
- Star product of a chiral and an anti chiral superfields:

$$\Phi \star \bar{\Phi} = \Phi\bar{\Phi} + \frac{\xi}{2} C^{\alpha\dot{\alpha}} \left(D_\alpha \Phi\right)\left(\bar{D}_{\dot{\alpha}} \bar{\Phi}\right) - \frac{\xi^2}{16} |C| \left(D^2 \Phi\right)\left(\bar{D}^2 \bar{\Phi}\right) + \mathcal{O}\left(\xi^3\right),$$

$$\bar{\Phi} \star \Phi = \bar{\Phi}\Phi + \frac{\xi}{2} C^{\alpha\dot{\alpha}} \left(\bar{D}_{\dot{\alpha}} \bar{\Phi}\right)\left(D_\alpha \Phi\right) - \frac{\xi^2}{16} |C| \left(\bar{D}^2 \bar{\Phi}\right)\left(D^2 \Phi\right) + \mathcal{O}\left(\xi^3\right).$$

- There is an ambiguity in generalising to the NAC superspace the standard WZ kinetic term $\int \mathrm{d}^8 z \Phi \bar{\Phi}$

NAC Wess-Zumino Model

- We choose a *symmetric* ordering:

$$\Phi \star \bar{\Phi} + \bar{\Phi} \star \Phi = 2\Phi\bar{\Phi} - \frac{\xi^2}{8}|C|\left(D^2\Phi\right)\left(\bar{D}^2\bar{\Phi}\right) + \mathcal{O}\left(\xi^3\right),$$

- We end up with

$$S = \int \mathrm{d}^8 z\left[\Phi\left(1 - \xi^2|C|\Box\right)\bar{\Phi} + \mathcal{O}\left(\xi^3\right)\right].$$

- Star product of two chiral and two anti chiral superfields reduces to the standard product, again due to ❊. Therefore, the potentials are not modified by the star product.

$$\int d^6 z\, V\left(\Phi\left(y, \theta\right)\right) + \int d^6 \bar{z}\, V\left(\bar{\Phi}\left(\bar{y}, \bar{\theta}\right)\right)$$

NAC Wess-Zumino Model

$$S = \int d^8z \left[\Phi \left(1 - \xi^2 \, |C| \Box \right) \bar{\Phi} + \mathcal{O}\left(\xi^3 \right) \right].$$

- Now this is interesting: this is *exactly* the supersymmetric Lee-Wick WZ model, as defined for example in *Antoniadis et. al., JHEP 03(2008)045*.
- So, at this approximation in ξ, this model is SUSY invariant. The linear in ξ terms in the $\star$ product are not SUSY invariant, but they cancel in the symmetric prescription we adopted.
- We can check that ξ^3 terms also cancel, while again ξ^4 terms sum up to a SUSY invariant expression. We conjecture the same happens in higher orders, but we did not attempted a general proof.

NAC Wess-Zumino Model

$$S = \int d^8z\left[\Phi\left(1 - \xi^2\,|C|\Box\right)\bar{\Phi} + \mathcal{O}\left(\xi^3\right)\right].$$

$$V_{WZ}(\Phi) = \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3$$

- As for the potential, it is not modified by the $\star$ product. In particular, this avoids a potential ambiguity in defining the interaction terms.
- That means the NAC WZ model turns out to be rather simple, at least at the classical level. Propagators can be easily calculated, being modified by the higher-derivative term, and vertices are as usual.
- Quantum corrections to this kind of models were studied, for example, in *Dias et.al., IJMP A30(2015) 1550113*.

Lee-Wick Theories

- Higher derivative theories have a long history. They usually have improved UV behaviour, but also several technical issues to be dealt with: unitarity is generally lost (ghosts), Wick rotation becomes non trivial due to the appearance of additional poles in the propagator, etc (see f.e. *Antoniadis et al, NPB 767(2007)29*).
- In 1970, Lee and Wick proposed a *finite* version of QED, based on the use of higher derivative operators to tame the UV divergences.
- In the last few years, Lee-Wick type of extensions of the Standard Model have been studied, and even compared with recent LHC results:
  - Grinstein et.al., PRD77(2008)025012
  - Carone et.al., PLB732(2014)122

Conclusions, mainly questions…

- It is difficult to deform SUSY models without spoiling its good properties. It is interesting to find another setting of NAC SUSY that is consistent, and relate to an important class of higher derivative models.
- It would be *more* interesting if this kind of NAC could be somewhat related to String Theory backgrounds. Non associative structures have appeared in such contexts (*Bagger et al, PRD 75(2007)045020*), but we do not know whether it has something to do with our model.
- We have also assumed that the star product does not produce non analytical effects at $\xi = 0$. This might not be true, as we learned in the case of canonical NC.
- It would be interesting to generalize to gauge models, and to study quantum corrections.