

Reconstructing CMSSM parameters at the LHC with $\sqrt{s} = 14$ TeV via the golden decay channel arXiv:1106.5117, Dec 2014

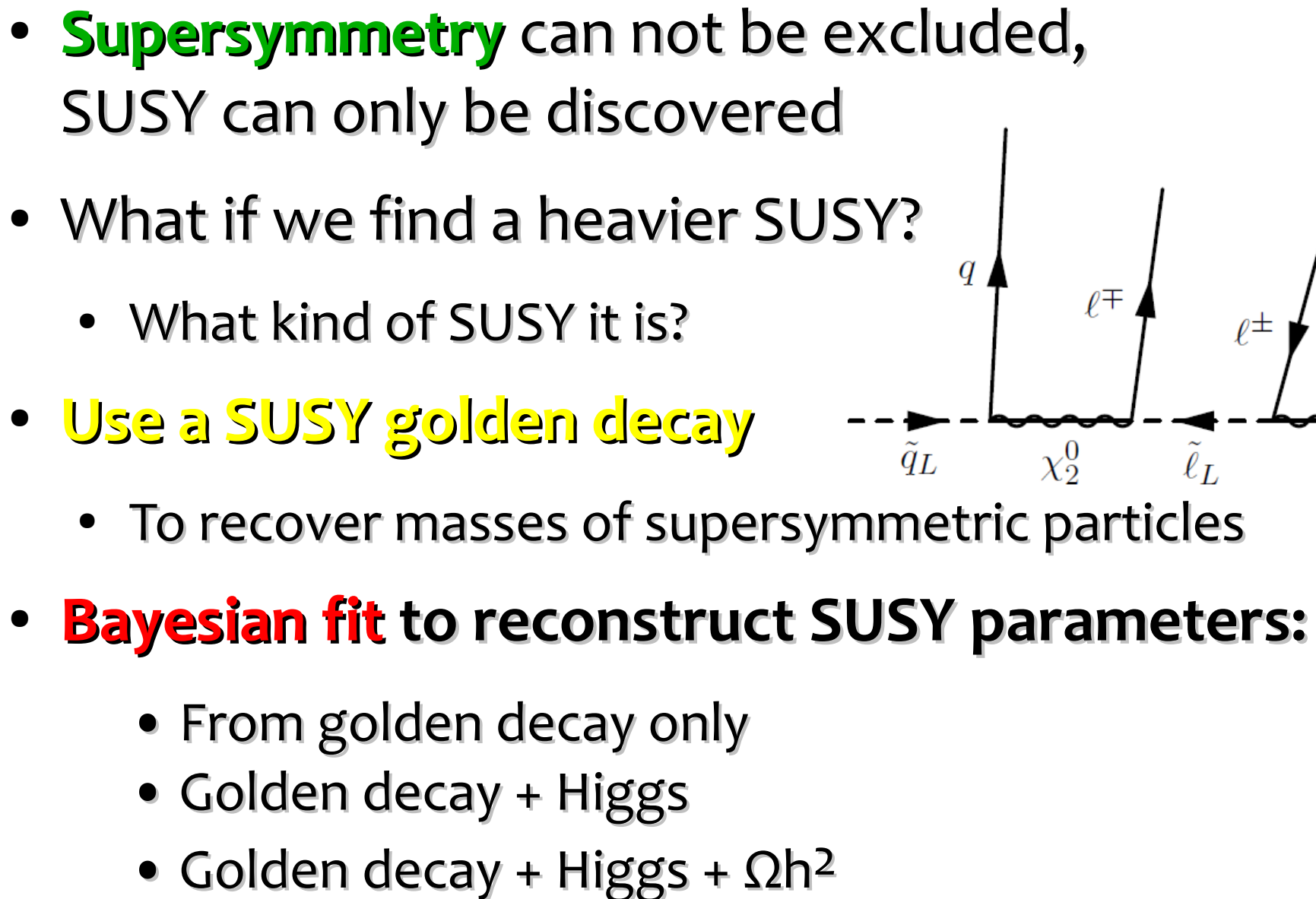
Andrew Fowlie, Małgorzata Kazana,
Leszek Roszkowski

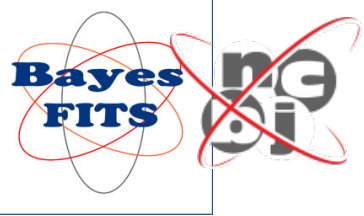
National Centre for Nuclear Research
Warsaw, Poland



SUSY 2015
Lake Tahoe, California
August 23 – 29

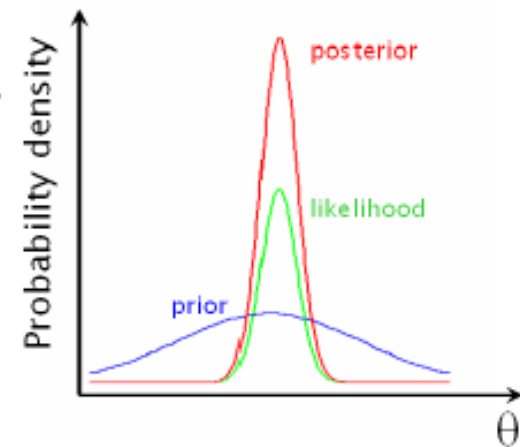
Motivation/Outline





Bayesian theorem

In Bayesian theory, our degree of belief in the preposition changes rationally the probability of the posterior observation.



- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta, \psi)}{p(d)}$$

- $p(d|\xi) = \mathcal{L}$: likelihood

- $\pi(\theta, \psi)$: prior pdf

- $p(d)$: evidence (normalization factor)

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

- usually marginalize over SM (nuisance) parameters $\psi \Rightarrow p(\theta | d)$

- $m = (\theta, \psi)$ – model's all relevant parameters

- model parameters θ

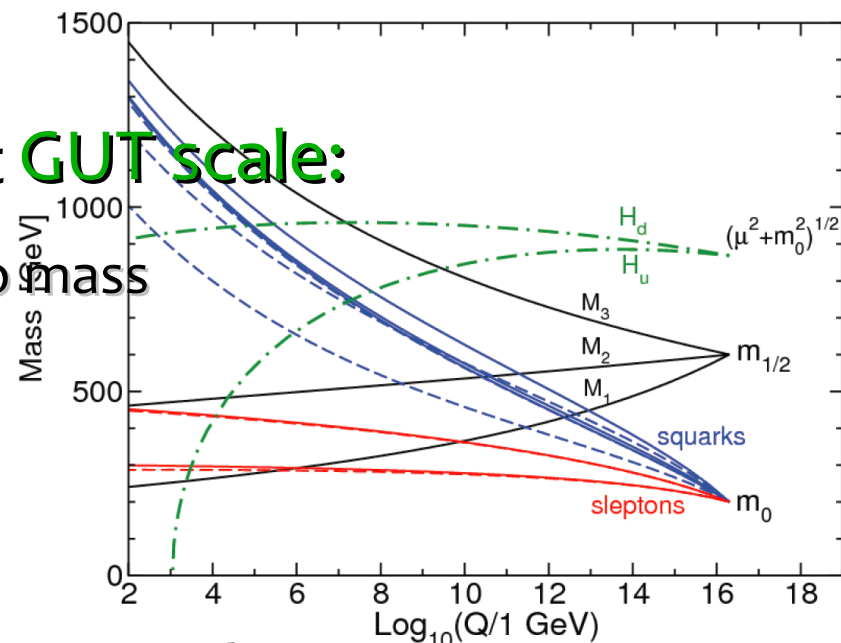
- relevant SM param's $\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{em}(M_Z)^{\overline{MS}}$

- $\xi = (\xi_1, \xi_2, \dots, \xi_m)$: set of derived variables (observables): $\xi(m)$

- d : data ($\Omega_{\text{CDM}} h^2, b \rightarrow s\gamma, m_h$, etc)

Focus on CMSSM

- **C**onstrained **M**inimal **S**upersymmetric **S**tandard **M**odel
- **Might appear less natural than before, however:**
 - It correctly reproduces both the Higgs boson mass and the DM relic abundance in the “unnatural” multi-TeV regions of mass parameters
 - It also remains compatible with all experimental data, **with the exception of $(g-2)_\mu$**
- **Unification of MSSM soft masses at GUT scale:**
 - **$m_{1/2} = M_1 = M_2 = M_3$** → Common gaugino mass
 - m_0** → Common scalar mass
 - A_0** → Common trilinear
 - $\tan \beta$** → Ratio of Higgs vevs
 - $\text{sgn } \mu$**
 - Run parameters to low scale with renormalisation group equations to calculate masses at 1 TeV scale

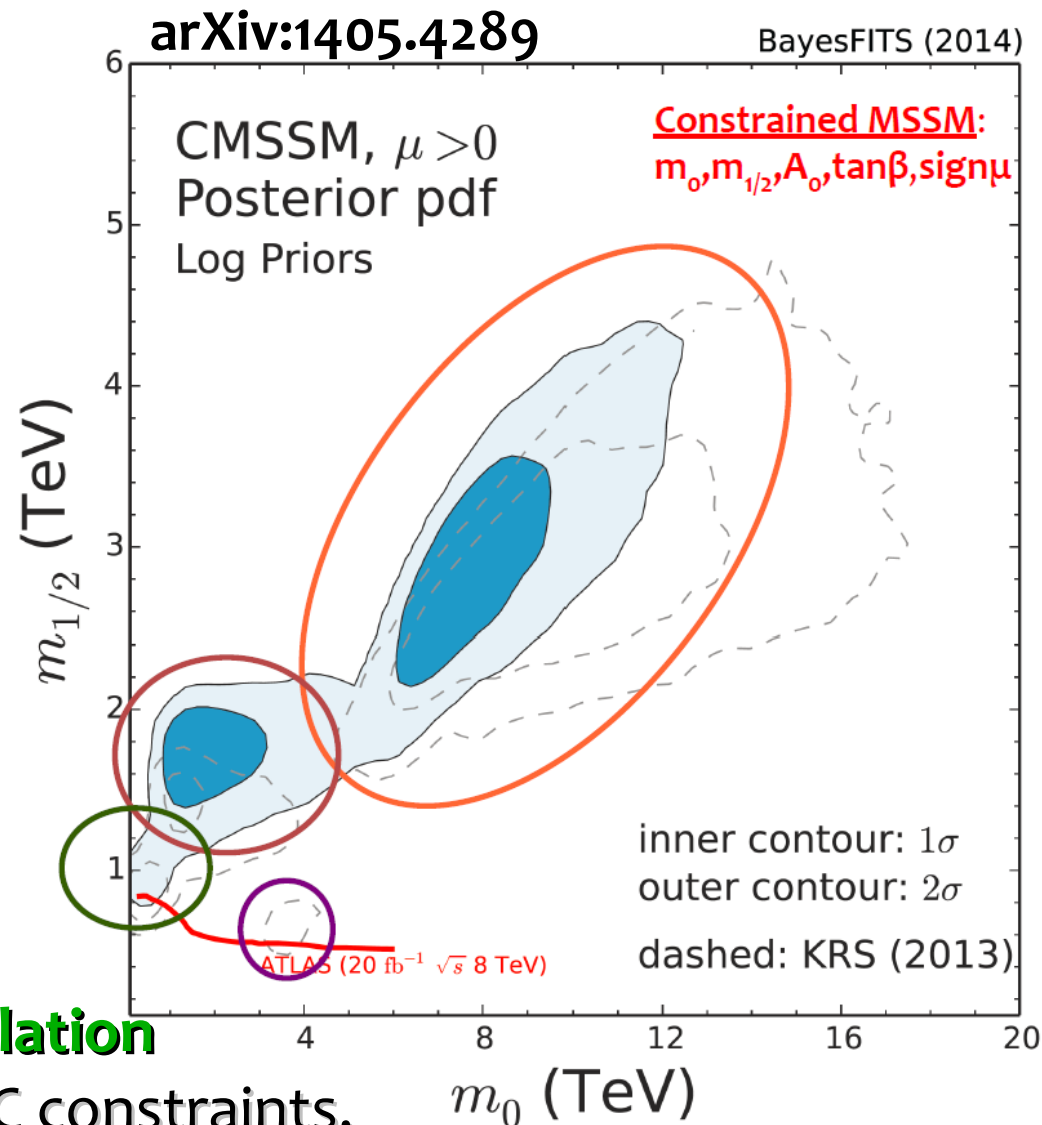


SUSY must be heavier

From our recent fit:

- $M_{\text{SUSY}} >$ than expected
- Decreased m_0 in **$\sim 1\text{TeV}$ higgsino region** due to higgs corrections
- Posterior in **A-funnel region** increases due to better fit to higgs mass
- **Focus point region** disfavoured by LUX
- Reduced posterior in **stau-coannihilation region** (Bino DM region) due to LHC constraints,

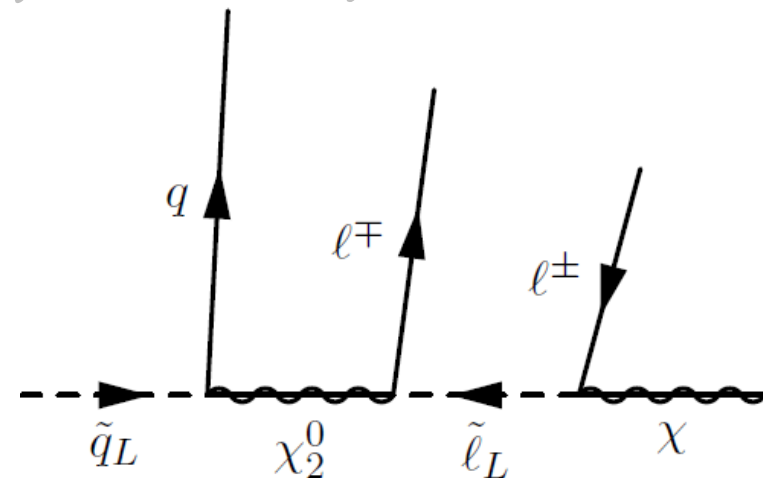
but $M_{\text{SUSY}} < 1\text{ TeV}$ still probable



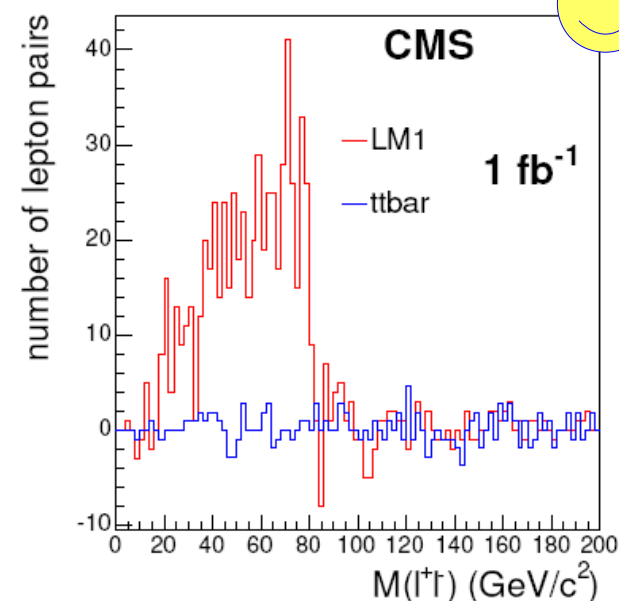
■/■ = 68%/95% region

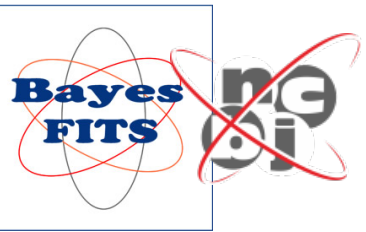
Signature of SUSY

- **Golden channel**
- Allows to reconstruct sparticle masses from kinematic edges
- Many studies made in pre-LHC era for light SUSY
 - Our previous analysis
(arXiv:1106.5117, arXiv:0907.0594)
- **Our goal:** check if we can recover sparticle masses from kinematic edges for higher SUSY and reconstruct model parameters ?



Example from the past





Methodology

- **Pick a CMSSM point allowed by experiments**
(e.g. m_h , Planck relic density Ωh^2 , direct searches)
- **Monte Carlo analysis for CMSSM point at 14 TeV**
 - Perform MC simplified (Gen Level) analysis to simulate sparticle mass measurements from golden decay
- **Bayesian reconstruction** of CMSSM parameters with simulated sparticle mass measurements

CMSSM point

- Golden decay requires hierarchy:

$$\tilde{q} > \tilde{\chi}_2^0 > \tilde{\ell}$$

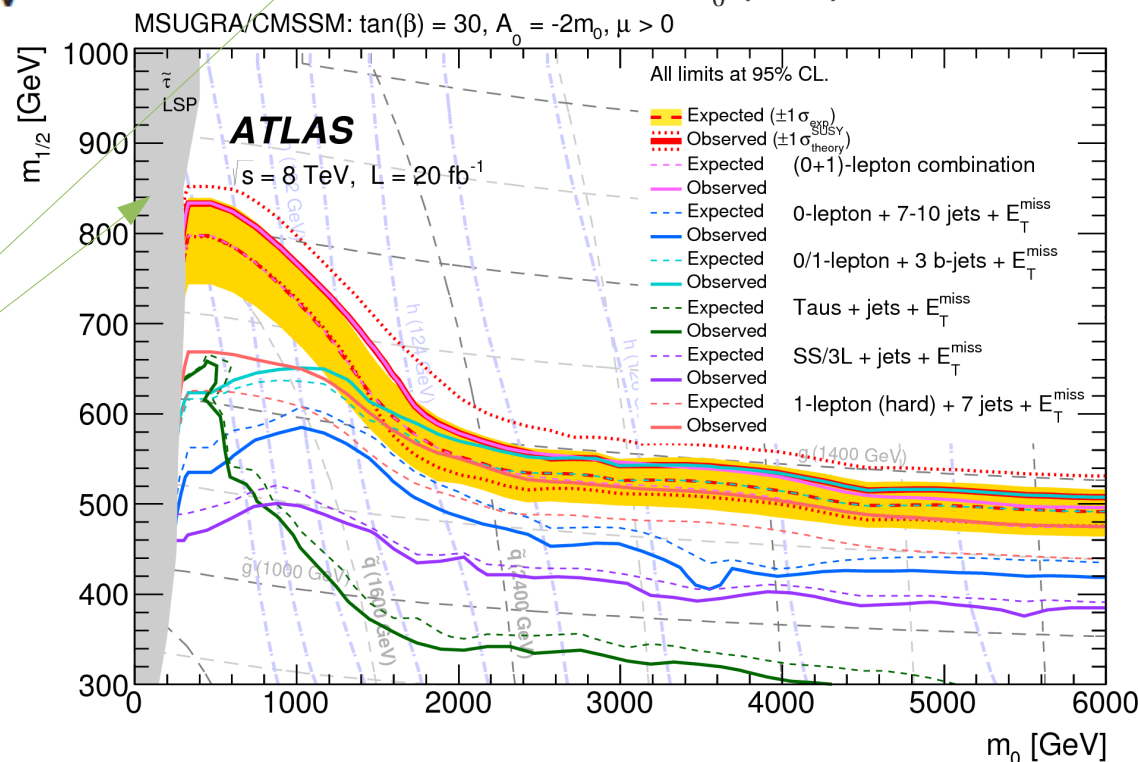
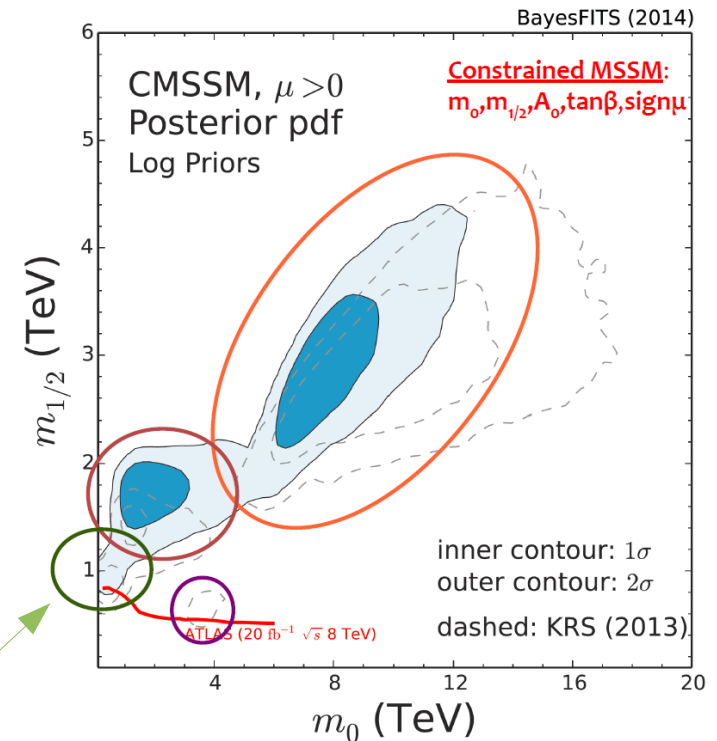
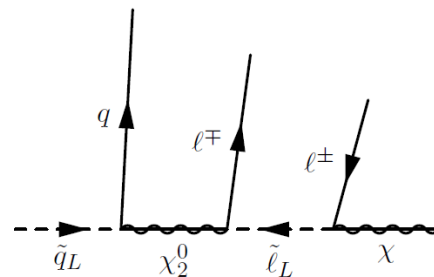
$\tilde{g} > \tilde{q}$ to shut $\tilde{q} \rightarrow \tilde{g}q$
spoiler

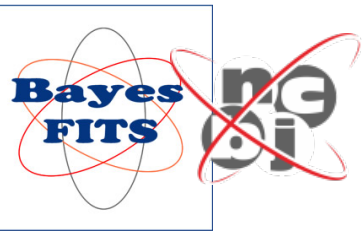
- Actually, $\tilde{\chi}_2^0 > \tilde{\ell} + 50 \text{ GeV}$

to avoid phase space suppression of BR

→ In CMSSM,
it means $m_{1/2} > m_0$

- Stau-coannihilation region** allowed in CMSSM with golden decay





Benchmark CMSSM point

- Search for a good point with golden decay
 - High mass, above LHC limits: $m_{1/2} = 900\text{GeV}$
- Minuit to find the point with:
 - $m_h = 125\text{GeV}$ within errors
 - $\Omega h^2 \approx \text{WMAP/PLANCK (0.1197)}$
 - Golden decay
- The benchmark CMSSM point:
 - satisfies reasonable
 - 9 measurements:
 - m_h , DM relic density,
 - $b \rightarrow s\gamma$, $B_u \rightarrow \tau\nu$, $B_s \rightarrow \mu^+\mu^-$,
 - m_W , $\sin^2 \theta_{\text{eff}}$, M_t , ΔM_{B_s}
- $\Omega h^2 = 0.1390 \sim$ agreement with the Planck measurements

CMSSM:

$m_{1/2} = 900\text{GeV}$

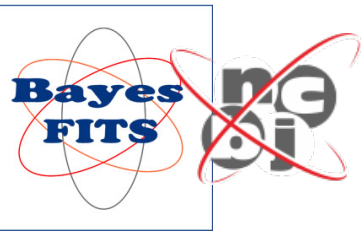
$m_0 = 315\text{GeV}$

$\tan \beta = 11$

$A_0 = -2550\text{GeV}$

$\text{sgn } \mu = +1$

Particle Mass (GeV):							
$\chi_1^0 = \chi$	382.8	\tilde{e}_L	679.8	\tilde{d}_L	1835	h	124.1
χ_2^0	728.7	\tilde{e}_R	463.4	\tilde{d}_R	1754	H	1741
χ_3^0	1645	$\tilde{\nu}_e$	675.1	\tilde{u}_L	1834	A	1742
χ_4^0	1649	$\tilde{\tau}_1$	384.6	\tilde{u}_R	1762	H^\pm	1744
χ_1^\pm	728.9	$\tilde{\tau}_2$	659.9	\tilde{b}_1	1509		
χ_2^\pm	1649	$\tilde{\nu}_\tau$	651.4	\tilde{b}_2	1726		
\tilde{g}	1985			\tilde{t}_1	984.1		
				\tilde{t}_2	1552		



MC Simuations

- **MC Pythia (Gen Level)** simulation for **@ 14 TeV**

- Particle mass spectrum from SoftSUSY

- **CMSSM $\sigma = 34.5/\text{fb}$**

- Run III LHC **$L = 300/\text{fb}$** $\rightarrow N_{\text{total}} = \sim 10.000$ events

- **Selection – classical SUSY with lepton cuts**

Very simplified assumptions:

- **Detector acceptance:**

- Isolated leptons:
 $pt > 10 \text{ GeV}$, $|\eta^e| < 2.4$, $|\eta^\mu| < 2.4$
- AntikT jets: $pt > 50 \text{ GeV}$, $|\eta^{\text{jet}}| < 5$

- Total selection efficiency: 0.10

- **Event selection:**

- At least 2 opposite sign leptons
- At least 4 jets
- $pt^{\text{1st jet}} > 100 \text{ GeV}$
- Z peak veto:
 $89 \text{ GeV} > \text{minv}_{\ell\ell} > 95 \text{ GeV}$

Endpoints

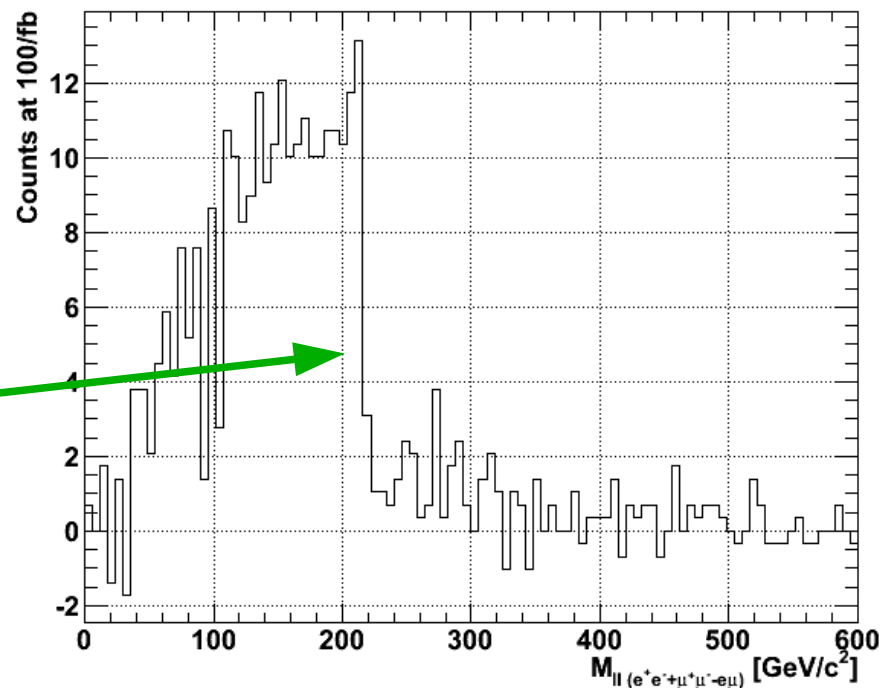
- Reconstruction of **mass invariant distribution with endpoints**:
 - 1 lepton pair ($\ell\ell$) OSSF ($ee+\mu\mu-\mu e$)
 - 2-3 each lepton with the jet (ℓq and $\ell' q$)
 - 4 the jet and both leptons ($\ell\ell q$)
 - 5 threshold $\ell\ell q$, with $\theta > \pi/2$ between leptons in slepton frame
- To recover masses from functions

$$m_{\ell\ell}^2 = \frac{(m_{\tilde{\chi}_2^0}^2 - m_l^2)(m_l^2 - m_{\tilde{\chi}_1^0}^2)}{m_l^2}$$

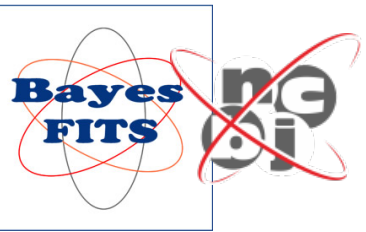
$$m_{\ell q, \text{near}}^2 = \frac{(m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_l^2)}{m_{\tilde{\chi}_1^0}^2}$$

$$m_{\ell q, \text{far}}^2 = \frac{(m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2)(m_l^2 - m_{\tilde{\chi}_1^0}^2)}{m_l^2}$$

$$m_{\ell q q}^2 = \max \left[\frac{(m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\chi}_2^0}^2}, \frac{(m_{\tilde{q}}^2 - m_l^2)(m_l^2 - m_{\tilde{\chi}_1^0}^2)}{m_l^2} \right]$$



arXiv:0410303



Sparticle mass recovery

- **Five mass invariant distribution fitted**
to the get endpoint position with the error in Root
- **Fit unknown sparticle masses to five endpoints** with Root
 - **Single solution for a sparticle mass with statistical errors**
- **Errors are correlated \Rightarrow covariance matrix**

- **C matrix basis in GeV^2**
 $(m_{\tilde{\chi}_1^0}, m_{\tilde{\ell}}, m_{\tilde{\chi}_2^0}, m_{\tilde{q}})$

$$\sigma = \begin{pmatrix} 132.0 & 18.4 & 31.9 & 175.8 \\ \cdot & 25.5 & 24.2 & 21.3 \\ \cdot & \cdot & 24.8 & 39.6 \\ \cdot & \cdot & \cdot & 401.1 \end{pmatrix}$$

Covariance matrix

- Covariance matrix is diagonalised to find errors:

$$V \sigma^{-1} V^T \approx \text{diag} [(0.3 \text{ GeV})^{-2}, (5.6 \text{ GeV})^{-2}, (7.5 \text{ GeV})^{-2}, (22.3 \text{ GeV})^{-2}]$$

[GeV]

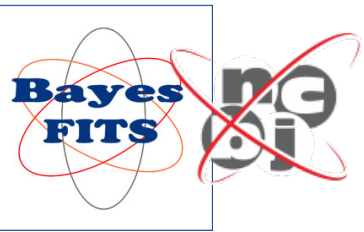
$$\mathbf{0.3} = 0.1 \cdot m_{\chi_1^0} + 0.7 \cdot m_{\tilde{\ell}} - 0.8 \cdot m_{\chi_2^0} + 0.0 \cdot m_{\tilde{q}} \approx \frac{1}{\sqrt{2}}(m_{\tilde{\ell}} - m_{\chi_2^0}),$$

$$\mathbf{5.6} = 0.6 \cdot m_{\chi_1^0} - 0.6 \cdot m_{\tilde{\ell}} - 0.4 \cdot m_{\chi_2^0} - 0.2 \cdot m_{\tilde{q}},$$

$$\mathbf{7.5} = 0.6 \cdot m_{\chi_1^0} - 0.4 \cdot m_{\tilde{\ell}} - 0.5 \cdot m_{\chi_2^0} + 0.4 \cdot m_{\tilde{q}},$$

$$\mathbf{22.3} = 0.4 \cdot m_{\chi_1^0} + 0.1 \cdot m_{\tilde{\ell}} + 0.1 \cdot m_{\chi_2^0} + 0.9 \cdot m_{\tilde{q}} \approx m_{\tilde{q}}.$$

- Two (one very) well determined directions $\sigma \leq (1) 10 \text{ GeV}$



Statistical method

- To **recover original CMSSM parameters** from simulated sparticle mass measurements
- Use Bayesian statistics . Bayes theorem:

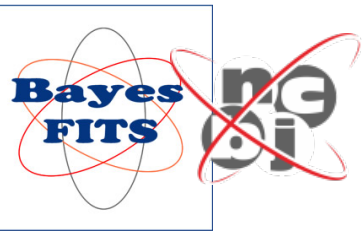
$$\underbrace{p(m_0, m_{1/2}, \tan \beta, A_0 | \mathbf{D})}_{\text{Posterior density}} \propto \underbrace{\mathcal{L}(\mathbf{D} | m_0, m_{1/2}, \dots)}_{\text{Likelihood}} \times \underbrace{\pi(m_0, m_{1/2}, \dots)}_{\text{Prior}}$$

- We want to find posterior density for CMSSM , given golden decay measurements
- Marginalise posterior, to remove parameter dependencies, e.g.,

$$p(m_0, m_{1/2} | \mathbf{D}) = \int p(m_0, m_{1/2}, \tan \beta, A_0 | \mathbf{D}) dA_0 d\tan \beta$$

- Find “**credible regions:**” Smallest region A such that

$$\int_A p(m_0, m_{1/2} | \mathbf{D}) dm_0 dm_{1/2} = 95\%$$



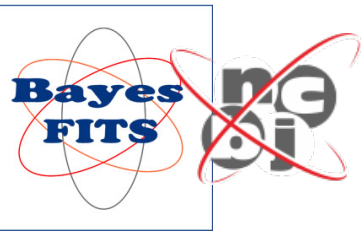
Statistical method

- Priors reflect “**prior belief**” in parameter space
- Choose **flat priors**, expect prior independence
- **Likelihood** is a multivariate Gaussian from our golden decay simulations

$$\mathcal{L}_{\text{golden decay}} = \exp \left[-\frac{1}{2} (M - M_{\text{benchmark}}) C^{-1} (M - M_{\text{benchmark}})^T \right]$$

- **M is a function of** $M = (m_{\tilde{\chi}_1^0}, m_{\tilde{\ell}}, m_{\tilde{\chi}_2^0}, m_{\tilde{q}})$ and C is a covariance matrix from our MC
- In the next step, apply **Gaussian likelihoods** for $\Omega h^2 = 0.1186 \pm 0.0031 \pm 10\%$ and $m_h = 125.8 \pm 0.5 \pm 3\text{GeV}$
- **Posteriors** are calculated with MultiNest from all priors and likelihoods

Parameter	Prior range	Distribution
m_0	(0.1, 4) TeV	Flat
$m_{1/2}$	(0.1, 2) TeV	Flat
A_0	(-4, 4) TeV	Flat
$\tan \beta$	(3, 62)	Flat
$\text{sign } \mu$	+1	Fixed
M_t	173.5 GeV	Fixed
$m_b(m_b)^{\overline{MS}}$	4.18 GeV	Fixed
$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}}$	127.944	Fixed
$\alpha_s(M_Z)^{\overline{MS}}$	0.1184	Fixed

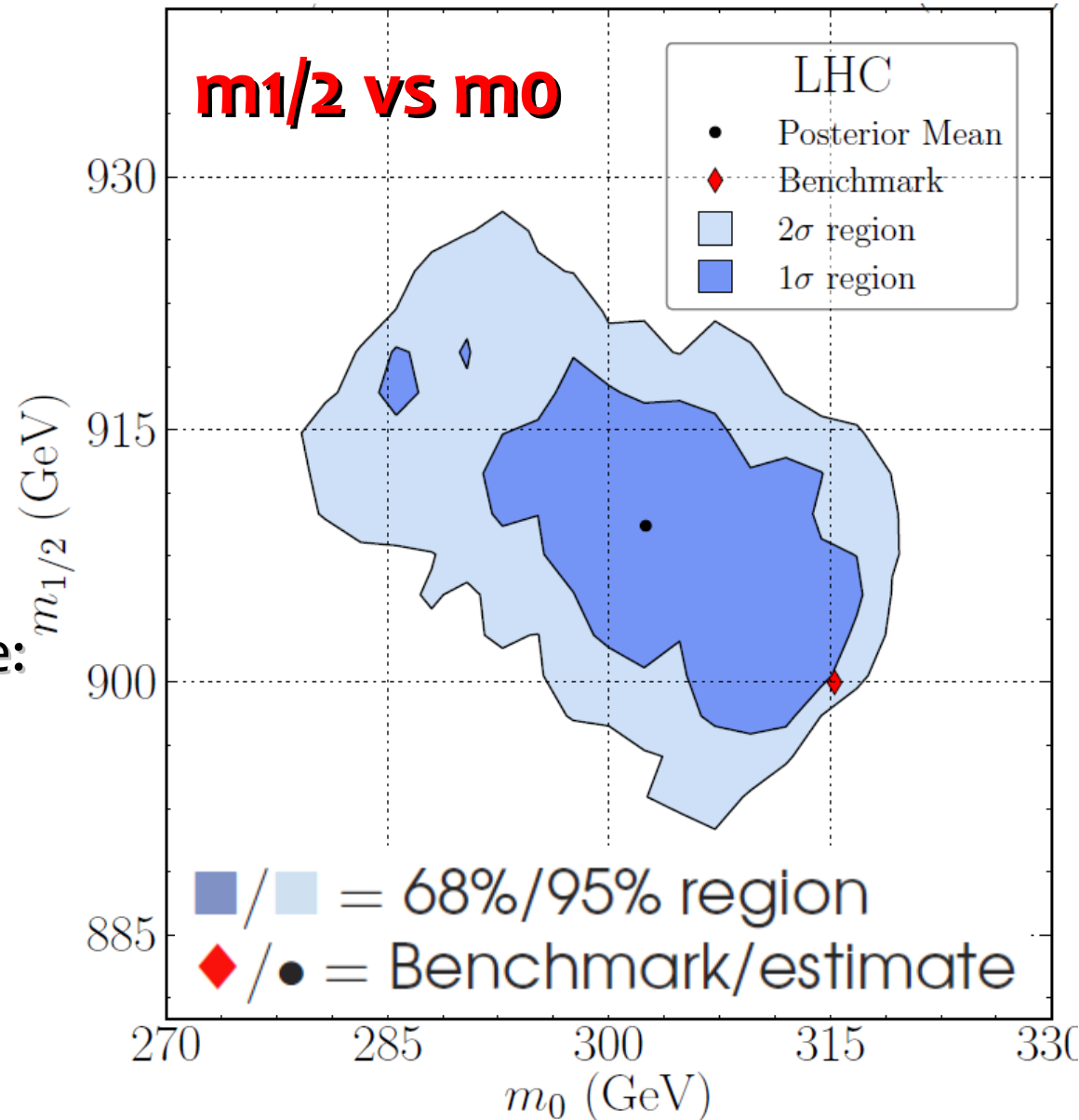


Our method

- [1] Assume SUSY **CMSSM benchmark point** is “true”
 - [2] Assume sparticle **masses measured by golden decay** at LHC $\sqrt{s} = 14$ TeV
 - [3] Find expected errors (**covariance matrix**) from MC
 - [4] Assume **flat priors** for CMSSM parameters $m_0, m_{1/2}, A_0, \tan \beta$
 - [5] Fit **CMSSM** to golden decay measurements **with Bayesian statistics**
 - [6] Afterwards, **add information** from m_h and Ωh^2 to see how much it improves recovery
- **How well do we recover the original benchmark parameters?**





Results – CMSSM reco

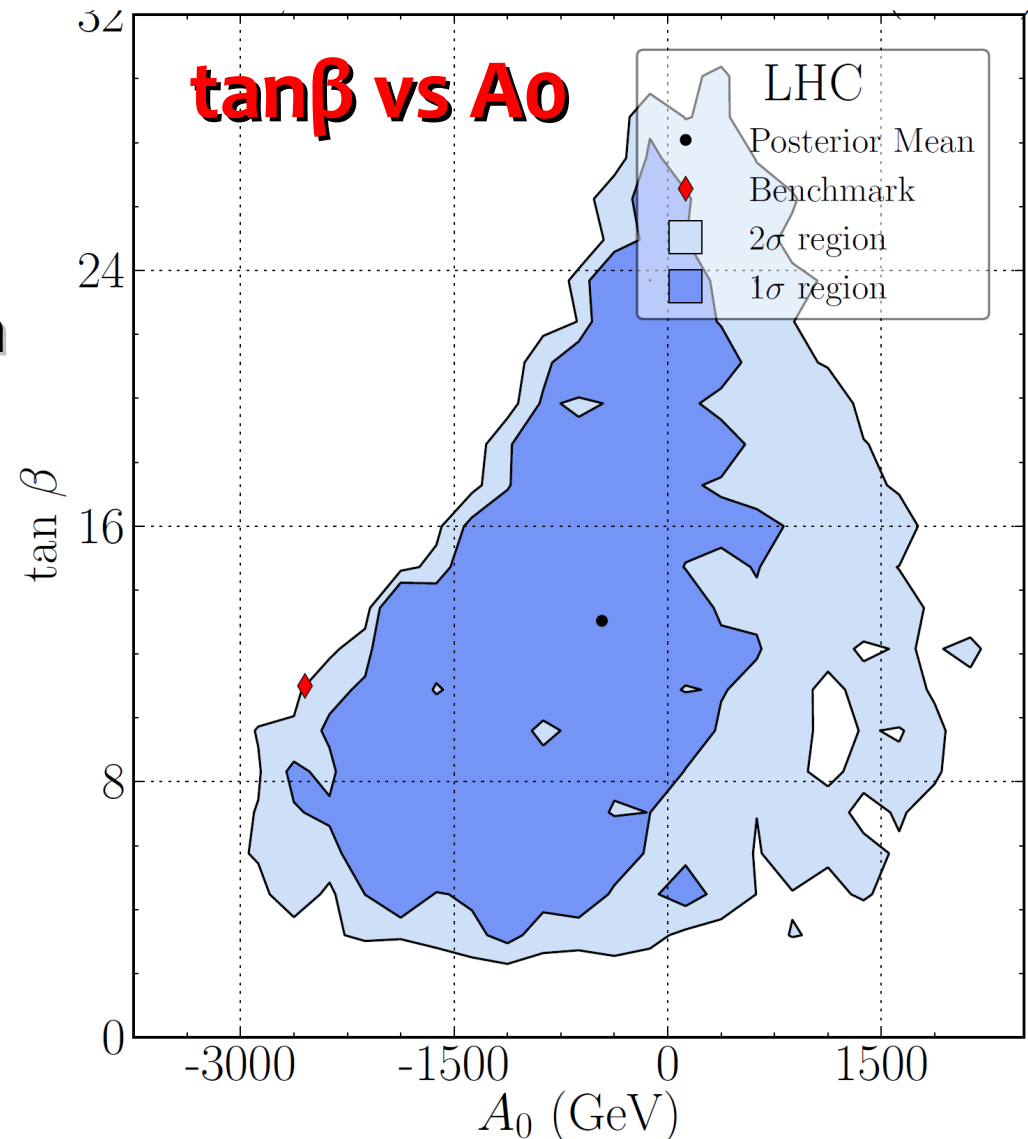
- With the endpoint information alone, **recovered “true” benchmark point**
- **Single correct solution found, the benchmark point is in 68% region**
- Two orthogonal directions in the parameter space are visible: **anti-diagonal m_0 - $m_{1/2}$** and **diagonal $m_0+m_{1/2}$** , which correspond to the first (0.3 GeV) and second (5.6 GeV) eigenvectors of C matrix



CMSSM reconstruction

- **A_0 is not well reconstructed**
- Broadened the credible region in the m_0 - $m_{1/2}$ direction
- **$\tan \beta$ determined to within a few units**

 /  = 68%/95% region
 /  = Benchmark/estimate



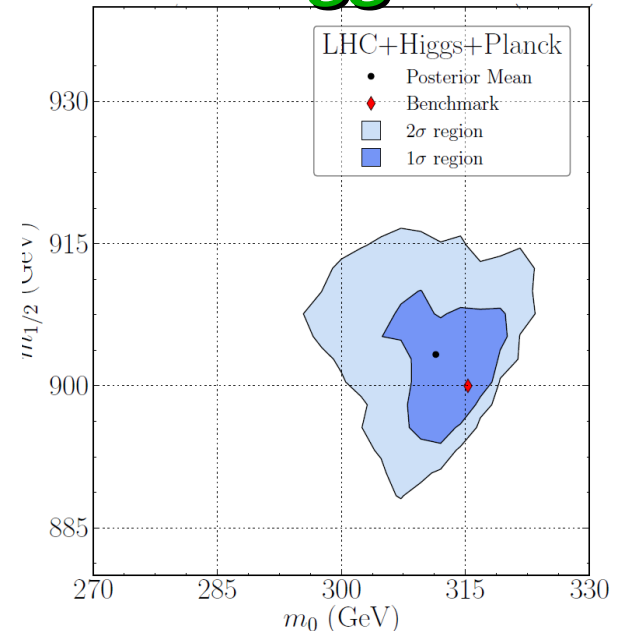
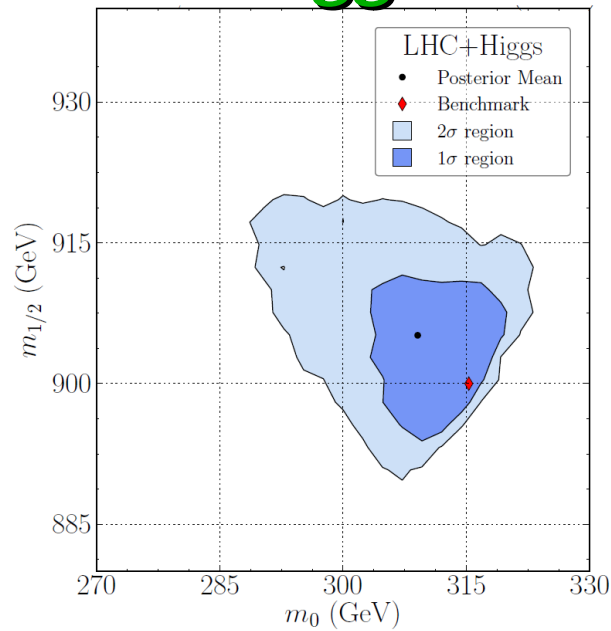
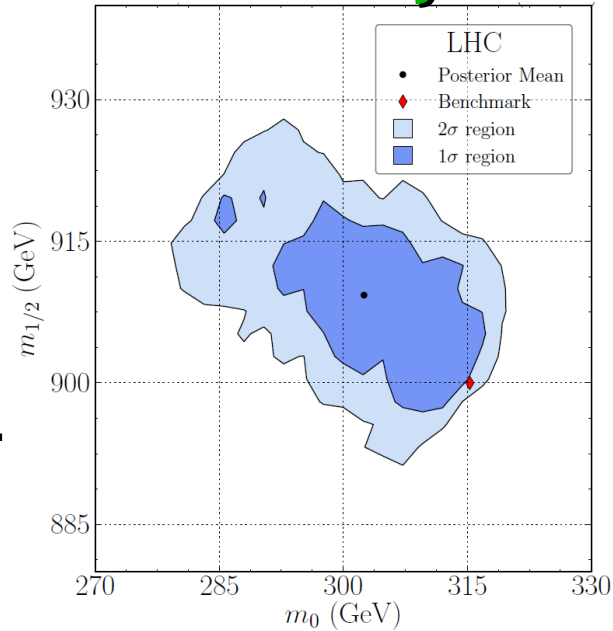
CMSSM reco

Gold decay

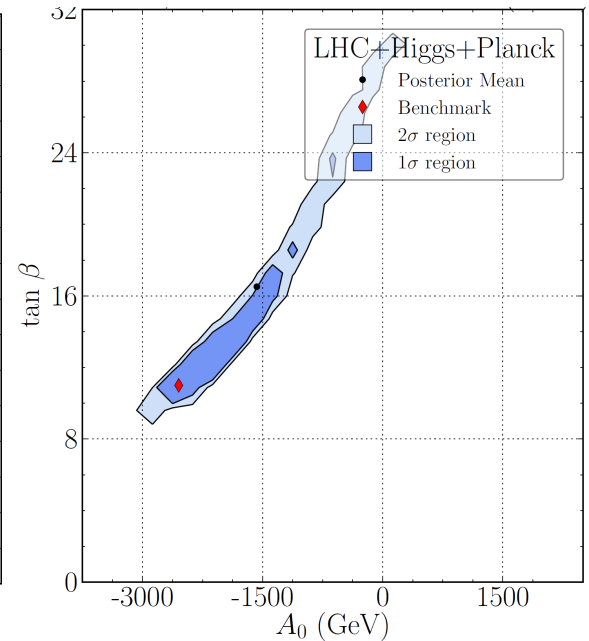
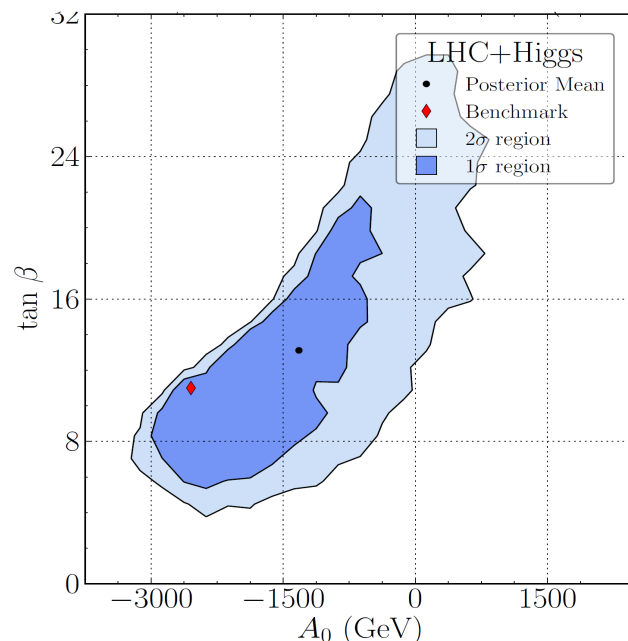
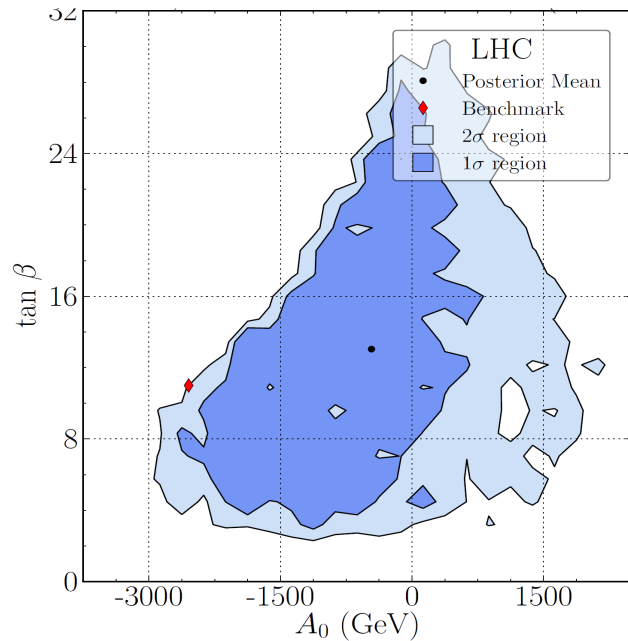
LHC+higgs

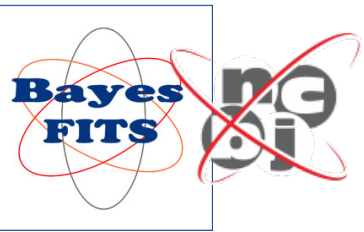
LHC+higgs+Planck

$m_{1/2}$ vs m_0



$\tan\beta$ vs A_0





Adding more data to fit

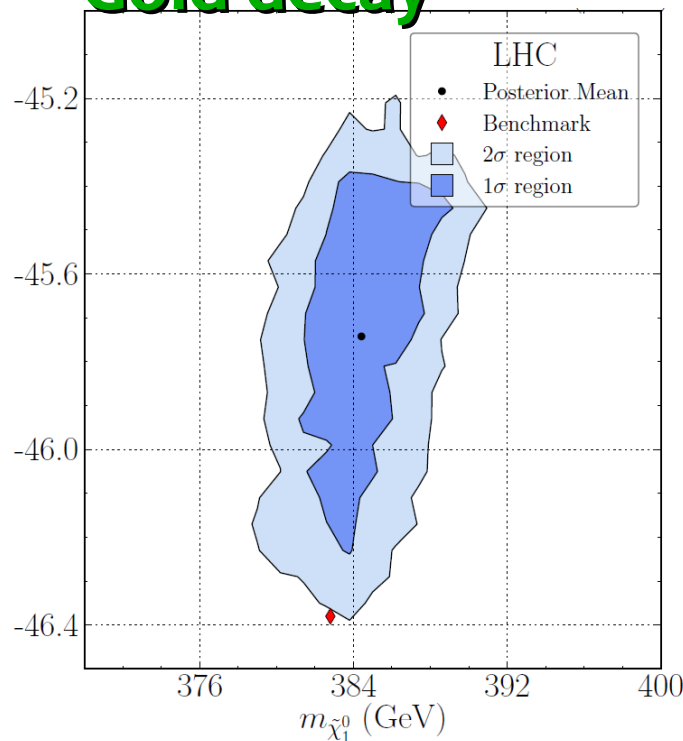
- The **credible regions shrink successively as the data is added**, though two orthogonal directions in the parameter space remain visible
 - The diagonal **$m_0+m_1/2$** direction of the is **only marginally shrunk**, whereas the anti-diagonal **$m_0-m_1/2$** direction is **squashed** for $(A_0, \tan \beta)$
- When we add **higgs**, A_0 must be < 0.5 TeV to increase the Higgs boson mass via maximal mixing $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$
Increases in Higgs boson mass from increasing $m_1/2$ and m_0 to increase stop masses are negligible
- When we add **Planck**, we enforce mass degeneracy so that staus and neutralinos coannihilate effectively and **reduce the relic density to the Planck value**
- **This is rather fortunate** – **higgs** and **Planck** constrain the direction of parameter space that was poorly constrained by LHC

Direct Dark Matter searches

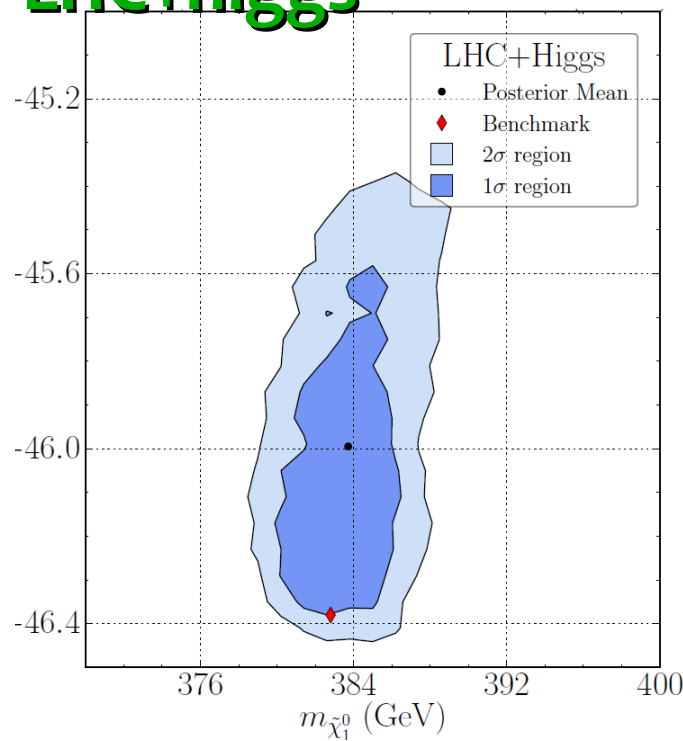
- LHC prediction indicate that in our discovery scenario the **DM might be within reach of direct detection experiments**
 - should be accessible at a 1-tonne detectors whose reach is expected to be $< 10^{-46} \text{ cm}^2$
- The resolution and bias of σ^{SI} improves slightly as data is added, especially Planck, but the resolution of the neutralino mass is not much improved

$\log \sigma^{\text{SI}}_{\text{P}} (\text{cm}^2) \text{ vs } m_{\text{LSP}} (\text{GeV})$

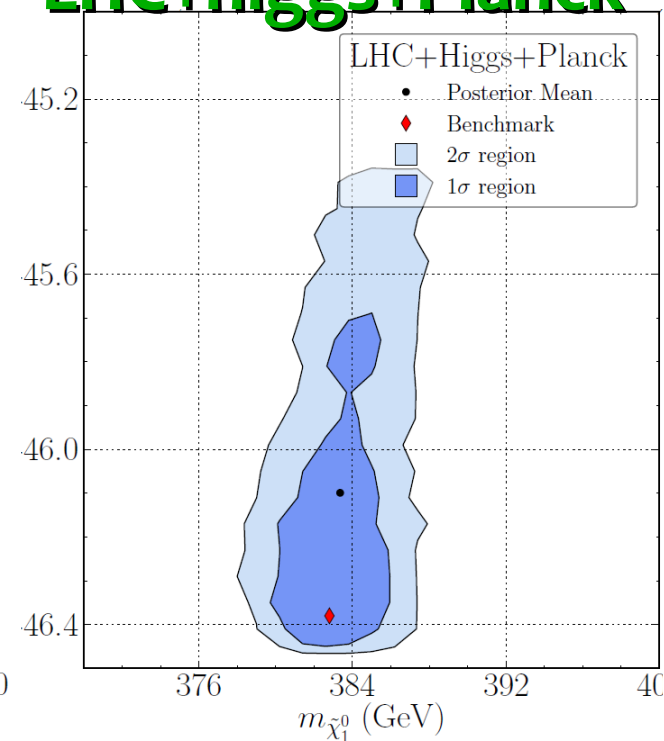
Gold decay

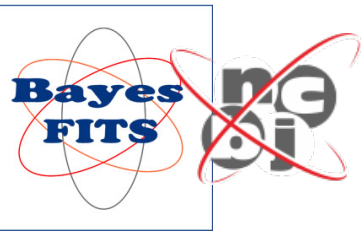


LHC+higgs



LHC+higgs+Planck





Conclusions

- **We demonstrated the possibility of reconstructing CMSSM parameters with Bayesian statistics**
- If SUSY is found in the LHC, we can check existence of the golden decay
- **We found that sparticle masses can be measured with good precision for high mass CMSSM benchmark point**
- **We found that CMSSM parameters can be well recovered**
 - Improved when additional information from Ωh^2 is added, but less so for m_h



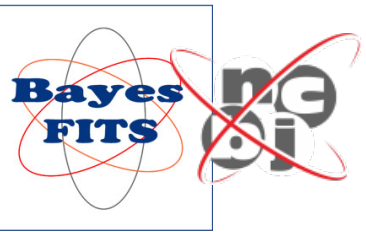
INNOVATIVE ECONOMY
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EUROPEAN UNION
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References:

- [1] A. Fowlie, M. Kazana, L. Roszkowski, *Reconstructing CMSSM parameters at the LHC with $\sqrt{s}=14$ TeV via the golden decay channel*, **arXiv:1106.5117, Dec 2014**