$\mathcal{N} = 4$ super Yang–Mills on a space-time lattice

David Schaich (Syracuse)

SUSY 2015, Lake Tahoe, 25 August

arXiv:1411.0166, arXiv:1505.03135, arXiv:1508.00884 & more to come
with Simon Catterall, Poul Damgaard, Tom DeGrand and Joel Giedt

Context: Why lattice supersymmetry

Lattice discretization provides non-perturbative,
gauge-invariant regularization of vectorlike gauge theories

Amenable to numerical analysis
$\longrightarrow$ complementary approach to study strongly coupled field theories

Proven success for QCD; many potential susy applications:

- Compute Wilson loops, spectrum, scaling dimensions, etc.,
complementing perturbation theory, holography, bootstrap, ...
- Further direct checks of conjectured dualities
- Predict low-energy constants from dynamical susy breaking
- Validate or refine AdS/CFT-based modelling
(e.g., QCD phase diagram, condensed matter systems)

Context: Why lattice supersymmetry

Lattice discretization provides non-perturbative,
gauge-invariant regularization of vectorlike gauge theories

Amenable to numerical analysis
$\longrightarrow$ complementary approach to study strongly coupled field theories

Proven success for QCD; many potential susy applications:

- Compute Wilson loops, spectrum, scaling dimensions, etc., complementing perturbation theory, holography, bootstrap, ...
- Further direct checks of conjectured dualities
- Predict low-energy constants from dynamical susy breaking
- Validate or refine AdS/CFT-based modelling (e.g., QCD phase diagram, condensed matter systems)

Many ideas probably infeasible... Relatively few have been explored

Context: Why not lattice supersymmetry

Problem: $\left\{Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}}\right\} = 2\delta^{\mathrm{IJ}}\sigma^{\mu}_{\alpha\dot{\alpha}}P_{\mu}$
but infinitesimal translations don't exist in discrete space-time

Broken algebra $\Longrightarrow$ relevant susy-violating operators
(typically many, especially with scalar fields)

Fine-tuning their couplings to restore supersymmetry
is generally not practical in numerical lattice calculations

Solution: Preserve (some subset of) the susy algebra on the lattice
Possible for 4d $\mathcal{N} = 4$ SYM, and lower-dim. systems

Same lattice formulation obtained from orbifolding / deconstruction
and from "topological" twisting — cf. arXiv:0903.4881 for review

Exact susy on the lattice

Intuitive picture of Geometric-Langlands twist for $\mathcal{N} = 4$ SYM

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}}\gamma_5$$
$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a \gamma_a + \mathcal{Q}_{ab}\gamma_a\gamma_b$$
with $a, b = 1, \cdots, 5$

$\mathcal{Q}$'s transform with **integer spin** under "twisted rotation group"

$$\text{SO}(4)_{tw} \equiv \text{diag}\left[\text{SO}(4)_{\text{euc}} \otimes \text{SO}(4)_R\right] \qquad\qquad \text{SO}(4)_R \subset \text{SO}(6)_R$$

This change of variables gives a susy subalgebra $\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$

This subalgebra can be exactly preserved on the lattice

Twisted $\mathcal{N} = 4$ SYM fields and $\mathcal{Q}$

Everything transforms with **integer spin** under $\mathrm{SO}(4)_{tw}$ — **no spinors**

$$Q^{\mathrm{I}}_{\alpha} \text{ and } \overline{Q}^{\mathrm{I}}_{\dot{\alpha}} \longrightarrow \mathcal{Q},\ \mathcal{Q}_a \text{ and } \mathcal{Q}_{ab}$$

$$\Psi^{\mathrm{I}} \text{ and } \overline{\Psi}^{\mathrm{I}} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab}$$

$$A_{\mu} \text{ and } \Phi^{\mathrm{IJ}} \longrightarrow \mathcal{A}_a = (A_{\mu}, \phi) + i(B_{\mu}, \overline{\phi}) \text{ and } \overline{\mathcal{A}}_a$$

The twisted-scalar supersymmetry $\mathcal{Q}$ acts as

$$\mathcal{Q}\,\mathcal{A}_a = \psi_a \qquad\qquad \mathcal{Q}\,\psi_a = 0$$

$$\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\,\overline{\mathcal{A}}_a = 0$$

$$\mathcal{Q}\,\eta = d \qquad\qquad \mathcal{Q}\,d = 0$$

bosonic auxiliary field with e.o.m. $d = \overline{\mathcal{D}}_a \mathcal{A}_a$

1. $\mathcal{Q}$ directly interchanges bosonic $\longleftrightarrow$ fermionic d.o.f.
2. The susy subalgebra $\mathcal{Q}^2 \cdot = 0$ is manifest

Lattice $\mathcal{N} = 4$ SYM

The lattice theory is nearly a direct transcription,

despite breaking the 15 $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

- Covariant derivatives $\longrightarrow$ finite difference operators
- Complexified gauge fields $\mathcal{A}_a \longrightarrow$ gauge links $\mathcal{U}_a$

$$\mathcal{Q}\,\mathcal{A}_a \longrightarrow \mathcal{Q}\,\mathcal{U}_a = \psi_a \qquad\qquad \mathcal{Q}\,\psi_a = 0$$

$$\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\,\overline{\mathcal{A}}_a \longrightarrow \mathcal{Q}\,\overline{\mathcal{U}}_a = 0$$

$$\mathcal{Q}\,\eta = d \qquad\qquad \mathcal{Q}\,d = 0$$

(Complexification $\Longrightarrow$ U(N) = SU(N) $\otimes$ U(1) gauge invariance)

- Supersymmetric lattice action ($\mathcal{Q}S = 0$)

  follows from $\mathcal{Q}^2 \cdot = 0$ and Bianchi identity

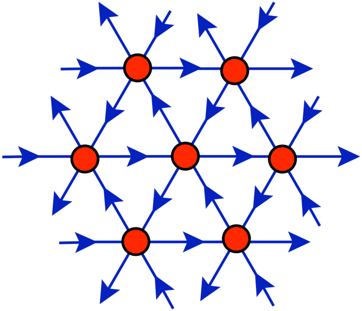
$$S = \frac{N}{2\lambda_{\text{lat}}}\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{N}{8\lambda_{\text{lat}}}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de}$$

Five links in four dimensions $\longrightarrow A_4^*$ lattice

—Can picture A_4^* lattice
as 4d analog of 2d triangular lattice

—Basis vectors are non-orthogonal
and linearly dependent

—Preserves S_5 point group symmetry



S_5 irreps precisely match onto irreps of twisted $\mathrm{SO}(4)_{tw}$

$$\mathbf{5} = \mathbf{4} \oplus \mathbf{1}: \quad \psi_a \longrightarrow \psi_\mu, \ \ \overline{\eta}$$

$$\mathbf{10} = \mathbf{6} \oplus \mathbf{4}: \quad \chi_{ab} \longrightarrow \chi_{\mu\nu}, \ \ \overline{\psi}_\mu$$

$S_5 \longrightarrow \mathrm{SO}(4)_{tw}$ in continuum limit restores the rest of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Twisted $\mathcal{N} = 4$ SYM on the A_4^* lattice

High degree of exact symmetry: gauge invariance + $\mathcal{Q}$ + S_5

Several important analytic consequences:

- Moduli space preserved to all orders of lattice perturbation theory $\longrightarrow$ no scalar potential induced by radiative corrections
- β function vanishes at one loop in lattice perturbation theory
- Real-space RG blocking transformations preserve $\mathcal{Q}$ and S_5
- Only one log. tuning to recover $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ in the continuum

Not quite suitable for numerical calculations

Exact zero modes and flat directions must be regulated,

especially important in U(1) sector

Stabilized lattice action

[arXiv:1505.03135]

$$S = \frac{N}{2\lambda_{\text{lat}}} \mathcal{Q} \left(\chi_{ab} \mathcal{F}_{ab} + \quad \downarrow \quad - \frac{1}{2} \eta d \right) - \frac{N}{8\lambda_{\text{lat}}} \epsilon_{abcde}\ \chi_{ab} \overline{\mathcal{D}}_c\ \chi_{de} + \mu^2 V$$

$$\eta \left(\overline{\mathcal{D}}_a \mathcal{U}_a + G \sum_{\mathcal{P}} \left[\det \mathcal{P} - 1 \right] \mathbb{I}_N \right)$$

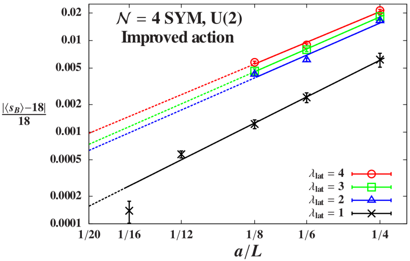
—Scalar potential $V = \frac{1}{2N\lambda_{\text{lat}}} \left(\text{Tr} \left[\mathcal{U}_a \overline{\mathcal{U}}_a \right] - N \right)^2$ lifts SU(N) flat directions

—Constraint on plaquette det. lifts U(1) zero mode & flat directions
Crucial that $\det \mathcal{P}$ deformation preserves $\mathcal{Q}$

Scalar potential **softly** breaks $\mathcal{Q}$

Ward identity violations $\langle \mathcal{Q}\mathcal{O} \rangle \propto a^2$
approaching $a \to 0$ continuum limit

Effective $\mathcal{O}(a)$ improvement
since $\mathcal{Q}$ forbids all dim-5 operators



Physics result: Static potential is Coulombic at all λ

Extract static potential $V(r)$ from rectangular $(r \times T)$ Wilson loops

$$W(r, T) \propto e^{-V(r)\,T}$$

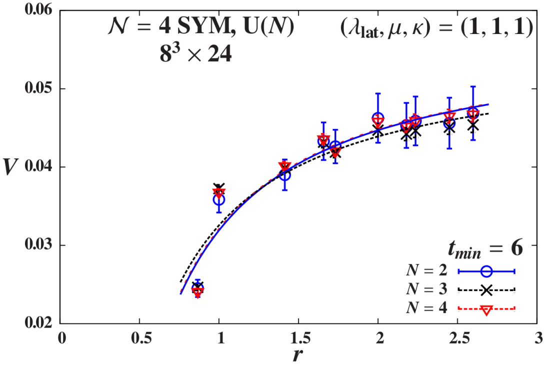
Fit $V(r)$ to Coulombic or confining form

$$V(r) = A - C/r$$

$$V(r) = A - C/r + \sigma r$$

C is Coulomb coefficient

σ is string tension

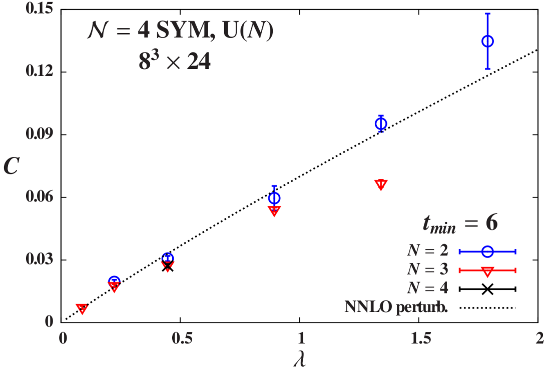


Fits to confining form always produce vanishing string tension $\sigma = 0$

Coupling dependence of Coulomb coefficient

Weak-coupling perturbation theory predicts $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

AdS/CFT predicts $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$, $\lambda \to \infty$, $\lambda \ll N$



$N = 2$ results agree with perturbation theory for $\lambda \lesssim 2$

$N = 3$ results bend down for $\lambda \gtrsim 1$ — approaching AdS/CFT?

Physics result: Konishi operator scaling dimension

The Konishi operator is the simplest conformal primary operator

$$\mathcal{O}_K = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}\Phi^{\mathrm{I}}\right] \qquad\qquad C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

There are many predictions for its scaling dim. $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$

- From weak-coupling perturbation theory,
  related to strong coupling by $\frac{4\pi N}{\lambda} \longleftrightarrow \frac{\lambda}{4\pi N}$ S duality
- From holography for $N \to \infty$ and $\lambda \to \infty$ but $\lambda \ll N$
- Upper bounds from the conformal bootstrap program

Only lattice gauge theory can access nonperturbative λ at moderate N

Konishi operator on the lattice

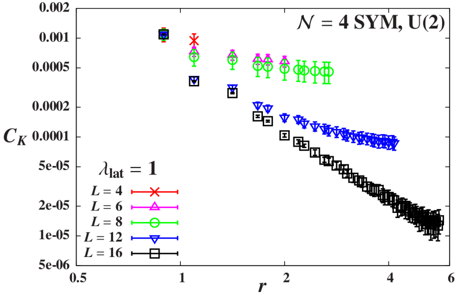
Extract scalar fields from polar decomposition of complexified links

$$\mathcal{U}_a \simeq U_a \left(\mathbb{I}_N + \varphi_a\right) \qquad \widehat{\mathcal{O}}_K = \sum_a \text{Tr}\left[\varphi_a \varphi_a\right] \qquad \overline{\mathcal{O}}_K = \widehat{\mathcal{O}}_K - \left\langle \widehat{\mathcal{O}}_K \right\rangle$$

$$\overline{C}_K(r) = \overline{\mathcal{O}}_K(x+r)\overline{\mathcal{O}}_K(x) \propto r^{-2\Delta_K}$$

Obvious sensitivity to volume
as desired for conformal system

Applicable lattice techniques:
—Finite-size scaling (FSS)
—Monte Carlo RG (MCRG)



Promising preliminary results for Δ_K from FSS and MCRG analyses. . .

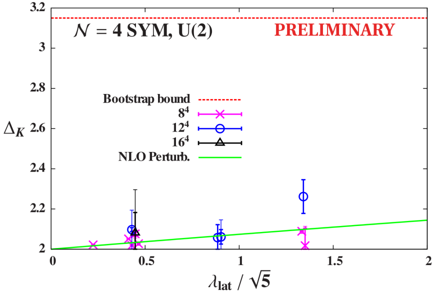
Konishi scaling dimension from Monte Carlo RG

Eigenvalues of MCRG stability matrix $\longrightarrow$ scaling dimensions

RG blocking parameter ξ set by matching plaquettes for L vs. $L/2$

Horizontally displaced points use different auxiliary couplings μ & G

Currently running larger λ_{lat} and larger $N = 3, 4$



Uncertainties from weighted histogram of results for...

- ⋆ 1 & 2 RG blocking steps
- ⋆ 1–5 operators in stability matrix
- ⋆ Blocked volumes 3^4 through 8^4

Recapitulation and outlook

Rapid recent progress in lattice supersymmetry

- Lattice promises non-perturbative insights from first principles
- Lattice $\mathcal{N} = 4$ SYM is practical thanks to exact $\mathcal{Q}$ susy
- Public code to reduce barriers to entry

Selected results from ongoing calculations

- Static potential is Coulombic at all couplings, $C(\lambda)$ confronted with perturbation theory and AdS/CFT
- Promising initial Konishi anomalous dimension at weak coupling

Many more directions are being — or can be — pursued

- Understanding the (absence of a) sign problem
- Systems with less supersymmetry, in lower dimensions, including matter fields, exhibiting spontaneous susy breaking, ...

Thank you!

Thank you!

Collaborators

Simon Catterall, Poul Damgaard, Tom DeGrand and Joel Giedt

Funding and computing resources

Supplement: Potential sign problem

In lattice gauge theory we compute operator expectation values

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O}\, e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]}\ \mathrm{pf}\, \mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

$\mathrm{pf}\, \mathcal{D} = |\mathrm{pf}\, \mathcal{D}| e^{i\alpha}$ can be complex for lattice $\mathcal{N} = 4$ SYM
$\longrightarrow$ Complicates interpretation of $\left[e^{-S_B}\ \mathrm{pf}\, \mathcal{D}\right]$ as Boltzmann weight

Instead compute phase-quenched (pq) observables and reweight:

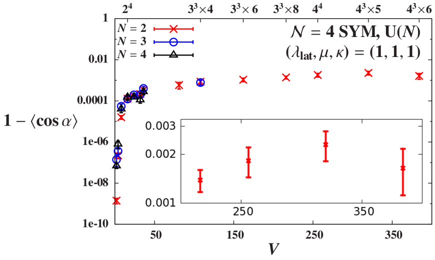
$$\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq} = \frac{1}{\mathcal{Z}_{pq}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O} e^{i\alpha}\, e^{-S_B}\, |\mathrm{pf}\, \mathcal{D}| \qquad \Longrightarrow \langle \mathcal{O} \rangle = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq}}{\left\langle e^{i\alpha} \right\rangle_{pq}}$$

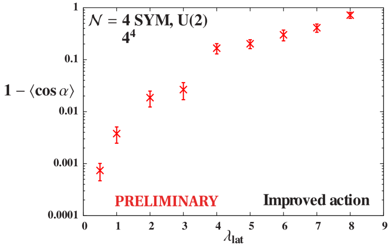
Sign problem: This breaks down if $\left\langle e^{i\alpha} \right\rangle_{pq}$ is consistent with zero

Pfaffian phase dependence on volume and coupling

Left: $1 - \langle\cos(\alpha)\rangle \ll 1$ independent of volume and N at $\lambda_{\rm lat} = 1$

Right: New 4^4 results at $4 \leq \lambda_{\rm lat} \leq 8$ show much larger fluctuations





Currently filling in more volumes and N

Extremely expensive analysis despite new parallel algorithm:
$\sim$40$\times$50 hours per 4^4 point; $\mathcal{O}(n^3)$ scaling

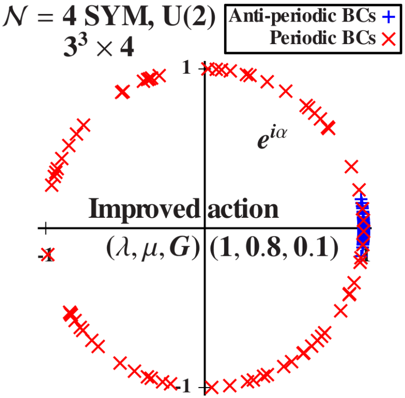
Two puzzles posed by the sign problem

- With periodic temporal boundary conditions for the fermions
  we have an obvious sign problem, $\left\langle e^{i\alpha} \right\rangle_{pq}$ consistent with zero
- With anti-periodic BCs and all else the same $e^{i\alpha} \approx 1$,
  phase reweighting has negligible effect

Why such sensitivity to the BCs?

Also, other observables
are nearly identical
for these two ensembles

Why doesn't the sign problem
have observable effects?



Backup: Public code for lattice $\mathcal{N} = 4$ SYM

so that the full improved action becomes

$$
\begin{aligned}
S_{\text{imp}} &= S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \qquad (3.10)\\
S'_{\text{exact}} &= \frac{N}{2\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) \\
&\qquad + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{\text{det}} \\
S_{\text{det}} &= \frac{N}{2\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right] \\
S_{\text{closed}} &= -\frac{N}{8\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right], \\
S'_{\text{soft}} &= \frac{N}{2\lambda_{\text{lat}}}\mu^2 \sum_n \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2
\end{aligned}
$$

The lattice action is obviously very complicated

(For experts: The fermion operator involves $\gtrsim$100 gathers)

To reduce barriers to entry our parallel code is publicly developed at

github.com/daschaich/susy

Evolved from MILC lattice QCD code, presented in arXiv:1410.6971

Backup: Failure of Leibnitz rule in discrete space-time

Given that $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu = 2i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu$ is problematic,

why not try $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2i\sigma^{\mu}_{\alpha\dot{\alpha}}\nabla_\mu$ for a discrete translation?

Here $\nabla_\mu \phi(x) = \frac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu \phi(x) + \frac{a}{2}\partial_\mu^2 \phi(x) + \mathcal{O}(a^2)$

Essential difference between ∂_μ and ∇_μ on the lattice, $a > 0$

$$
\begin{aligned}
\nabla_\mu \left[\phi(x)\chi(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\chi(x + a\widehat{\mu}) - \phi(x)\chi(x)\right] \\
&= \left[\nabla_\mu \phi(x)\right]\chi(x) + \phi(x)\nabla_\mu \chi(x) + a\left[\nabla_\mu \phi(x)\right]\nabla_\mu \chi(x)
\end{aligned}
$$

We only recover the Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ “Discrete supersymmetry” breaks down on the lattice

(Dondi & Nicolai, “Lattice Supersymmetry”, 1977)

Backup: Twisting $\longleftrightarrow$ Kähler–Dirac fermions

The Kähler–Dirac representation is related to the spinor $Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{I}}_{\dot\alpha}$ by

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}}\gamma_5$$

$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a \gamma_a + \mathcal{Q}_{ab}\gamma_a\gamma_b$$

with $a, b = 1, \cdots, 5$

The 4×4 matrix involves R symmetry transformations along each row and (euclidean) Lorentz transformations along each column

$\Longrightarrow$ Kähler–Dirac components transform under "twisted rotation group"

$$\mathrm{SO}(4)_{tw} \equiv \mathrm{diag}\left[\mathrm{SO}(4)_{\mathrm{euc}} \otimes \underbrace{\mathrm{SO}(4)_R}_{\uparrow}\right]$$

only $\mathrm{SO}(4)_R \subset \mathrm{SO}(6)_R$

Backup: A_4^* lattice with five links in four dimensions

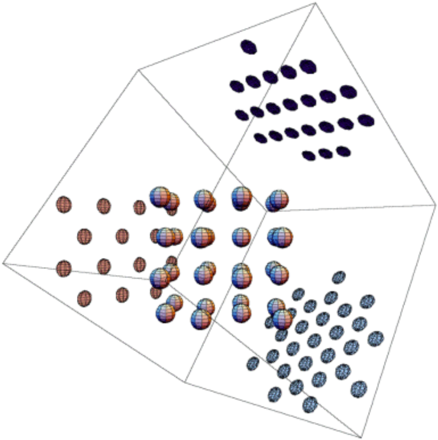
$A_a = (A_\mu, \phi)$ may remind you of dimensional reduction

On the lattice we want to treat all five $\mathcal{U}_a$ symmetrically
to obtain $S_5 \longrightarrow \text{SO}(4)_{tw}$ symmetry

—Start with hypercubic lattice
in 5d momentum space

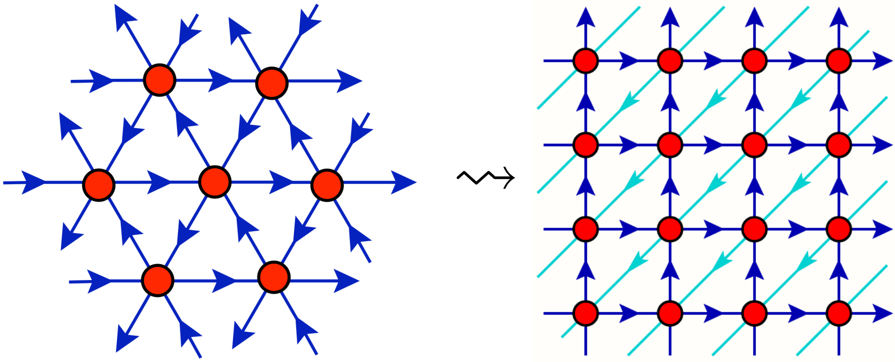
—**Symmetric** constraint $\sum_a \partial_a = 0$
projects to 4d momentum space

—Result is A_4 lattice
$\longrightarrow$ dual A_4^* lattice in real space



Backup: Hypercubic representation of A_4^* lattice

In the code it is very convenient to represent the A_4^* lattice
as a hypercube with a backwards diagonal



Backup: More on flat directions

1. Complex gauge field $\Longrightarrow$ U(N) $=$ SU(N) $\otimes$ U(1) gauge invariance
   U(1) sector decouples only in continuum limit

2. $\mathcal{Q}\,\mathcal{U}_a = \psi_a \Longrightarrow$ gauge links must be elements of algebra
   Resulting flat directions required by supersymmetric construction
   but must be lifted to ensure $\mathcal{U}_a = \mathbb{I}_N + \mathcal{A}_a$ in continuum limit

We need to add two deformations to regulate flat directions

SU(N) scalar potential $\propto \mu^2 \sum_a \left(\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - N\right)^2$

U(1) plaquette determinant $\sim G \sum_{a \neq b} \left(\det \mathcal{P}_{ab} - 1\right)$

Scalar potential **softly** breaks $\mathcal{Q}$ supersymmetry

susy-violating operators vanish as $\mu^2 \to 0$

Plaquette determinant can be made $\mathcal{Q}$-invariant [arXiv:1505.03135]

Backup: One problem with flat directions

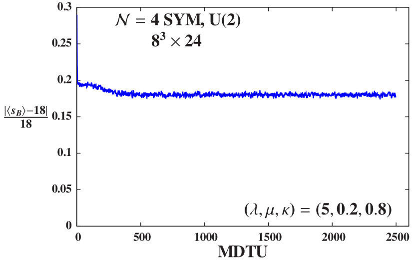
Gauge fields $\mathcal{U}_a$ can move far away from continuum form $\mathbb{I}_N + \mathcal{A}_a$

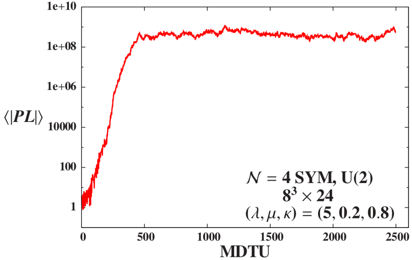
if $N\mu^2/(2\lambda_{\text{lat}})$ becomes too small

Example for two-color $(\lambda_{\text{lat}}, \mu, \kappa) = (5, 0.2, 0.8)$ on $8^3 \times 24$ volume

Left: Bosonic action is stable ~18% off its supersymmetric value

Right: Polyakov loop wanders off to $\sim 10^9$

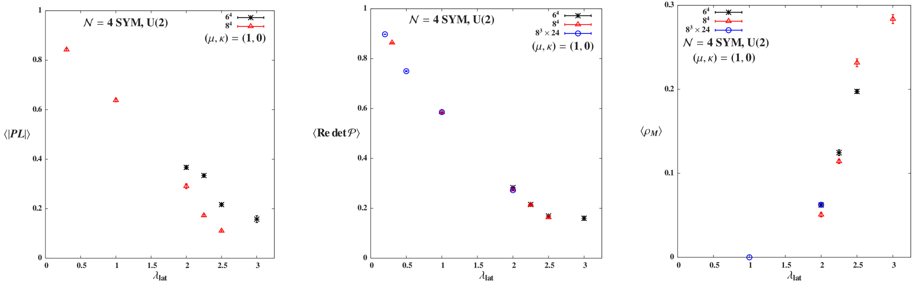




Backup: Another problem with U(1) flat directions

Flat directions in U(1) sector can induce transition to confined phase

This lattice artifact is not present in continuum $\mathcal{N} = 4$ SYM



Around the same $\lambda_{\text{lat}} \approx 2\ldots$

Left: Polyakov loop falls towards zero

Center: Plaquette determinant falls towards zero

Right: Density of U(1) monopole world lines becomes non-zero

Backup: Soft susy breaking

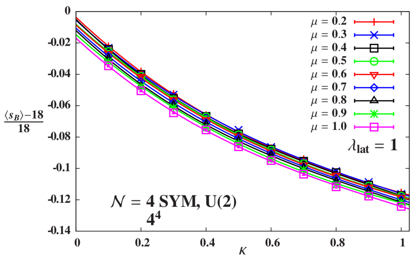
Before 2015 we added the $\det \mathcal{P}$ constraint directly to the lattice action

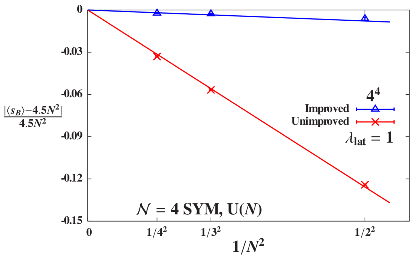
$$S_{soft} = \frac{N}{2\lambda_{\text{lat}}}\mu^2 \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 + \kappa \left|\det \mathcal{P}_{ab} - 1\right|^2$$

Both terms explicitly break $\mathcal{Q}$ but $\det \mathcal{P}_{ab}$ effects dominate

Left: The breaking is **soft** — guaranteed to vanish as $\mu, \kappa \longrightarrow 0$

Right: Soft $\mathcal{Q}$ breaking is also suppressed $\propto 1/N^2$





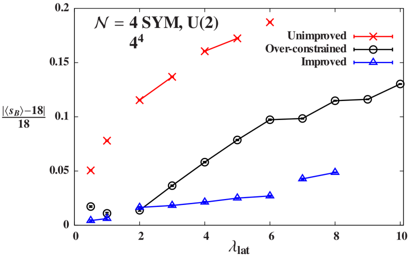
Backup: More on supersymmetric constraints

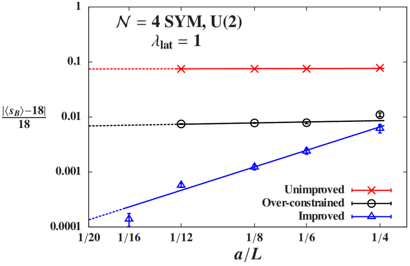
arXiv:1505.03135 introduces method to impose $\mathcal{Q}$-invariant constraints

Basic idea: Modify aux. field equations of motion $\longrightarrow$ moduli space

$$d(n) = \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) \longrightarrow \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) + G\mathcal{O}(n)\mathbb{I}_N$$

Applied to plaquette determinant, $\mathcal{O}(n) = \sum_{a \neq b}\left[\det \mathcal{P}_{ab}(n) - 1\right]$,
produces much smaller violations of $\mathcal{Q}$ Ward identity $\langle s_B \rangle = 9N^2/2$





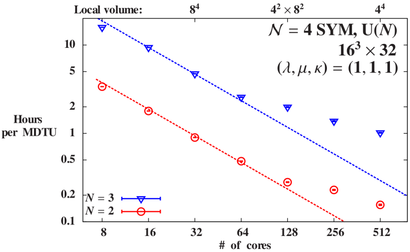
Backup: Code performance—weak and strong scaling

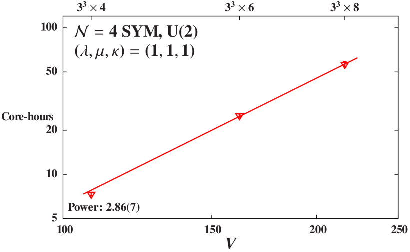
These results from arXiv:1410.6971 use an old (“unimproved”) action

Left: Strong scaling for U(2) and U(3) $16^3 \times 32$ RHMC

Right: Weak scaling for $\mathcal{O}(n^3)$ pfaffian calculation (fixed local volume)

$n \equiv 16N^2L^3N_T$ is number of fermion degrees of freedom





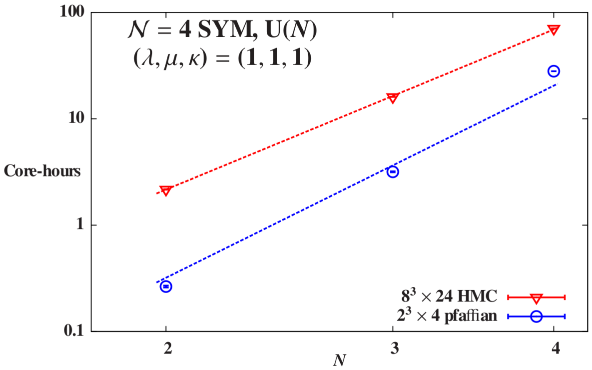
Both plots on log–log axes with power-law fits

Backup: Numerical costs for $N = 2$, 3 and 4 colors

Red: Find RHMC cost scaling $\sim N^5$ (recall adjoint fermion d.o.f. $\propto N^2$)

Blue: Pfaffian cost scaling consistent with expected N^6

Additional factor of $\sim 2\times$ from new improved action, but same scaling

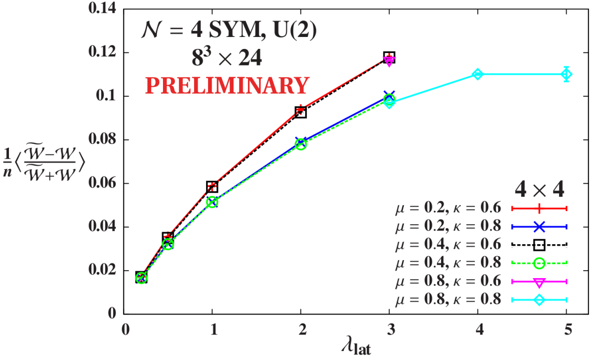


Backup: Restoration of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ supersymmetries

Results from arXiv:1411.0166 to be revisited with the new action

Restoration of the other 15 $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ in the continuum limit follows from restoration of R symmetry (motivation for A_4^* lattice)

Modified Wilson loops test R symmetries at non-zero lattice spacing

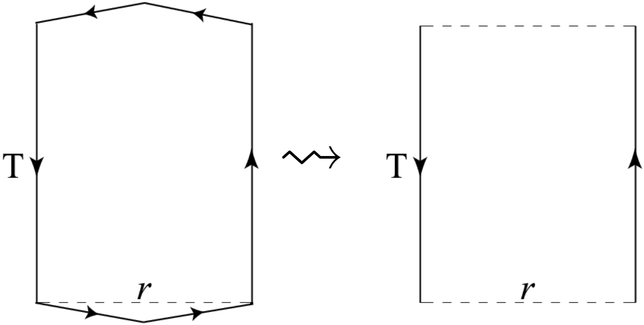


Backup: $\mathcal{N} = 4$ static potential from Wilson loops

Extract static potential $V(r)$ from $r \times T$ Wilson loops

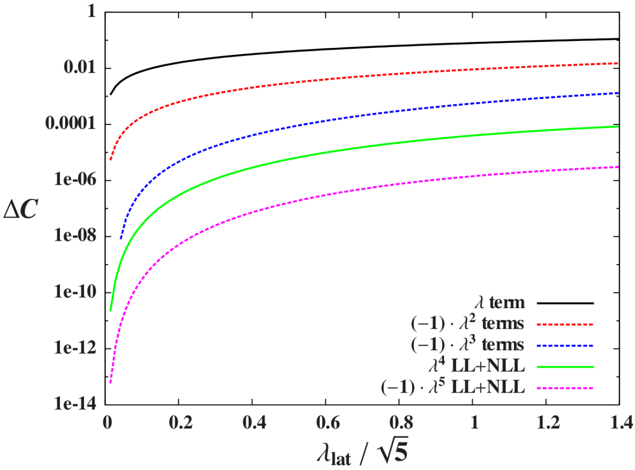
$$W(r, T) \propto e^{-V(r)\ T} \qquad\qquad V(r) = A - C/r + \sigma r$$

Coulomb gauge trick from lattice QCD reduces A_4^* lattice complications



Backup: Perturbation theory for Coulomb coefficient

For range of couplings currently being studied
(continuum) perturbation theory for $C(\lambda)$ is well behaved

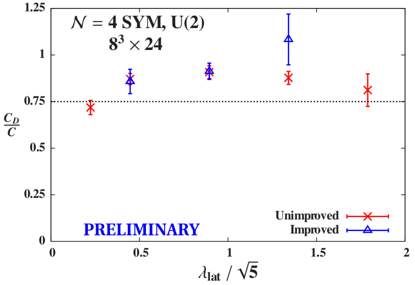


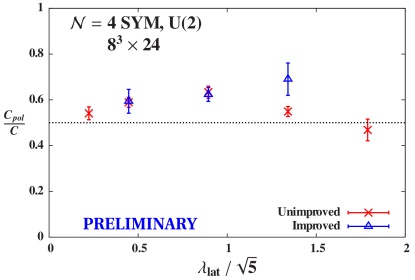
Backup: More tests of the U(2) static potential

Left: Projecting Wilson loops from U(2) $\longrightarrow$ SU(2)

$\Longrightarrow$ factor of $\frac{N^2-1}{N^2} = 3/4$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$





Some results slightly above expected factors,

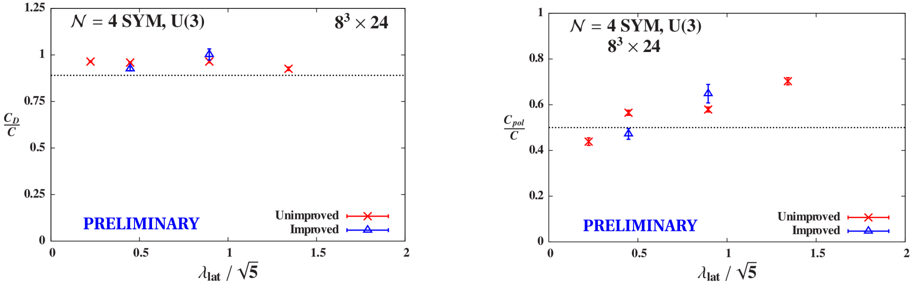
may be related to non-zero auxiliary couplings μ and κ / G

Backup: More tests of the U(3) static potential

Left: Projecting Wilson loops from U(3) $\longrightarrow$ SU(3)

$\Longrightarrow$ factor of $\frac{N^2-1}{N^2} = 8/9$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$



Some results slightly above expected factors,

may be related to non-zero auxiliary couplings μ and κ / G

Backup: Smearing for Konishi analyses

—As in glueball analyses, operator basis enlarged through smearing

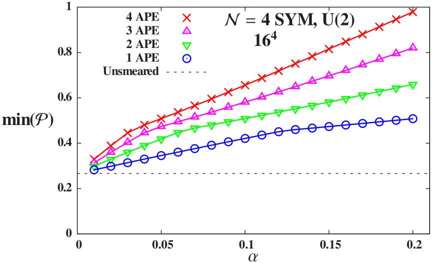
—Use APE-like smearing $(1-\alpha)\,\text{—} + \frac{\alpha}{8}\sum \sqcap$,

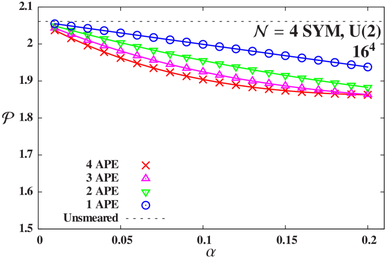
with staples built from unitary parts of links but no final unitarization

(unitarized smearing — e.g. stout — doesn't affect Konishi)

—Average plaquette is stable upon smearing (**right**)

while minimum plaquette steadily increases (**left**)





Backup: Scaling dimensions from Monte Carlo RG

Write system as (infinite) sum of operators $\mathcal{O}_i$ with couplings c_i

Couplings c_i flow under RG blocking transformation R_b

n-times-blocked system is $H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}$

Consider linear expansion around fixed point $H^\star$ with couplings $c_i^\star$

$$c_i^{(n)} - c_i^\star = \sum_j \left. \frac{\partial c_i^{(n)}}{\partial c_j^{(n-1)}} \right|_{H^\star} \left(c_j^{(n-1)} - c_j^\star \right) \equiv \sum_j T_{ij}^\star \left(c_j^{(n-1)} - c_j^\star \right)$$

$T_{ij}^\star$ is the stability matrix

Eigenvalues of $T_{ij}^\star \longrightarrow$ scaling dimensions of corresponding operators