Precise knowledge of the Higgs boson mass in the SM and in the MSSM

Pietro Slavich

(LPTHE Paris)

SUSY 2015

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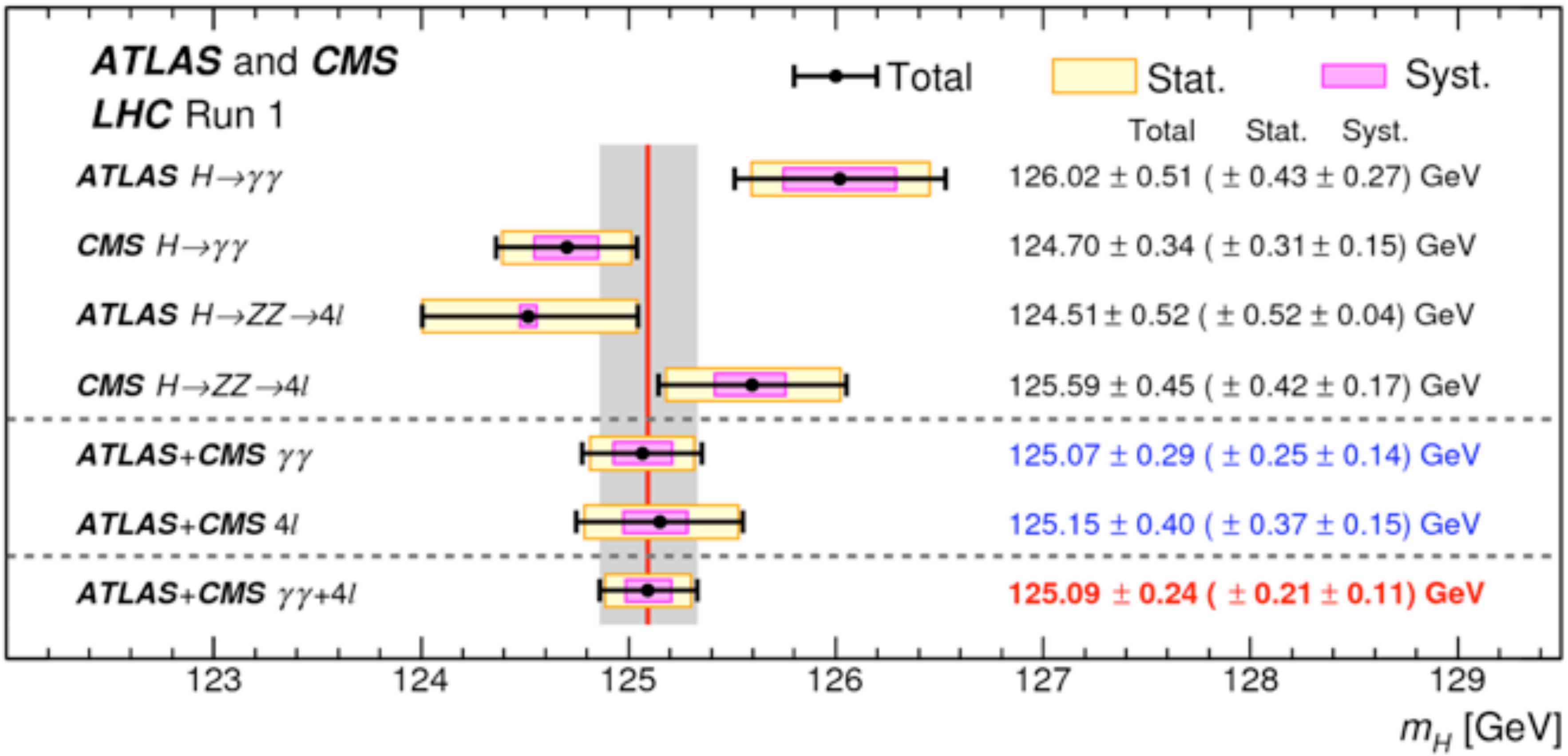
An amazing feat by ATLAS and CMS

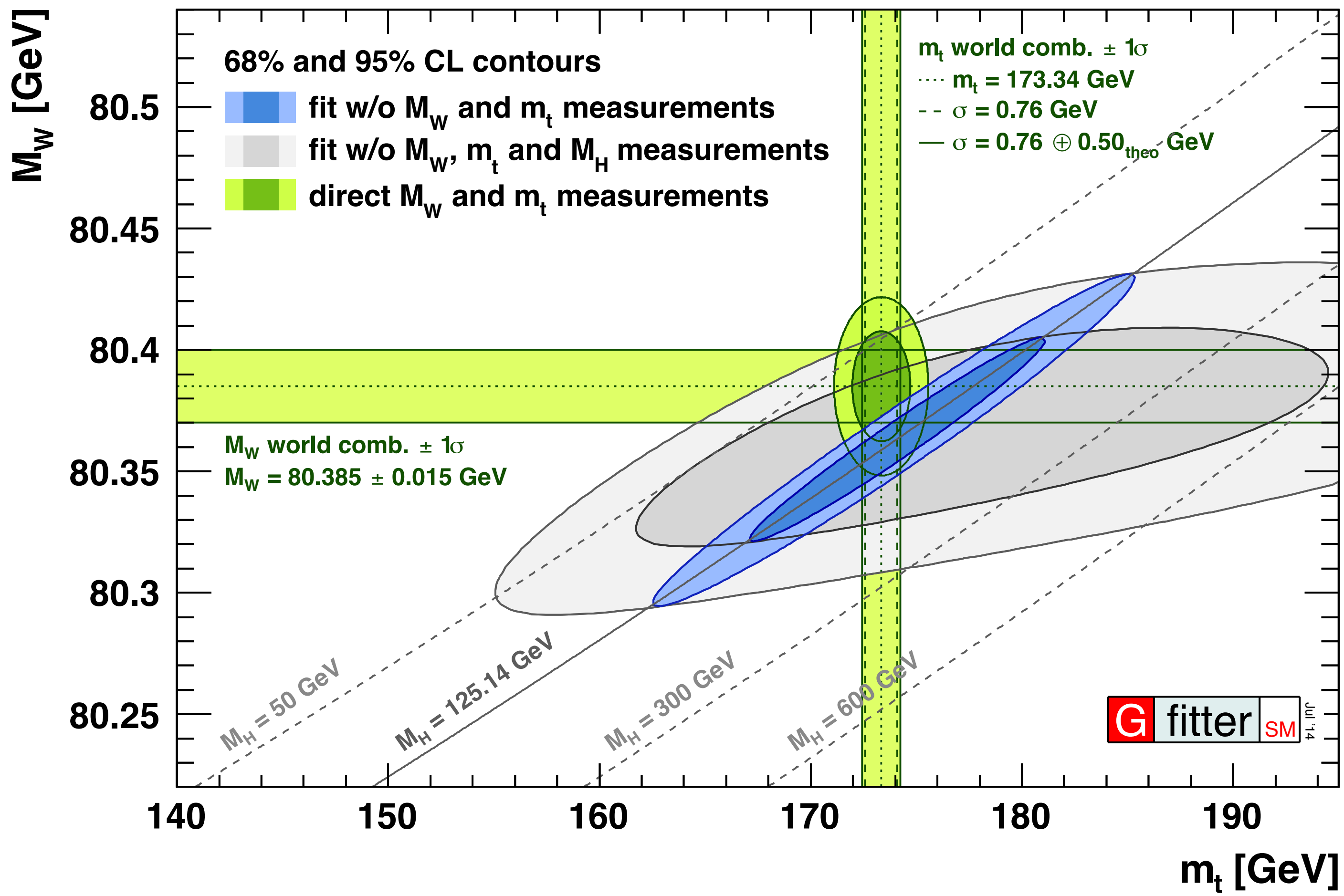


$\Delta M_H / M_H \approx 0.2\%$, still dominated by statistics!

1) *What do we learn from such a precise measurement of the Higgs boson mass?*
2) *How does this amazing feat compare with the accuracy of theory calculations?*

Part 1) The Standard Model Higgs

First message: the SM fit remains in good health



Full 2-loop + dominant 3-loop prediction for M_W, th. uncertainty $\approx$ 4 – 6 MeV

[Summarized in Awramik *et al.*, hep-ph/0311148; dominant 4-loop (< 1 MeV) in Boughezal *et al.*, hep-ph/0606232; 2-loop recently revisited in Degrassi *et al.*, 1411.7040 and Martin, 1503.03782]

Limited sensitivity to the Higgs mass:

$$\frac{\delta M_W^2}{M_W^2} \approx \frac{3\,\alpha}{16\pi\cos 2\theta_W}\left(\frac{M_t^2}{M_Z^2\,\sin^2\theta_W} + \log\frac{M_H^2}{M_W^2} + \ldots\right)$$

Precise determination of the SM Lagrangian parameters

Knowing the Higgs mass, we can finally access all of the fundamental parameters of the SM

$$m^2,\ \lambda,\ g,\ g',\ g_s,\ y_f$$

The parameters are computed in the $\overline{\text{MS}}$ scheme at some low reference scale, e.g., $Q = M_t$.
They can then be evolved to a higher scale, to be matched with those of a BSM Lagrangian or to study the stability of the SM scalar potential at large values of the Higgs field

A few details:

- Minimizing the scalar potential we can trade m^2 for the (gauge-dependent) vev v
- Yukawa couplings other than y_t have tiny impact on the results. Their effect may well be included at the lowest order only (and will be neglected in the rest of this talk)
- Need six "physical" inputs (different options): $\alpha\,,\ \alpha_s\,,\ G_\mu,\ M_H,\ M_t,\ M_Z,\ M_W$

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- Need six “physical” inputs (different options): $\alpha\,,\; \underbrace{\alpha_s\,,\; G_\mu,\; M_H,\; M_t,\; M_Z,\; M_W}_{\text{Vacuum stability analyses}}$

- Extract $\overline{\text{MS}}$ parameters at $Q = M_t$ from physical inputs, including loop corrections

[Recent state-of-the-art analyses: Buttazzo *et al.*, 1307.3536v4; Kniehl *et al.*, 1503.02138]

$$p_i \;\equiv\; (\alpha_s\,,\; G_\mu,\; M_H,\; M_t,\; M_Z,\; M_W)$$

$$m^2(Q) \;=\; -\,\frac{M_H^2}{2} \;+\; \Delta_m(p_i,Q)$$

or

$$v(Q,\xi) = \frac{1}{(\sqrt{2}\,G_\mu)^{1/2}} \;+\; \Delta_v(p_i,Q,\xi)$$

$$\lambda(Q) \;=\; \frac{G_\mu}{\sqrt{2}}\,M_H^2 \;+\; \Delta_\lambda(p_i,Q)$$

$$g(Q) \;=\; 2(\sqrt{2}\,G_\mu)^{1/2}\,M_W \;+\; \Delta_g(p_i,Q)$$

$$g'(Q) \;=\; 2(\sqrt{2}\,G_\mu)^{1/2}\,\sqrt{M_Z^2 - M_W^2} \;+\; \Delta_{g'}(p_i,Q)$$

$$y_t(Q) \;=\; 2\left(\frac{G_\mu}{\sqrt{2}}\,M_t^2\right)^{1/2} \;+\; \Delta_{y_t}(p_i,Q)$$

$$g_s(Q) \;=\; \alpha_s(M_Z)^{\overline{\rm MS},\,5f} \;+\; \Delta_{g_s}(p_i,Q)$$

- The results can be given as interpolating formulae. E.g., this is from Buttazzo *et al.*

$$\lambda(M_t) \;=\; 0.12604 \;+\; 0.00206\left(\frac{M_H}{\rm GeV} - 125.15\right) \;-\; 0.00004\left(\frac{M_t}{\rm GeV} - 173.34\right) \;\pm\; 0.00030_{\rm th}$$

- The parameters are then evolved to higher scales with the full 3-loop RGE of the SM

[Mihaila *et al.*, 1201.5868 & 1208.3357; Chetyrkin+Zoller, 1205.2892 & 1303.2890; Bednyakov *et al.*, 1212.6829 & 1303.4364]

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full 2-loop

$$y_t(Q) \;=\; 2\left(\frac{G_\mu}{\sqrt{2}}\,M_t^2\right)^{1/2} \;+\; \Delta_{y_t}(p_i, Q)$$

full 2-loop + 3-loop QCD

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3-loop matching $2_{\text{EW}}/4_{\text{QCD}}$-loop running

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(0.2%)

scale dependence of unknown 3-loop

- The parameters are then evolved to higher scales with the full 3-loop RGE of the SM

[Mihaila *et al.*, 1201.5868 & 1208.3357; Chetyrkin+Zoller, 1205.2892 & 1303.2890; Bednyakov *et al.*, 1212.6829 & 1303.4364]

The SM Higgs mass beyond 2-loop

[Martin, 1310.7553 & 1508.00912; Martin+Robertson, 1407.4336]

A different approach: Take the $\overline{\text{MS}}$ values of $\lambda,\ g,\ g',\ g_s,\ y_t$ and $v^{\,\xi=0}$ directly as inputs at the scale Q (they can be separately computed from physical quantities)

The public code SMH by Martin and Robertson computes:

$$M_H \;=\; F(\lambda,\ v,\ g,\ g',\ g_s,\ y_t,\ Q)$$

or

$$\lambda(Q) \;=\; F(M_H,\ v,\ g,\ g',\ g_s,\ y_t,\ Q)$$

Full 2-loop computation of the Higgs mass:

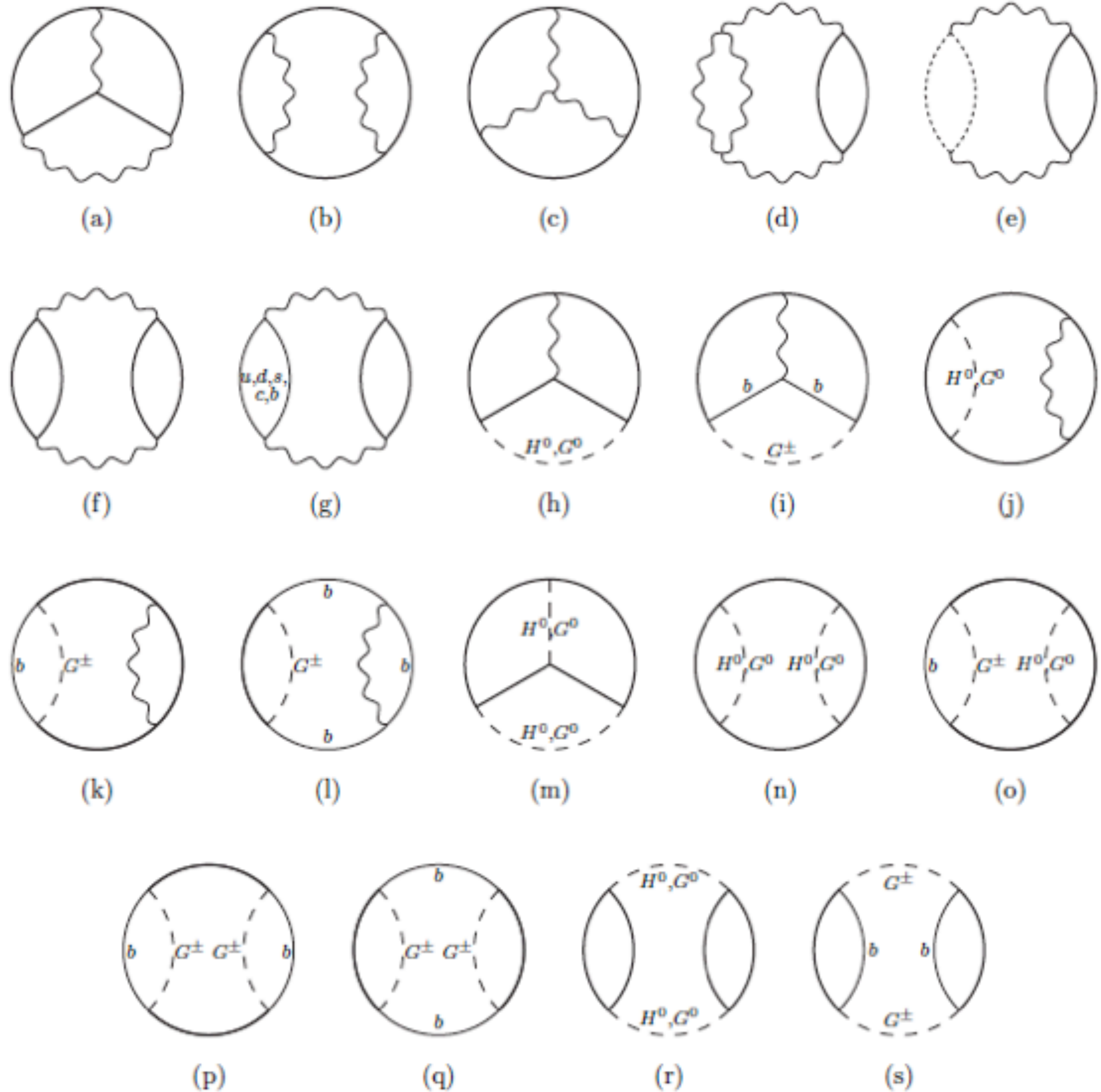
$$M_H^2 - i\,\Gamma_H\,M_H \;\equiv\; s_{\rm pole} \;=\; m_B^2 + 3\,\lambda_B\,v_B^2 + \frac{1}{16\pi^2}\Pi^{(1)}(s_{\rm pole}) + \frac{1}{(16\pi^2)^2}\Pi^{(2)}(s_{\rm pole})$$

$$=\; 2\,\lambda\,v^2 + \frac{1}{16\pi^2}\Delta^{(1)}_{M_H^2} + \frac{1}{(16\pi^2)^2}\left[\Delta^{(2),{\rm QCD}}_{M_H^2} + \Delta^{(2),{\rm non-QCD}}_{M_H^2}\right]$$

Inclusion of leading 3-loop effects of order $g_s^4\,y_t^2 m_t^2,\ g_s^2\,y_t^4 m_t^2,\ y_t^6 m_t^2$ in the effective-potential approximation:

$$\Delta M_H^2 \;=\; \left[\frac{\partial^2}{\partial v^2} - \frac{1}{v}\,\frac{\partial}{\partial v}\right]\,\Delta V_{\rm eff}^{3\ell}$$

Martin, 1310.7553 [see also 1508.00912 for 4-loop $O(g_s^6)$ terms]



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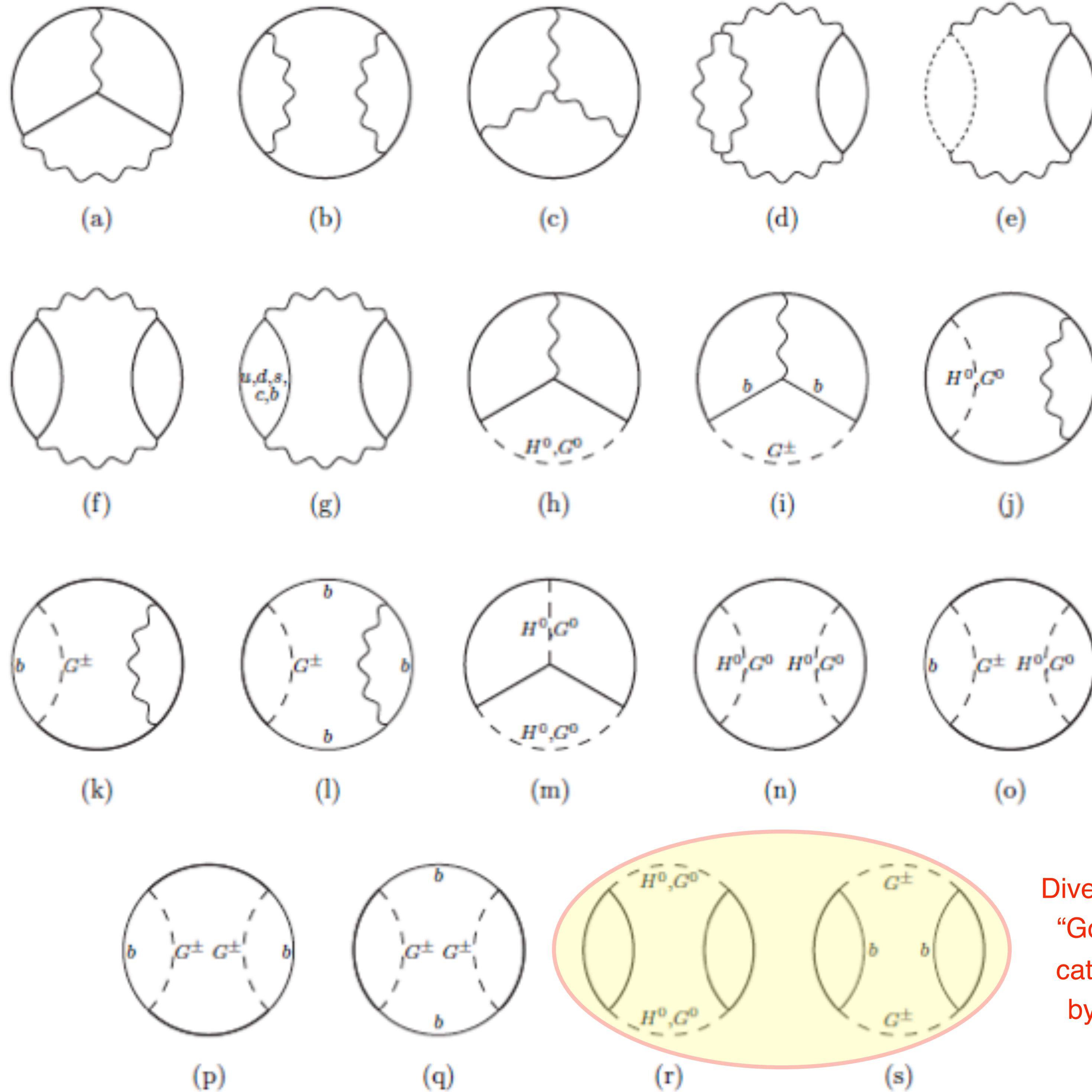


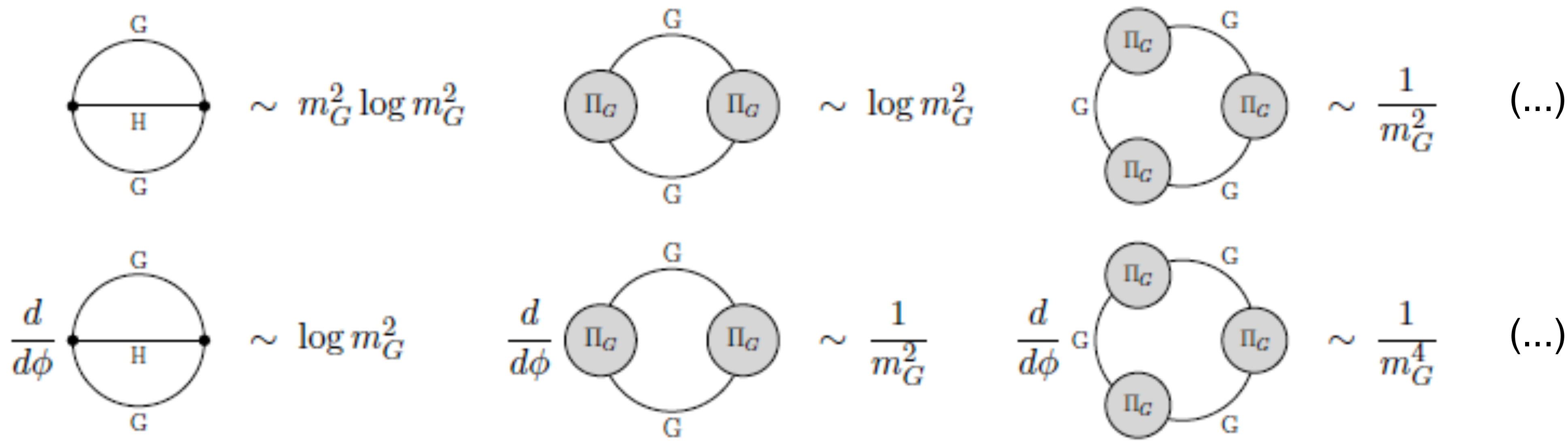
Divergent for $m_G=0$!
"Goldstone boson catastrophe" fixed by resummation

Fixing the Goldstone boson catastrophe

[Martin 1406.2355; Elias-Miró *et al.*, 1407.4336; see also Pilaftsis+Teresi, 1502.07986]

"Goldstone-boson ring" diagrams make $V_{\rm eff}$ and its derivative IR divergent for $m_G^2 \to 0$.
The troubles start at 3-loop order for the potential and 2-loop order for the derivative:

Borrowed from 1502.07986



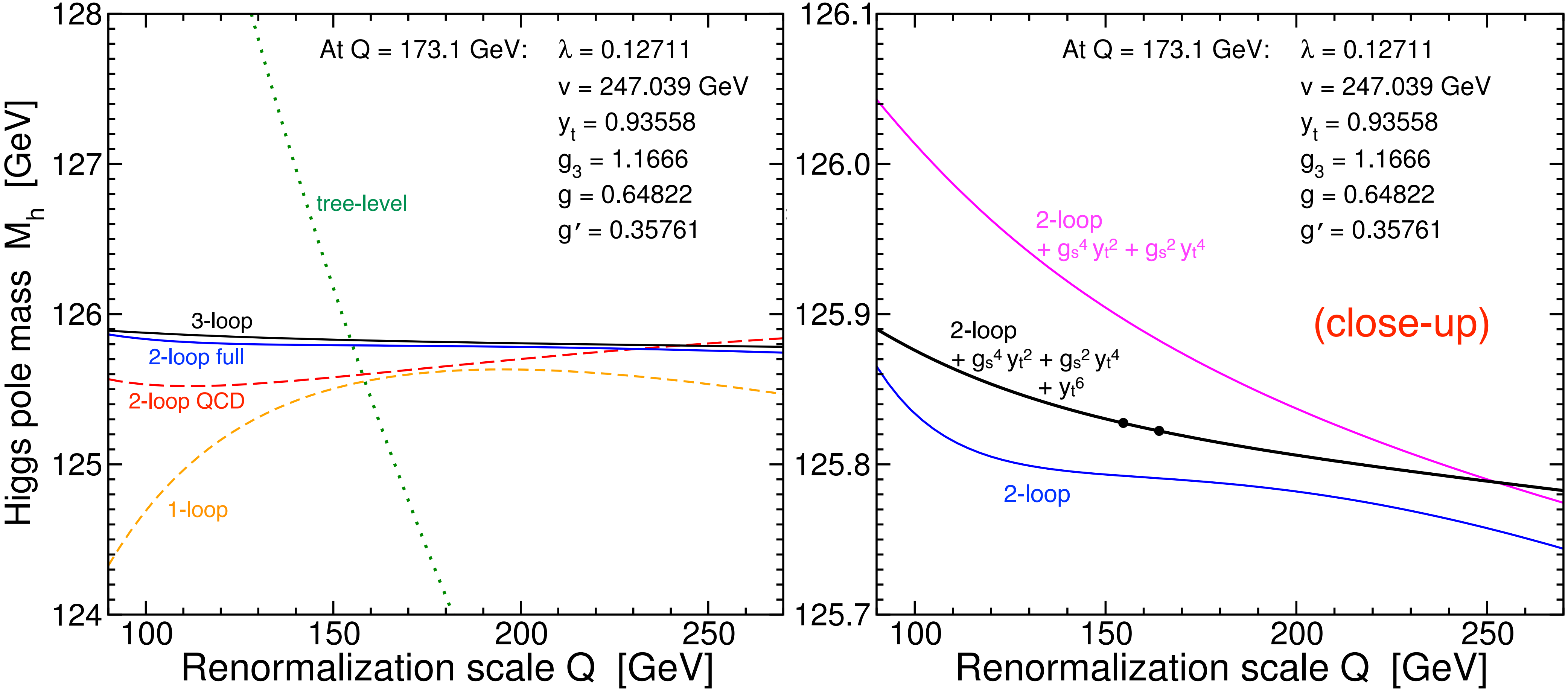
The divergent terms can all be absorbed in the 1-loop Goldstone contribution to the potential:

$$V^{1\ell}_{{\rm eff},G} = \frac{3\,m_G^4}{64\,\pi^2}\left(\log\frac{m_G^2}{Q^2}-\frac{3}{2}\right) \longrightarrow \frac{3\,(m_G^2+\Pi_g)^2}{64\,\pi^2}\left(\log\frac{m_G^2+\Pi_g}{Q^2}-\frac{3}{2}\right) - \left(\text{double-counted diagrams}\right)$$

Where $\Pi_g \equiv \Pi_G(0) - \left(\text{Goldstone-only contributions}\right)$ is the contribution from massive particles to the zero-momentum self-energy of the Goldstone

This "resummed" potential is IR-safe for both $m_G^2 \to 0$ and $m_G^2 + \Pi_g \to 0$

Full 2-loop + dominant 3-loop Higgs-mass calculation: Martin+Robertson, 1407.4336



Theory error estimated from scale variation: ≈100 MeV (0.1%)

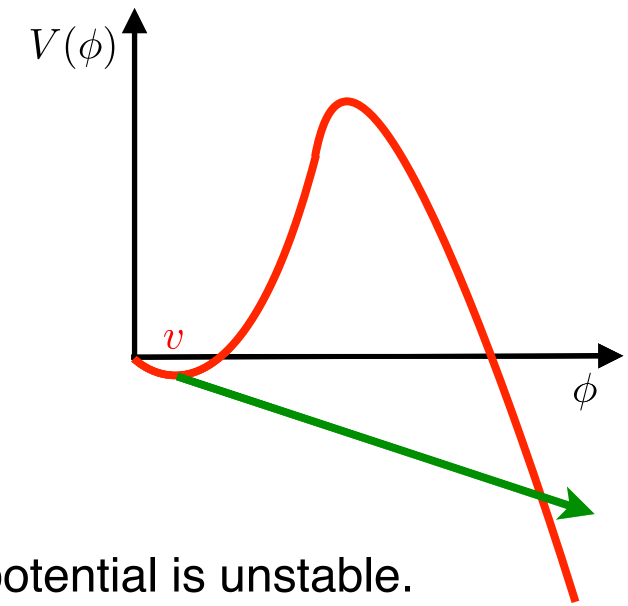
similar accuracy when extracting λ from M_H

Stability of the electroweak vacuum

The "fate of the SM" is determined by the values of $\lambda, \; y_t$ *etc. (RG-evolved to the large scale)*

Quantum corrections to the Higgs potential may induce a deeper vev at large values of the field:

$$V_{\rm eff}(\phi) \;=\; m^2(Q)\,|\phi|^2 \,+\, \lambda(Q)\,|\phi|^4 \,+\, \Delta V^{\rm loop}$$



At large ϕ, the potential is dominated by the quartic term:

$$V_{\rm eff}(\phi \gg v) \;\approx\; \lambda_{\rm eff}(Q \approx \phi)\,|\phi|^4$$

($\lambda_{\rm eff}$ includes corrections from ΔV)

If the quartic coupling turns negative at some large scale, the potential is unstable. This is still OK if the lifetime T_{EW} of the EW vacuum is larger than T_U *(metastability)*

NOTE: the value of ϕ at which the instability occurs is gauge-dependent (as are $V_{\rm eff}$ and v)

Nielsen identity (1975)

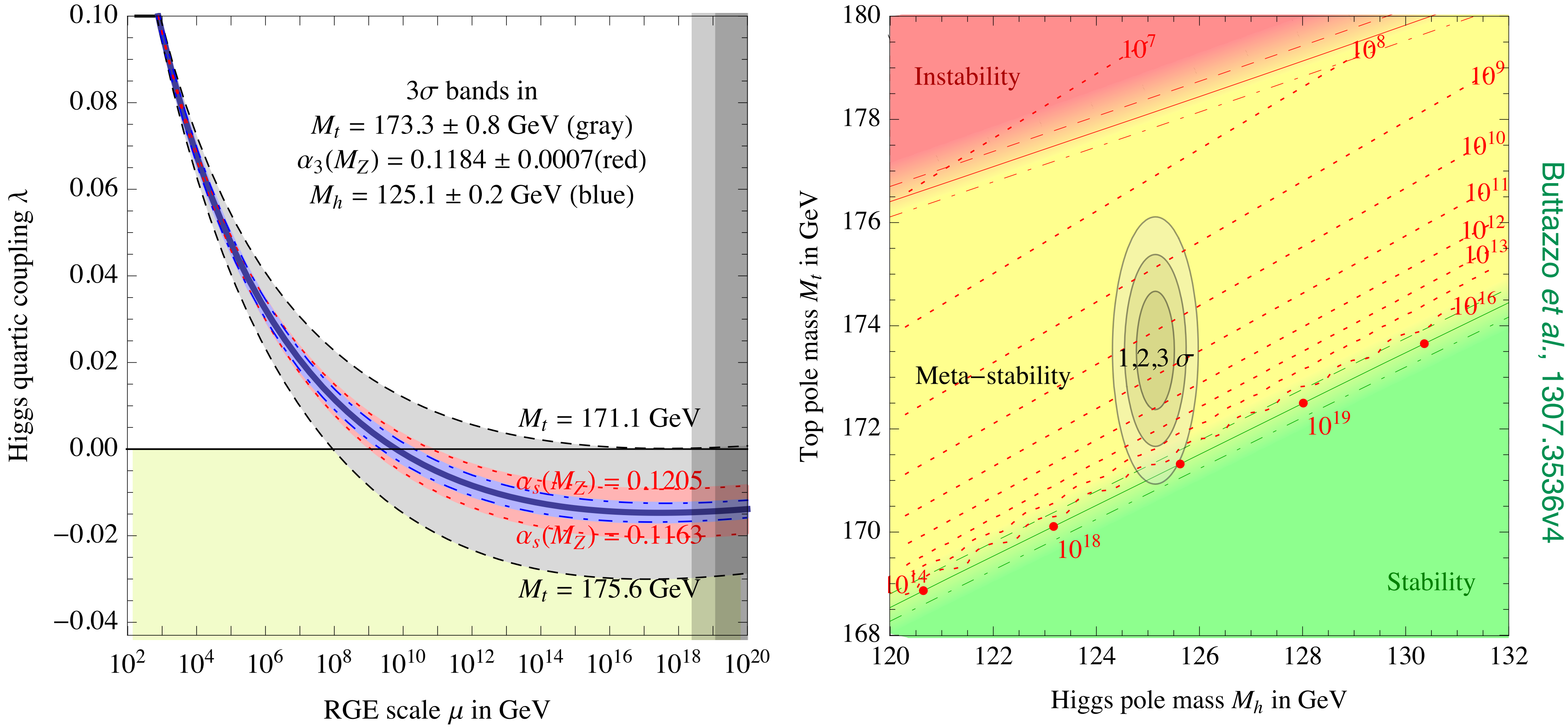
$$\left[\xi\,\frac{\partial}{\partial\xi} \,+\, C(\phi,\xi)\,\frac{\partial}{\partial\phi}\right] V_{\rm eff}(\phi,\xi) \;=\; 0$$

The values of $V_{\rm eff}$ at its extrema and the tunnelling rate between different minima are gauge invariant *[this holds only at all orders in perturbation theory]*

Various strategies for a consistent definition of the instability scale were discussed recently

[Di Luzio+Mihaila, 1404.7450; Nielsen, 1406.0788; Andreassen *et al.*, 1408.0292; Espinosa *et al.*, 1505.04825; Bednyakov *et al.*, 1507.08833]

$M_H \approx 125$ GeV is close to the boundary between the stability and metastability regions



Stability condition:

$M_H > (129.6 \pm 1.5)$ GeV for $M_t^{pole} = (173.34 \pm 0.76_{exp} \pm 0.3_{th})$ GeV

$M_t^{pole} < (171.53 \pm 0.42)$ GeV for $M_H = (125.15 \pm 0.24)$ GeV

NOTE: questions on the identification of M_t^{pole} with the " M_t^{MC} " provided by the experiments

Alternative determinations from $\sigma_{t\bar{t}}$, e.g. $M_t^{pole} = (171.2 \pm 2.4)$ GeV [Alekhin *et al.*, 1310.3059]

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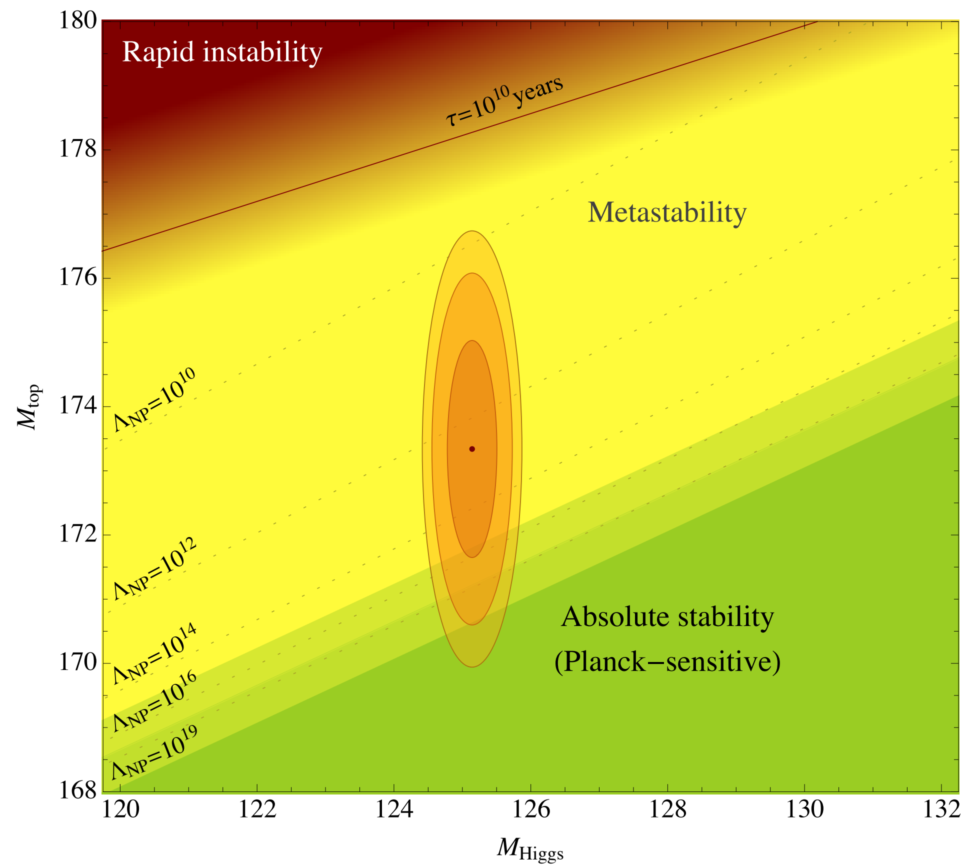
[Rearranging the Goldstone contribution to V_{eff}, to cure the gauge dependence of the stability bound order-by-order]



Andreassen *et al.*, 1408.0292

Stability condition:

$M_H > (129.4 \pm 1.5)$ GeV for $M_t^{\text{pole}} = (173.34 \pm 0.76_{\text{exp}} \pm 0.3_{\text{th}})$ GeV

$M_t^{\text{pole}} < (171.22 \pm 0.42)$ GeV for $M_H = (125.15 \pm 0.24)$ GeV

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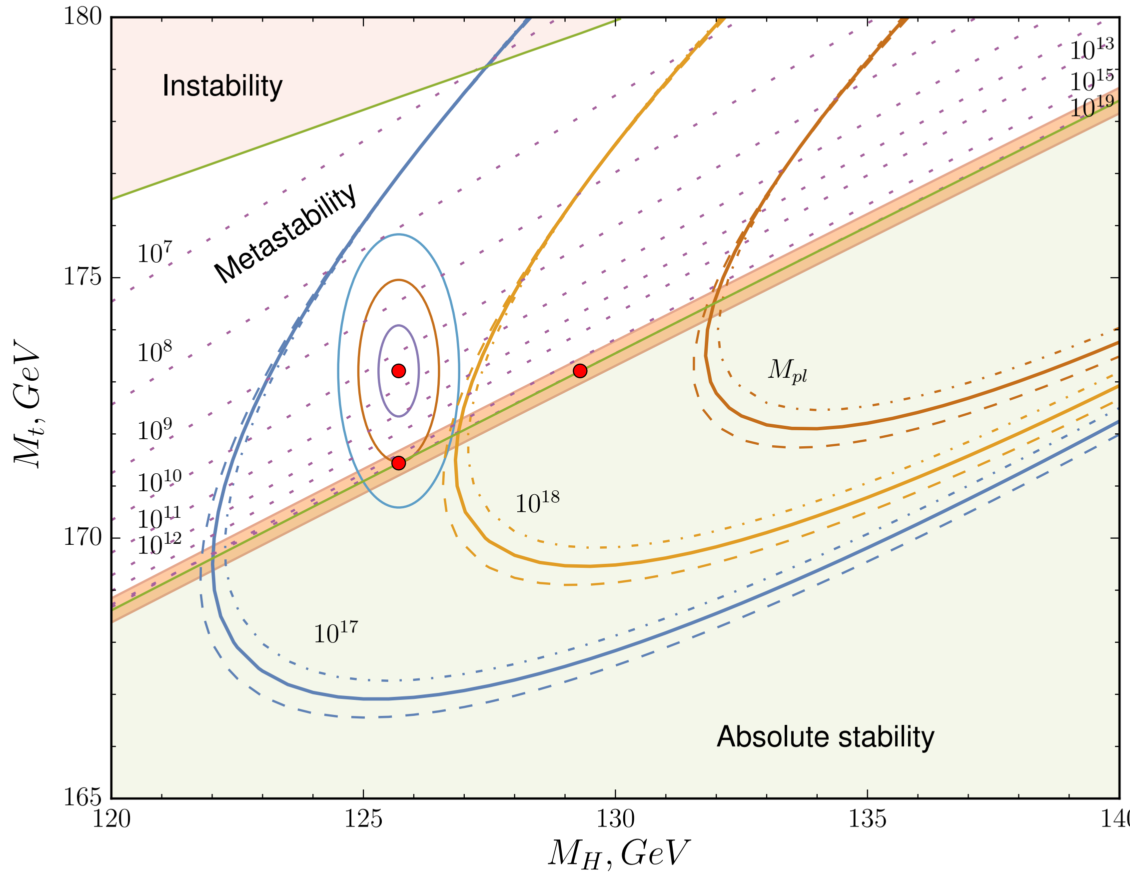
$M_H \approx 125$ GeV is close to the boundary between the stability and metastability regions

[Alternative determination of the SM parameters at the low scale]



Bednyakov *et al.*, 1507.08833

Stability condition: $M_t^{pole} < (171.54^{+0.4}_{-0.5})$ GeV for $M_H = (125.7 \pm 0.4)$ GeV

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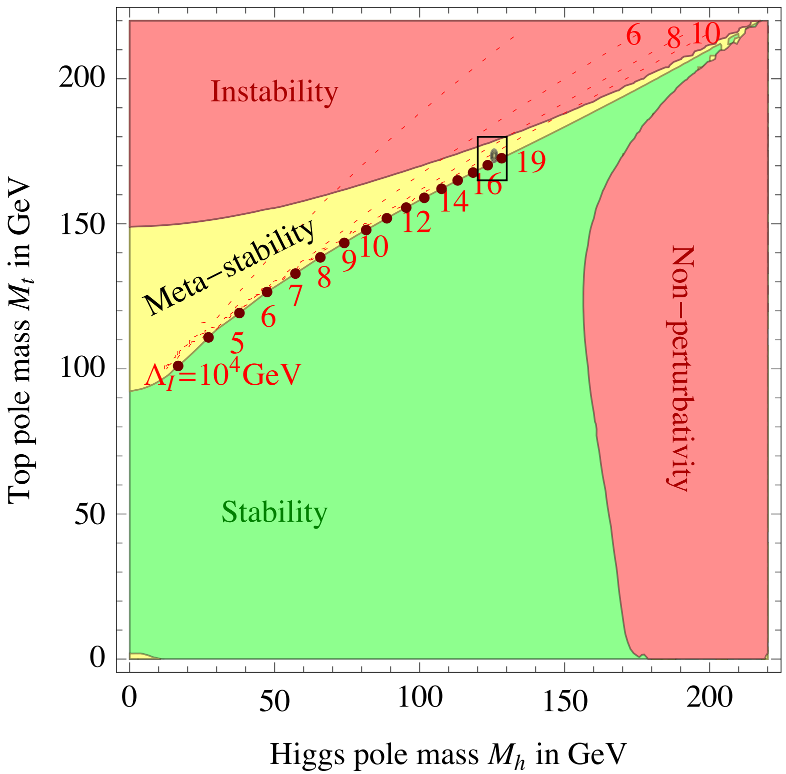
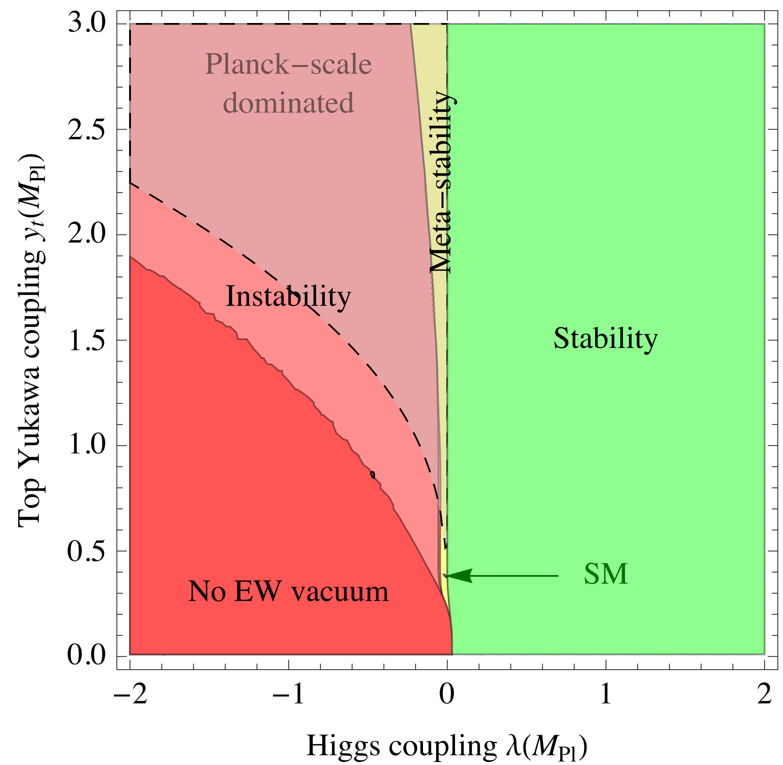
Nature appears to have made rather special choices for the SM parameters





Buttazzo *et al.*, 1307.3536

- Speculation #1: *metastability as a critical phenomenon* [Buttazzo *et al.*, 1307.3536]
  [the multiverse pushes for small λ, but the universes where it is too small don't survive]

- Speculation #2: *metastability required by quantum gravity* [Espinosa *et al.*, 1505.04825]
  [QG cannot be consistently defined in a (dS) vacuum with positive cosmological constant; the decay to another (AdS) vacuum with negative cosmological constant offers a way out]

All these "conclusions" can be altered if we introduce any New Physics below the Planck scale

[see also Branchina+ Messina, 1307.5193 and 1507.08812]

Part 2) The MSSM Higgs(es)

The Higgs sector of the MSSM

- Two Higgs doublets H_1 and H_2 to give mass to up- and down-type fermions; five physical states: two scalars (h, H) a pseudoscalar (A) two charged ($H^\pm$)

- The Higgs quartic couplings are not free parameters as in the SM, they are fixed in terms of the EW gauge couplings g and g'. This induces a tree-level bound on the mass of the lightest scalar: $M_h < M_Z\,|\cos 2\beta|$ (where $\tan\beta = v_2/v_1$)

- In the decoupling limit $M_A >> M_Z$, the lightest scalar has SM-like couplings to fermions and gauge bosons, and saturates the mass bound, $M_h \approx M_Z\,|\cos 2\beta|$, while the other Higgses form a heavy and mass-degenerate “exotic” multiplet.

- For lower M_A both scalars share the role of the SM Higgs, and the mass of the lightest scalar is pushed down by mixing effects.

- Radiative corrections can raise the MSSM prediction for the lightest scalar mass and allow for $M_h \approx 125$ GeV.

Radiative corrections to the light-Higgs mass in the MSSM

The dominant one-loop corrections to the Higgs masses are due to the particles with the strongest couplings to the Higgs bosons: the top (and bottom) quarks and squarks

$$(\Delta M_h^2)^{1-\mathrm{loop}} \simeq \frac{3\,M_t^4}{2\,\pi^2\,v^2}\left(\ln\frac{M_S^2}{M_t^2}+\frac{X_t^2}{M_S^2}-\frac{X_t^4}{12\,M_S^4}\right)-\frac{y_b^4\,\mu^4\,\tan^4\beta\,v^2}{32\,\pi^2\,M_S^4}$$

(decoupling limit, M_S = average stop mass, $X_t = A_t - \mu \cot\beta$ = L-R stop mixing)

- "Maximal-mixing" scenarios ($X_t \approx \sqrt{6}\, M_S$) can work with stops around the TeV (but only if $\tan\beta$ and M_A are large enough that $M_h \approx M_Z$ at tree level)
- Small-mixing ($X_t \ll M_S$) or small $\tan\beta$ (or M_A) require multi-TeV stop masses

A quarter-century of calculations gave us full 1-loop, almost-full 2-loop and partial 3-loop results

Bagger Borowka Brignole Carena Casas Chankowski Dabelstein Dedes Degrassi DiVita Ellis Espinosa Haber Hahn Harlander Heinemeyer Heinrich Hempfling Hoang Hollik Kant Martin Matchev Mihaila Navarro Okada Pierce Pokorski Quiros Ridolfi Riotto Rosiek Rzehak Slavich Steinhauser Wagner Weiglein Yamaguchi Yanagida Zhang Zwirner

1991 – 2015

[Apologies if I forgot anybody – and this is just for radiative corrections in the CP-conserving MSSM !!!]

How well can we predict M_h in the MSSM with TeV-scale SUSY?

Simplified benchmark point: $\tan\beta = 20$, all SUSY masses = 1 TeV, X_t varied to maximize M_h

Public code	M_h [GeV]
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SuSpect 2.43	125.8
SoftSUSY 3.6.2	124.3
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All of these codes include full 1-loop + dominant (strong+Yukawa) 2-loop corrections to M_h

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Same $\overline{DR}$ calculation of the Higgs mass, differences in determination of top Yukawa (SPheno, SuSpect, SoftSUSY, NMSSMTools)

OS calculation of Higgs mass (FeynHiggs)

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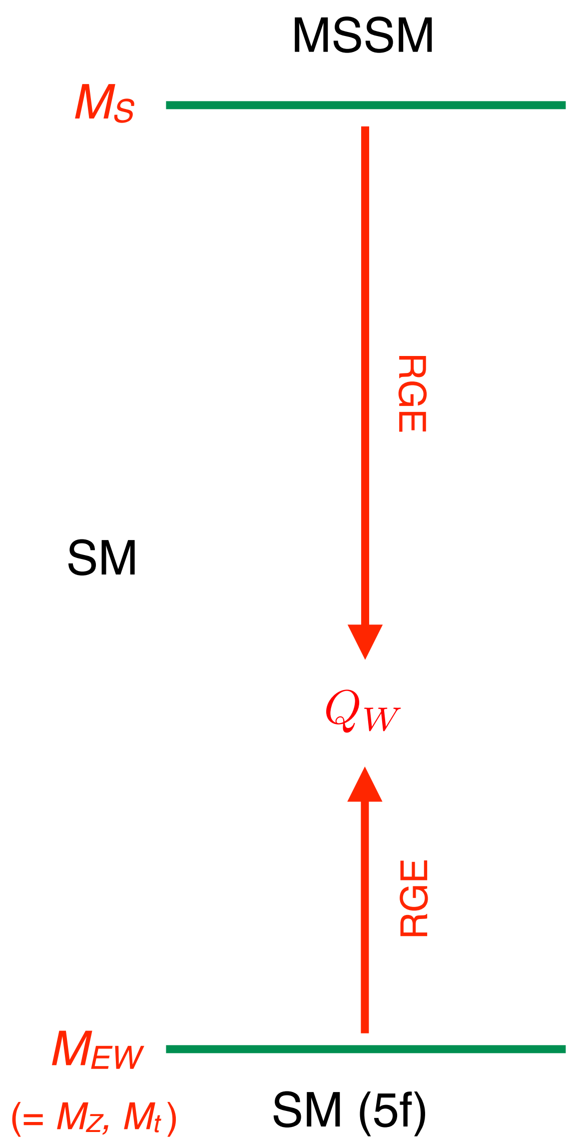
All of these codes include full 1-loop + dominant (strong+Yukawa) 2-loop corrections to M_h

With great corrections comes great uncertainty!

For multi-TeV SUSY masses, *log(M_S/M_EW)* terms must be resummed in an EFT approach



$$\lambda(M_S) = \frac{1}{8}\left(g^2 + g'^2\right)\cos^2 2\beta \; + \; \Delta\lambda$$

$$M_H^2 = \frac{\sqrt{2}\,\lambda(Q_W)}{G_F}\,(1+\delta_H)$$

$$g,\; g',\; g_s,\; y_t \;=\; (\ldots)$$

Resums large logarithms but neglects effects of $\mathcal{O}(v^2/M_S^2)$

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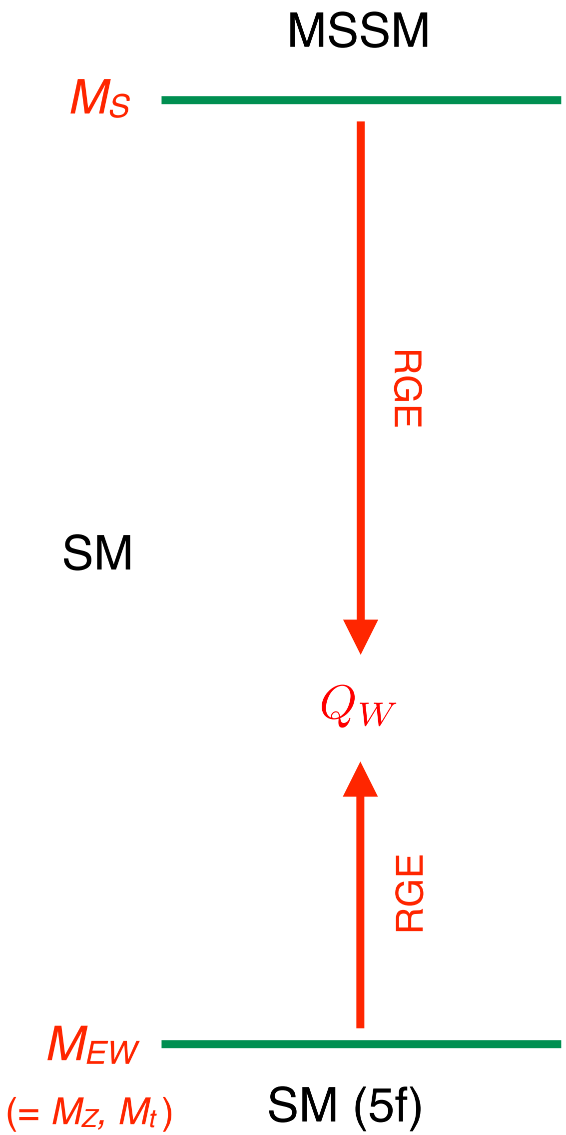

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(full 1-loop + dominant 2-loop SUSY threshold corrections)

(3-loop RGE for the SM)

$$M_H^2 = \frac{\sqrt{2}\,\lambda(Q_W)}{G_F}\,(1+\delta_H)$$

(2-loop "diagrammatic" calculations in the SM)

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For multi-TeV SUSY masses, *log(M_S/M_{EW})* terms must be resummed in an EFT approach

MSSM

M_S

$$\lambda(M_S) = \frac{1}{8}\left(g^2 + g'^{\,2}\right)\cos^2 2\beta \;+\; \Delta\lambda$$

$$\left[\; \tilde{g}_u,\ \tilde{g}_d,\ \tilde{g}'_u,\ \tilde{g}'_d \;=\; (\ldots) \;\right]$$

(full 1-loop + dominant 2-loop SUSY threshold corrections)

RGE

SM

(2-loop RGE for Split SUSY)

$$\left[+ \begin{array}{c} \tilde{b},\ \tilde{w},\ \tilde{g}, \\ \tilde{h}_1,\ \tilde{h}_2 \end{array}\right]$$

Q_W

$$M_H^2 = \frac{\sqrt{2}\,\lambda(Q_W)}{G_F}\,(1 + \delta_H)$$

RGE

(2-loop “diagrammatic” calculations in the SM + leading -ino contributions)

M_{EW}

(= M_Z, M_t)

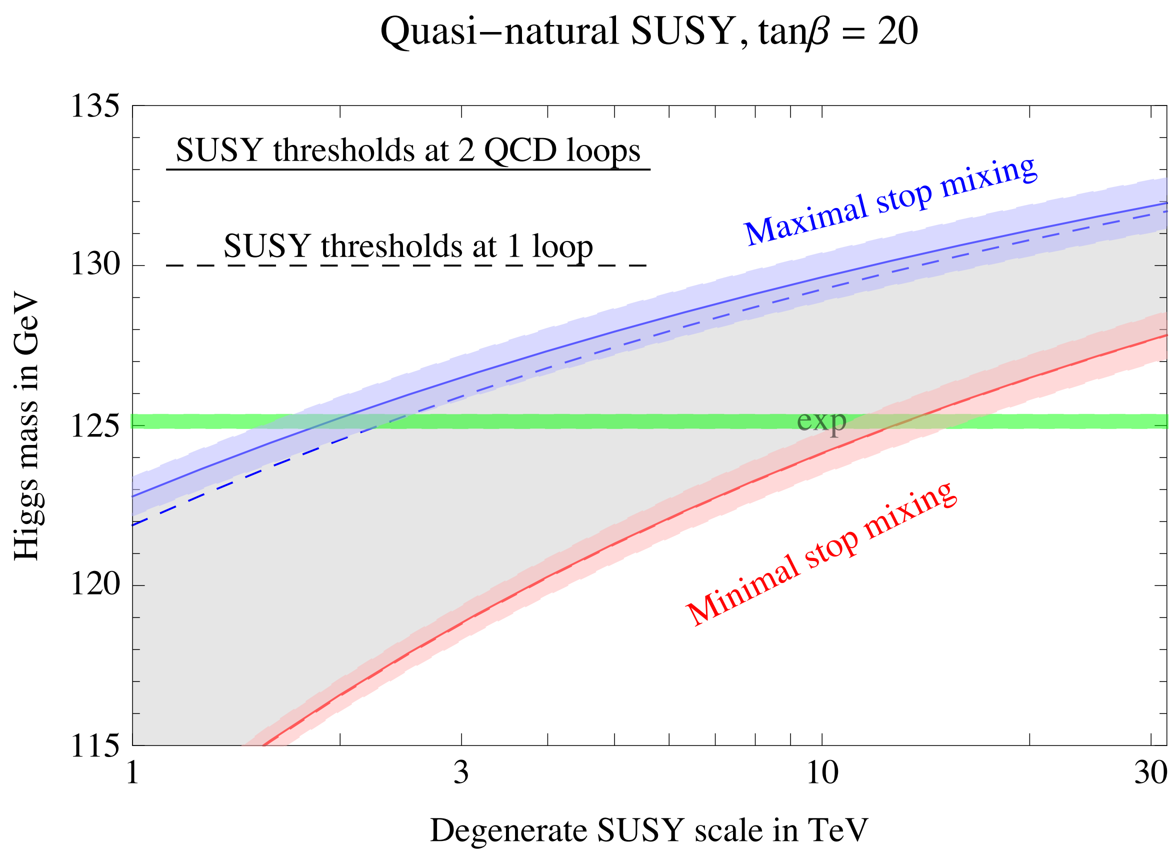
SM (5f)

$$g,\ g',\ g_s,\ y_t \;=\; (\ldots)$$

Resums large logarithms but neglects effects of $\mathcal{O}(v^2/M_S^2)$

Recent incarnations of the decades-old EFT approach:

Hahn et al. (FeynHiggs), 1312.4937; Draper et al., 1312.5743;
Bagnaschi et al., 1407.4081; PardoVega+Villadoro (SusyHD) 1504.05200



Bagnaschi *et al.*, 1407.4081

Simple test-point:

M_S = 10 TeV,

X_t = 0, $\tan\beta$ = 20

Draper *et al* : M_h = 123.2 GeV

Bagnaschi *et al* : M_h = 123.6 GeV

SusyHD: M_h = 123.6 GeV

FeynHiggs: M_h = 126.5 GeV

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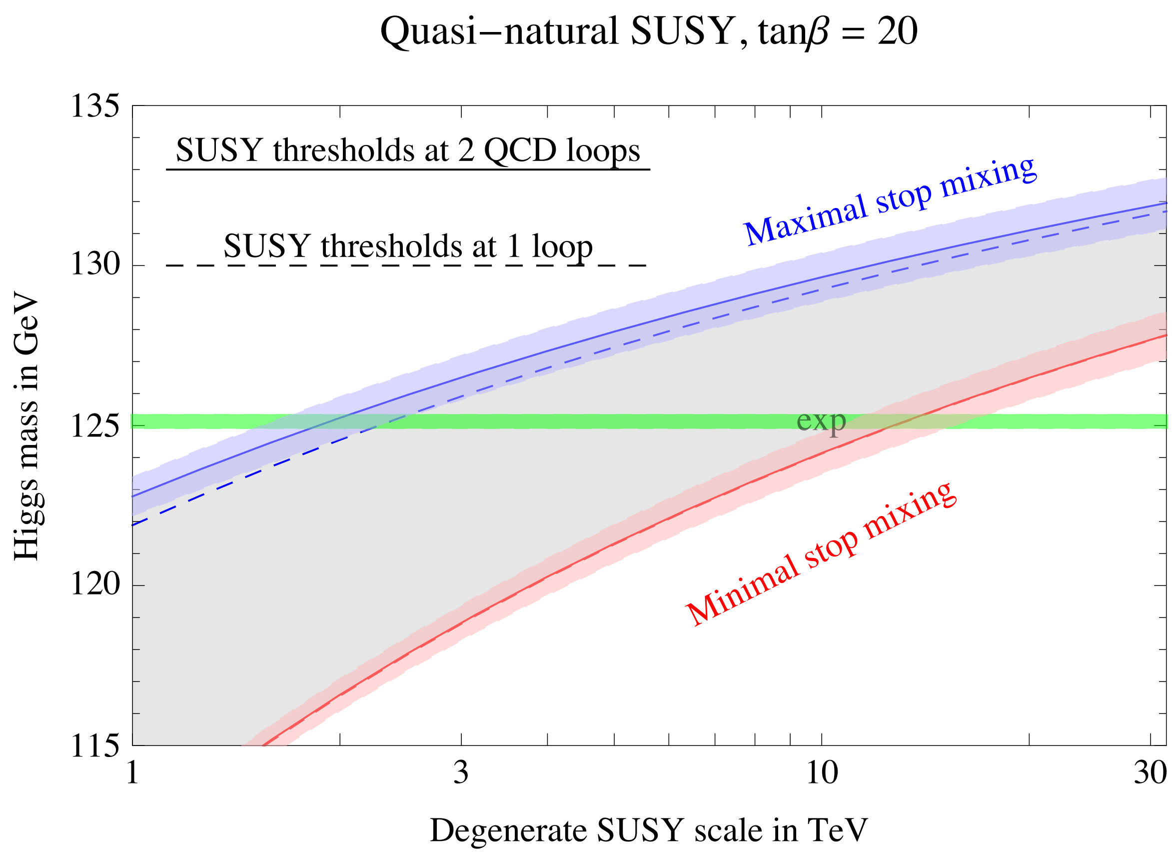

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SusyHD: $M_h = 123.6$ GeV

Pure EFT calculations; theoretical uncertainty estimated as < 1 GeV *(in this point!!!)*

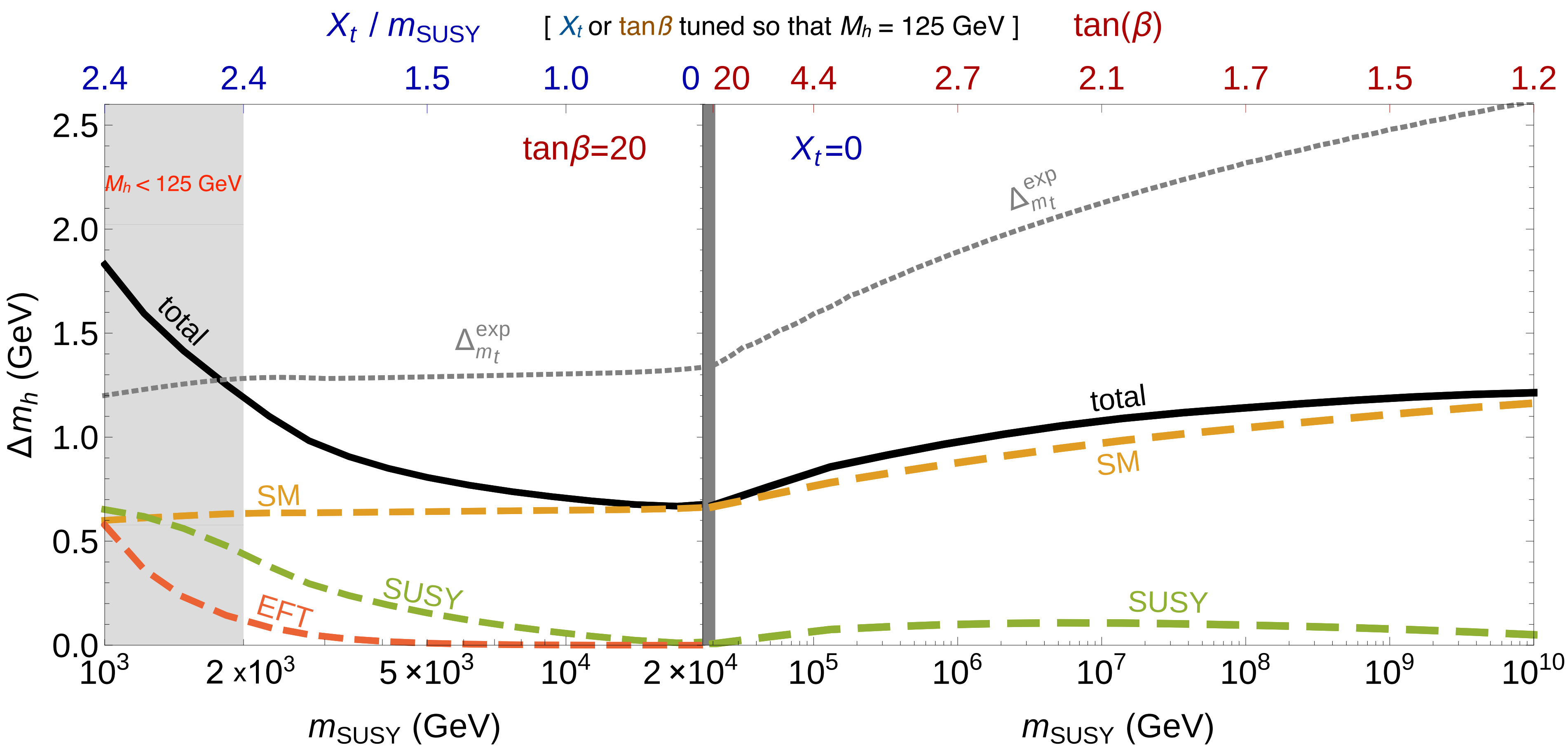
FeynHiggs: $M_h = 126.5$ GeV

"hybrid" calculation: 2-loop diagrammatic + partial resummation

Again, part of the discrepancy is related to the determination of y_t

Uncertainties of the EFT calculation

[PardoVega+Villadoro (SusyHD) 1504.05200]

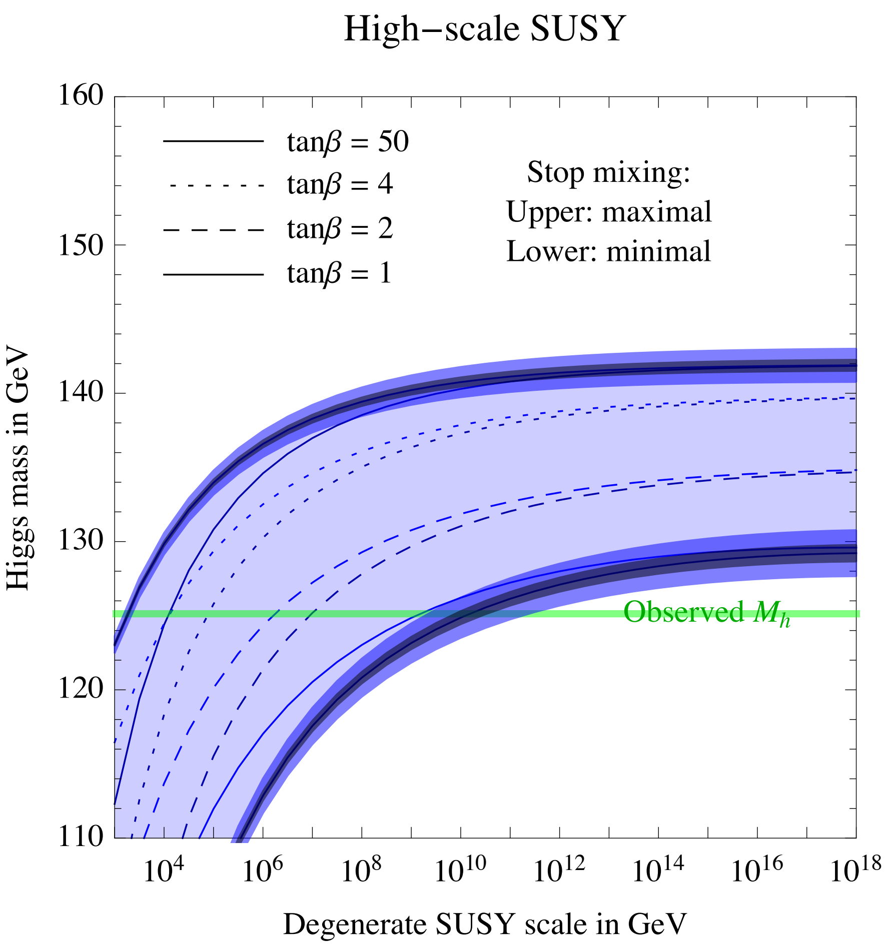


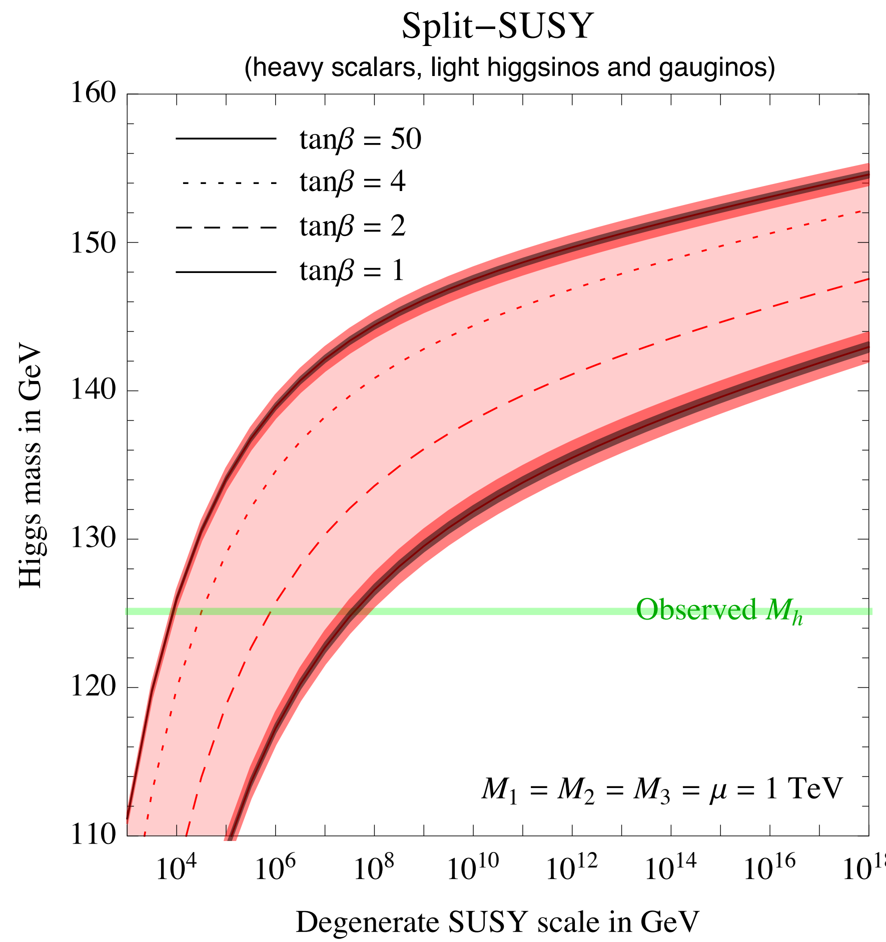
SM uncertainty: from the SM calculation (mostly from 3-loop QCD effects in y_t)

SUSY uncertainty: estimated varying the SUSY matching scale by a factor 1/2 or 2

EFT uncertainty: estimated replacing $\Delta\lambda \rightarrow \Delta\lambda\left(1+v^2/M_S^2\right)$ *(optimistic?)*

Pushing un-naturalness: High-scale SUSY and Split SUSY





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- The prediction depends on the high-scale parameter *tanβ* (and X_t in HSS)
- The observed M_h determines an upper bound on the SUSY-breaking scale

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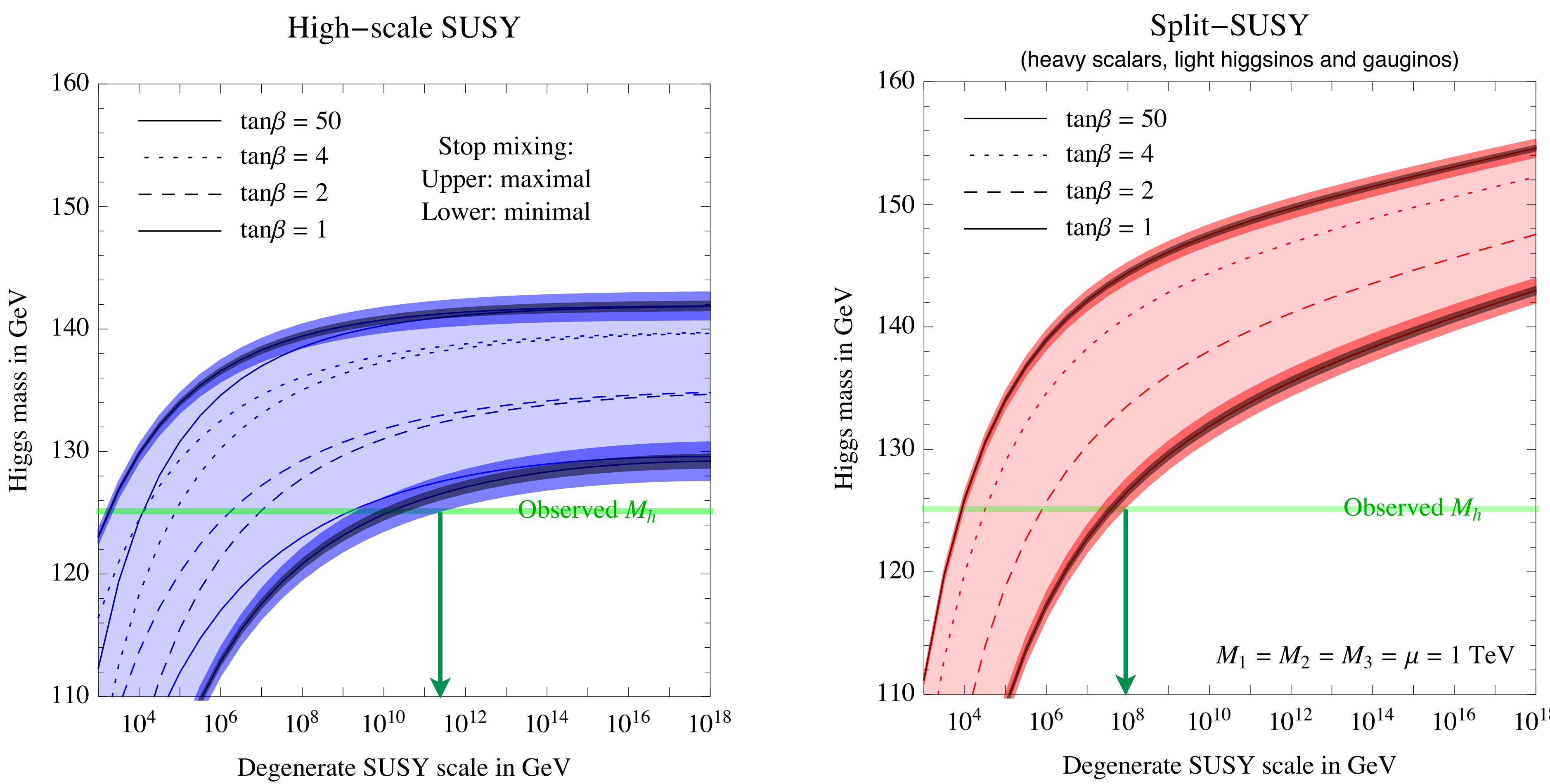

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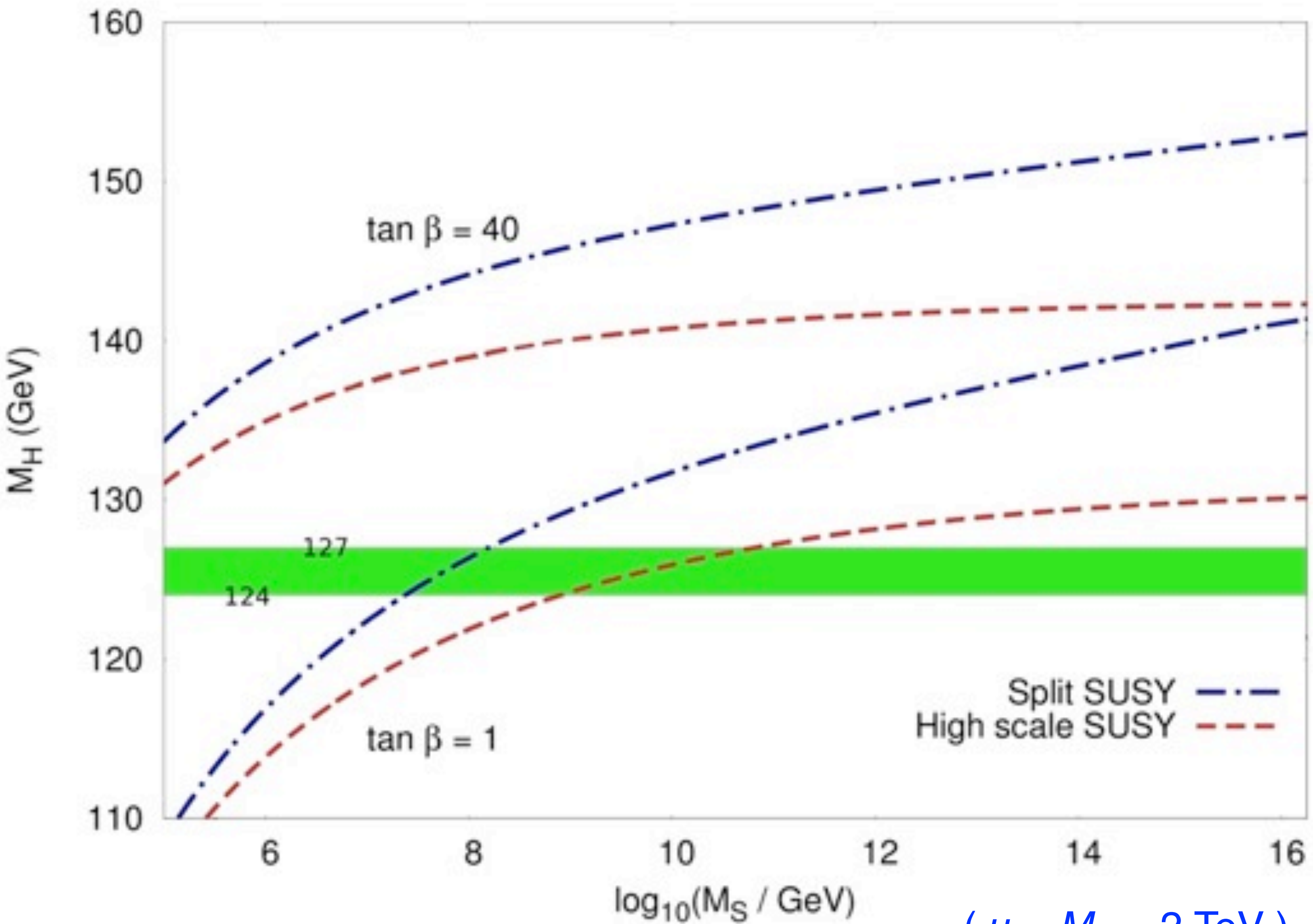
Self-promotion: the Fake Split-SUSY Model (FSSM)

[Benakli *et al.*, 1312.5220; also Benakli+Darme+Goodsell, 1508.02534]



Benakli *et al.*, 1312.5220

($\mu = M_{g'} = 2$ TeV)

Inspired by models with Dirac gauginos: higgsinos and gauginos replaced by "fake" counterparts that do not couple to the SM-like Higgs boson

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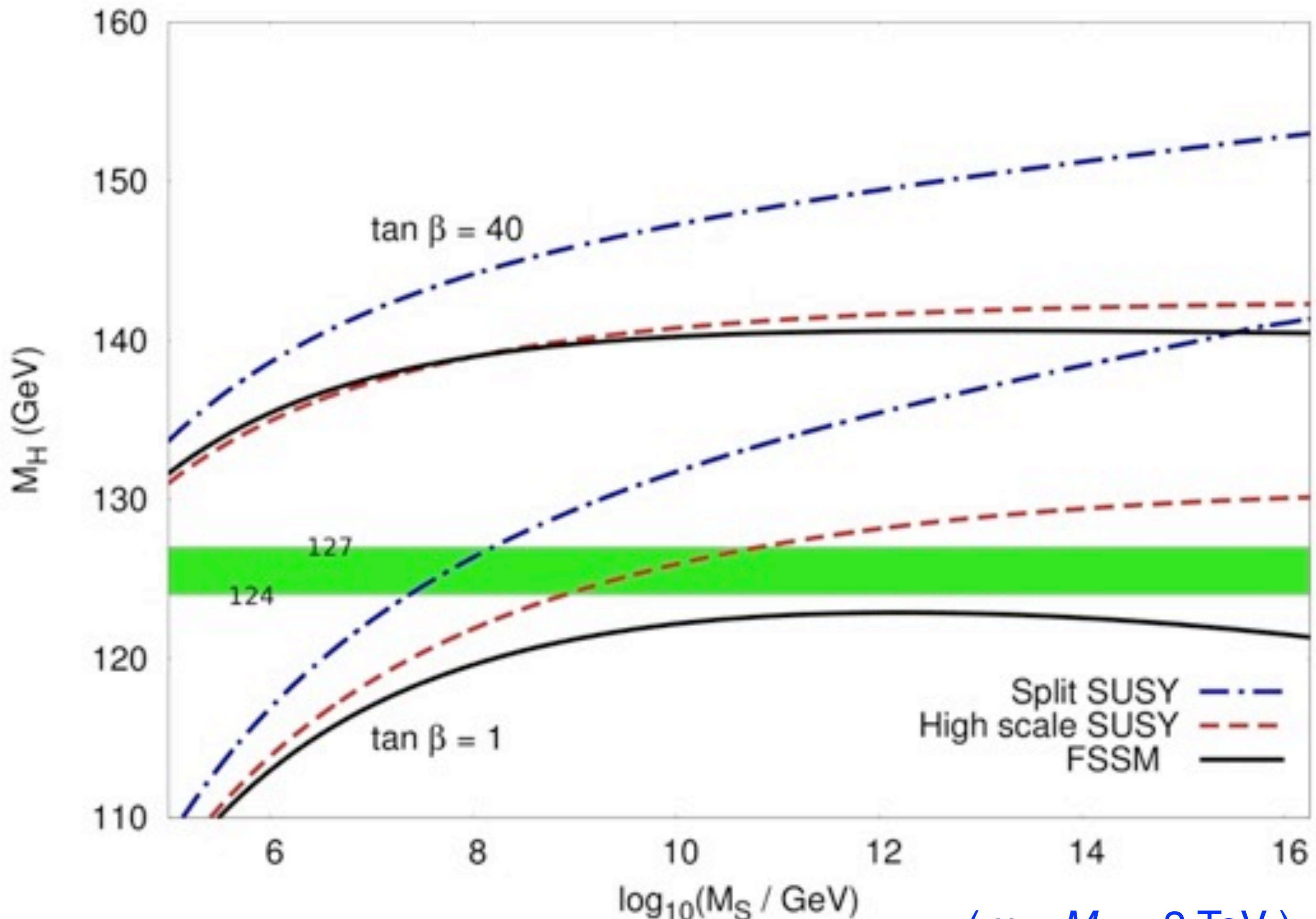

Benakli *et al.*, 1312.5220

($\mu = M_{g'} = 2$ TeV)

Inspired by models with Dirac gauginos: higgsinos and gauginos replaced by "fake" counterparts that do not couple to the SM-like Higgs boson

In the FSSM there is no upper bound on the SUSY-breaking scale

Reopening the low (M_A, $\tan\beta$) window

[see e.g.: Arbey *et al.*, 1303.7450; Djouadi+Quevillon, 1304.1787]

Appeal of the low (M_A, $\tan\beta$) region:

- For low M_A, extended Higgs sector potentially accessible at the LHC
- For low $\tan\beta$, not yet ruled out by the *H, A* —> tau tau searches
- Away from the decoupling limit, sizable couplings of *H, A* to gauge bosons and *h*

Interesting Higgs phenomenology: H —> hh, H —> WW, H —> ZZ, A —> Zh

However...

- At low $\tan\beta$, $M_h \approx 125$ GeV requires large stop masses M_S:
  - For $M_A \approx M_S$, $\tan\beta = 1$ implies $M_S \approx 10^8 - 10^{10}$ GeV

    At low M_A we might need an even larger M_S

This calls for the resummation of large logarithms in the EFT approach

Effective THDM with heavy SUSY

[Haber+Hempfling, early 90s, (...), Lee+Wagner, 1508.00576]

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 \ + \ \text{h.c.} \right]$$
$$+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$
$$+ \left\{ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) \ + \ \text{h.c.} \right\}$$

1) SUSY boundary conditions at the scale M_S :

$$\lambda_1 = \lambda_2 = -(\lambda_3 + \lambda_4) = \frac{1}{4} (g^2 + g'^2) ,$$
$$\lambda_4 = -\frac{g^2}{2} , \quad \lambda_5 = \lambda_6 = \lambda_7 = 0$$

(NOTE: loop corrections)

2) RG evolution of all seven lambdas from M_S to the weak scale;

3) scalar mass matrix in terms of the weak-scale lambdas:

$$M_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta \, c_\beta \\ -s_\beta \, c_\beta & c_\beta^2 \end{pmatrix} + v^2 \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix}$$

$$L_{11} = \lambda_1 \, c_\beta^2 + 2 \, \lambda_6 \, s_\beta \, c_\beta + \lambda_5 \, s_\beta^2$$
$$L_{12} = (\lambda_3 + \lambda_4) \, s_\beta \, c_\beta + \lambda_6 \, c_\beta^2 + \lambda_7 \, s_\beta^2$$
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$$\lambda_1 = \frac{1}{4}(g^2+g'^2) + \frac{2\,N_c}{(4\pi)^2}\Big(y_b^4 \frac{A_b^2}{M_S^2}(1-\frac{A_b^2}{12M_S^2}) - y_t^4 \frac{\mu^4}{12M_S^4}\Big)$$

$$\lambda_2 = \frac{1}{4}(g^2+g'^2) + \frac{2\,N_c}{(4\pi)^2}\Big(y_t^4 \frac{A_t^2}{M_S^2}(1-\frac{A_t^2}{12M_S^2}) - y_b^4 \frac{\mu^4}{12M_S^4}\Big)$$

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$$\lambda_4 = -\frac{1}{2}g^2 + \frac{2\,N_c}{(4\pi)^2}\Big(-y_b^2 y_t^2 \frac{A_{tb}}{2} + y_t^4(\frac{\mu^2}{4M_S^2} - \frac{\mu^2 A_t^2}{12M_S^4}) + y_b^4(\frac{\mu^2}{4M_S^2} - \frac{\mu^2 A_b^2}{12M_S^4})\Big)$$

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$$\lambda_6 = \frac{2\,N_c}{(4\pi)^2}\Big(y_b^4 \frac{\mu A_b}{M_S^2}(-\frac{1}{2} + \frac{A_b^2}{12M_S^2}) + y_t^4 \frac{\mu^3 A_t}{12M_S^4}\Big),$$

$$\lambda_7 = \frac{2\,N_c}{(4\pi)^2}\Big(y_t^4 \frac{\mu A_t}{M_S^2}(-\frac{1}{2} + \frac{A_t^2}{12M_S^2}) + y_b^4 \frac{\mu^3 A_b}{12M_S^4}\Big),$$

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$$
\begin{aligned}
V \;=\;& m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \Big[m_{12}^2 \Phi_1^\dagger \Phi_2 \;+\; \text{h.c.} \Big] \\
&+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
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\end{aligned}
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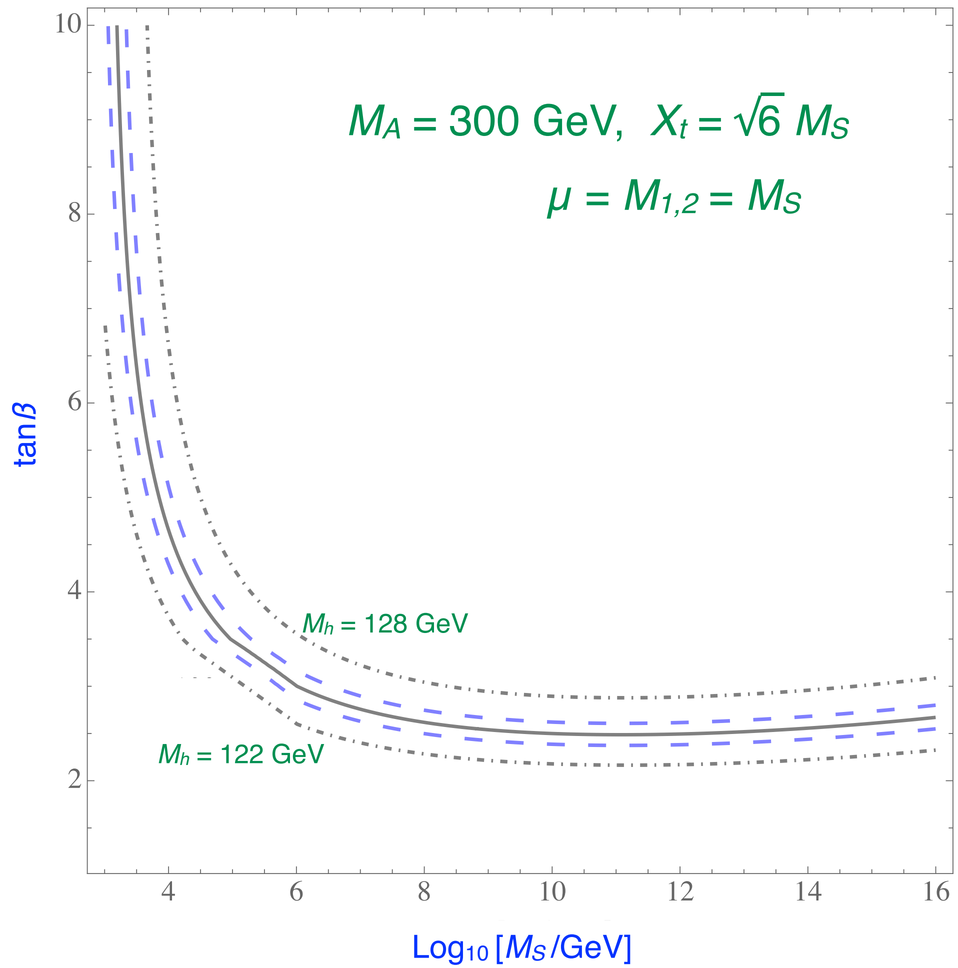
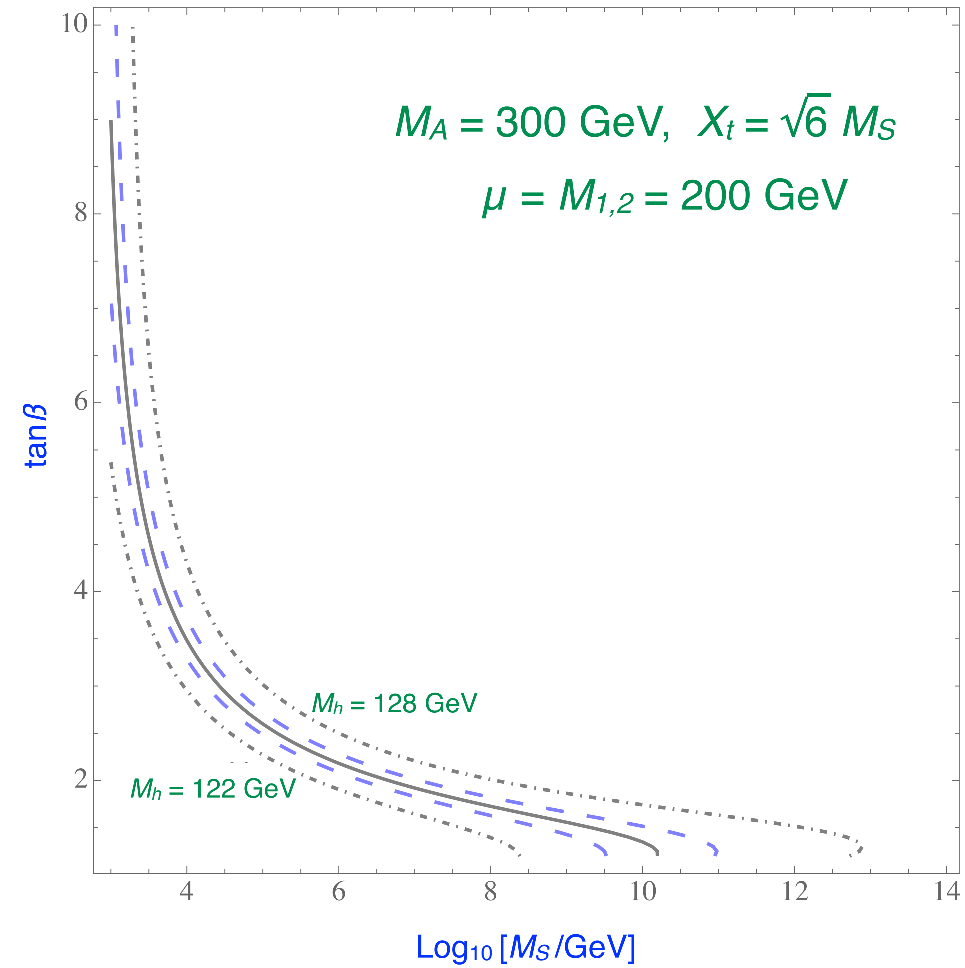
\end{aligned}
$$

For very low M_A and $\tan\beta$, $M_h = 125$ GeV can only be reached with light EW-inos!





Lee+Wagner, 1508.00576

See Gabriel Lee's talk in Friday's SUSY / Higgs session

An alternative approach: the hMSSM

[Maiani *et al.*, 1305.2172; Djouadi *et al.*, 1307.5205 and 1502.05653]

The dominant corrections affect mostly the (2,2) element of the scalar mass matrix.
We can trade it for the known M_h, and get formulae for M_H and for the scalar mixing angle:

$$M_H^2 \;=\; \frac{\mathcal{M}_{11}^2(\mathcal{M}_{11}^2 - M_h^2) + (\mathcal{M}_{12}^2)^2}{\mathcal{M}_{11}^2 - M_h^2}\,, \qquad \tan\alpha \;=\; -\frac{\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2 - M_h^2}$$

Setting the (1,1) and (1,2) elements to their tree-level values *(good approximation?)*
we obtain formulae that depend only on M_h, M_Z, M_A and $\tan\beta$

$$M_H^2 \;=\; \frac{(M_Z^2 + M_A^2 - M_h^2)(M_Z^2 c_\beta^2 + M_A^2 s_\beta^2) - M_A^2 M_Z^2 c_{2\beta}^2}{M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 - M_h^2}$$

$$\tan\alpha \;=\; -\frac{(M_Z^2 + M_A^2)\, c_\beta s_\beta}{M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 - M_h^2}$$

This allows for a “model independent” analysis with only two input parameters
(assuming no direct corrections from SUSY particles to the Higgs couplings)

EFT comparison:
[Lee+Wagner, 1508.00576]

Good agreement (few %) for M_H and mixing as long as the corrections to the (1,1) and (1,2) elements are suppressed (in particular, for $\mu X_t / M_S^2 \lesssim 1$)

LHCHXSWG-2015-002

August 1, 2015

LHC Higgs Cross Section Working Group

Public Note

Benchmark scenarios for low $\tan\beta$ in the MSSM

Emanuele Bagnaschi[1,a], Felix Frensch[2,b], Sven Heinemeyer[3,c], Gabriel Lee[4,d], Stefan Liebler[1,e], Margarete Mühlleitner[2,f] Allison Mc Carn[5,g], Jérémie Quevillon[6,h], Nikolaos Rompotis[7,i], Pietro Slavich[8,j], Michael Spira[9,k], Carlos E.M. Wagner[10,11,l], and Roger Wolf[2,m]

Conclusions

- The discovery of the Higgs and the accurate measurement of its mass have focused the theorists' attention on precision calculations in the Higgs sector
- The accuracy of the prediction for M_H from the SM Lagrangian parameters (or rather the extraction of said parameters from the known value of M_H) is now of order (0.1–0.2)%, comparable with the experimental accuracy
- In the MSSM, the measured Higgs mass calls for large radiative corrections; the accuracy of Higgs-mass calculations appears to be still of order "few %"
- Several hints point to scenarios with heavy superpartners; in that case, large logarithmic corrections need to be resummed in an effective-theory approach
- If the EFT valid at the weak scale is the SM, part of the corrections can be borrowed from the SM calculation, reducing the uncertainty to less than 1%
- Of course, much more interesting phenomenology in scenarios where the EFT valid at the weak scale is *not* the SM (e.g., light -inos and/or light THDM)

Thank you!!!