Lattice gauge theory for composite Higgs

David Schaich (Syracuse)

SUSY 2015, Lake Tahoe, 28 August

Motivations

Lattice discretization provides non-perturbative,
gauge-invariant regularization of vectorlike gauge theories

Amenable to numerical analysis
$\longrightarrow$ complementary approach to study strongly coupled field theories

Proven success for QCD

Composite Higgs requires non-QCD-like strong dynamics,
probably featuring approximate conformality

Lattice calculations become more crucial
when we can't exploit intuition from QCD

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Plan for this talk

Main goal

"Demystify" some aspects of lattice calculations
to encourage more interplay between lattice and continuum pheno

1. Essential features of the lattice approach
2. Selected applications to composite Higgs framework

—Composite spectrum

($\mathcal{O}$(100 GeV) Higgs boson with 2–3 TeV resonances)

—Low-energy constants of effective theory

(S parameter smaller than scaled-up QCD)

—Anomalous dimensions (Large $\gamma_m(\mu)$ over wide range of scales)

Additional resources

There are typically 20–30 BSM talks at the annual Lattice conference plus a plenary review of progress and prospects

An extended version of the most recent review appeared in the Snowmass 2013 proceedings [arXiv:1309.1206]

PROCEEDINGS OF SCIENCE

a service of SISSA

The 32nd International Symposium on Lattice Field Theory

LATTICE2014 - (other lattice conferences)

23-28 June, 2014
Columbia University New York, NY

Physics Beyond the Standard Model

Models of Walking Technicolor on the Lattice
PoS(LATTICE2014)239 pdf *D. Sinclair and J.B. Kogut*

Phase Structure Study of SU(2) Lattice Gauge Theory with 8 flavours
PoS(LATTICE2014)240 pdf *C.Y.H. Huang, C.J.D. Lin, K. Ogawa, H. Ohki and E. Rinaldi*

SU(2) gauge theory with many flavors of domain-wall fermions
PoS(LATTICE2014)241 pdf *H. Matsufuru, K. Nagai and N. Yamada*

Approaching Conformality
PoS(LATTICE2014)242 pdf *M.P. Lombardo, K. Miura, T. Nunes da Silva and E. Pallante*

Walking technicolor: testing infra-red conformality with exact results in two dimensions
PoS(LATTICE2014)243 pdf *O. Akerlund and P. de Forcrand*

Toward the minimal realization of a light composite Higgs
PoS(LATTICE2014)244 pdf *J. Kuti, Z. Fodor, K. Holland, S. Mondal, D. Nogradi and C.H. Wong*

arXiv.org > hep-lat > arXiv:1309.1206

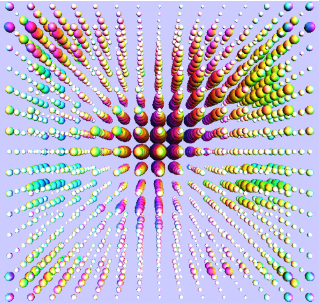
Search or Article-id

High Energy Physics - Lattice

Lattice Gauge Theories at the Energy Frontier

Thomas Appelquist, Richard Brower, Simon Catterall, George Fleming, Joel Giedt, Anna Hasenfratz, Julius Kuti, Ethan Neil, David Schaich

Essence of numerical lattice calculations



Evaluate observables from functional integral via importance sampling Monte Carlo

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \;\; \mathcal{O}(U) \;\; e^{-S[U]}}{\int \mathcal{D}U \;\; e^{-S[U]}}$$

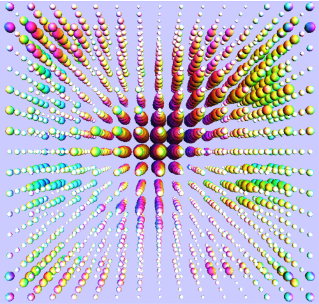
$$\longrightarrow \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U_n)$$

U are field configurations in discretized euclidean spacetime

$S[U]$ is the action, which should be real and positive so that e^{-S} can be treated as a probability distribution

The hybrid Monte Carlo algorithm samples U with probability $\propto e^{-S}$

Essence of numerical lattice calculations



Evaluate observables from functional integral
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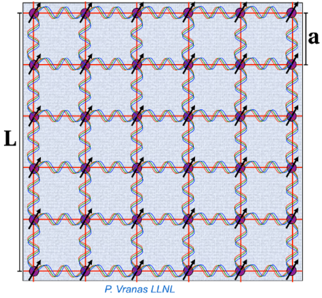
$$\longrightarrow \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U_n)$$

After generating and saving an ensemble $\{U_n\}$ distributed $\propto e^{-S}$
it is usually quick and easy to measure many observables $\langle \mathcal{O} \rangle$

However, changing the action S requires generating a new ensemble

Lattice action and physical limit



S includes **only** the new strong sector
at UV cutoff scale $\frac{1}{a}$ (inverse lattice spacing)

Electroweak (&c.) couplings incorporated
after lattice calculation

The UV cutoff $\frac{1}{a}$ is removed in the continuum limit $a \to 0$

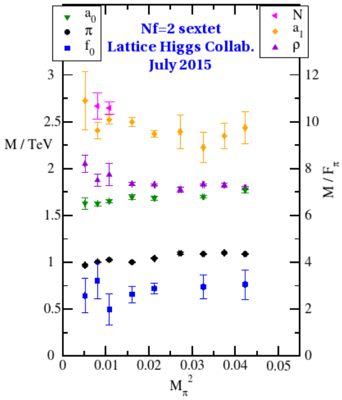
No electroweak $\longrightarrow$ uneaten Goldstones $\longrightarrow$ IR divergences

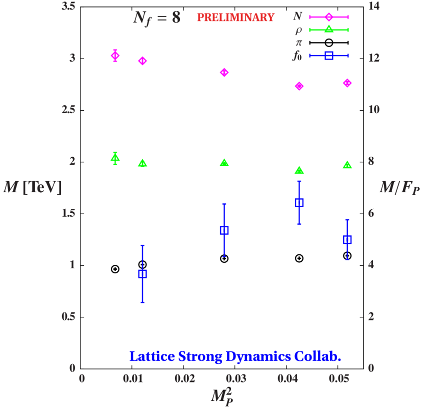
Typically regulate through bare fermion mass m

Requires extrapolation to chiral limit $m \to 0$,
which proves especially challenging in the composite Higgs context

Light Higgs complicates chiral extrapolations

Recent lattice studies find light composite scalars
in several near-conformal systems [arXiv:1403.5000, arXiv:1502.00028]



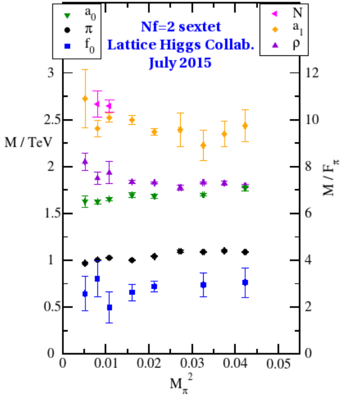


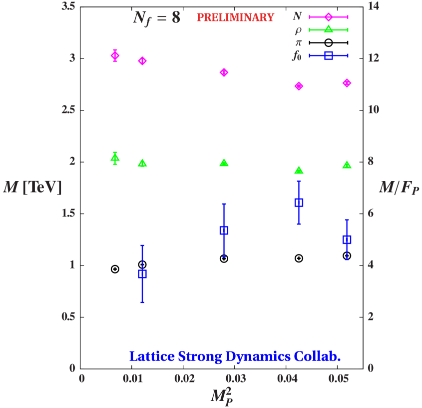
Dramatically different from QCD

Scalar lighter than or degenerate with pion!

Light Higgs complicates chiral extrapolations

Typical chiral extrapolation integrates out everything except pions,
can't reliably be applied to these data





Work in progress to extend chiral effective theory,
but no solid approach yet

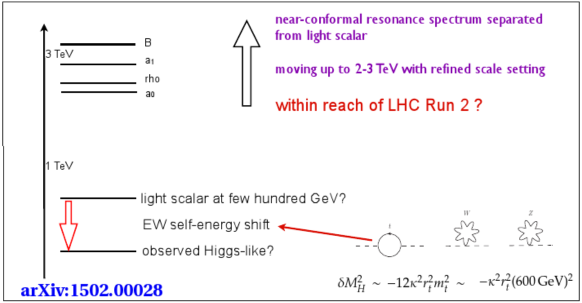
Status of light composite Higgs from lattice

Without reliable chiral extrapolation we can only estimate
$M_H \sim$ few hundred GeV with uncontrolled uncertainty

Much lighter than scaled-up QCD, still somewhat far from 125 GeV

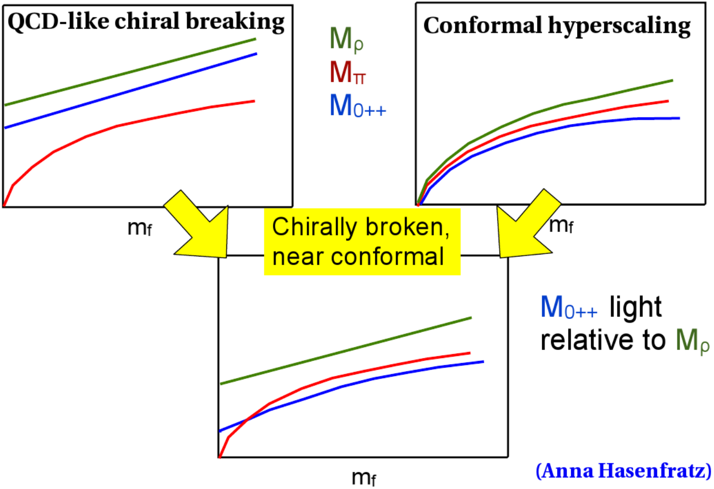
Of course, we **shouldn't** get exactly 125 GeV
since we haven't yet incorporated electroweak & top corrections

These should reduce M_H, but not yet consensus on size of effect...

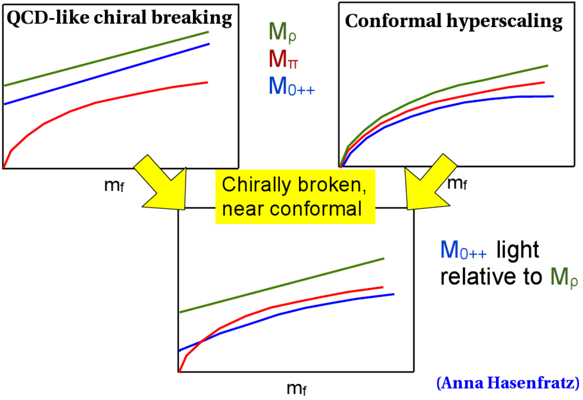


Emerging picture of spectrum from lattice calculations

Light scalar likely related to near-conformal dynamics
(unconfirmed interpretation as PNGB of approx. scale symmetry)



Emerging picture of spectrum from lattice calculations



Even if particular models considered so far are ruled out
experience with near-conformal strong dynamics may prove invaluable

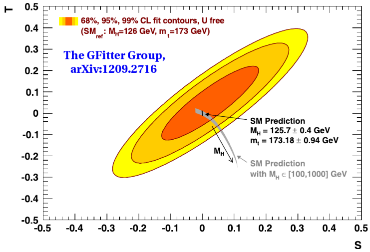
E.g.: Same UV theories can provide $SU(N_F)^2/SU(N_F)$ PNGB Higgs [arXiv:1506.00623]

Low-energy constants (LECs) of effective theory

Setting aside complications from the light Higgs,
let's consider the traditional electroweak chiral lagrangian $\mathcal{L}_\chi$

Predicting LECs of the low-energy effective theory
is a standard application of lattice gauge theory

Composite Higgs examples: the S parameter [arXiv:1405.4752]
and WW scattering parameters [arXiv:1201.3977]



S remains an important constraint
on new strong dynamics

Experiment: $S = 0.03 \pm 0.10$

Scaled-up QCD: $S \approx 0.43$

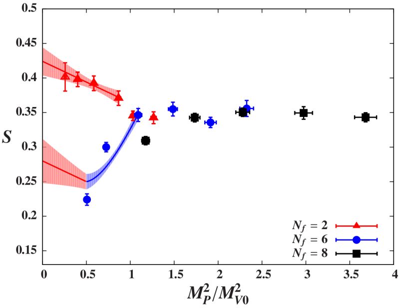
The S parameter on the lattice

$$\mathcal{L}_\chi \ni \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu} \right] \longrightarrow$$

$$S = -16\pi^2 \alpha_1 = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

Lattice provides strong dynamics vacuum polarization $\Pi_{V-A}(Q^2)$,

N_D and $\Delta S_{SM}(M_H)$ incorporate coupling to electroweak



$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

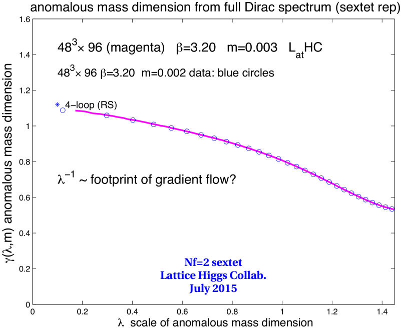
Significant reduction for larger N_F
approaching conformality

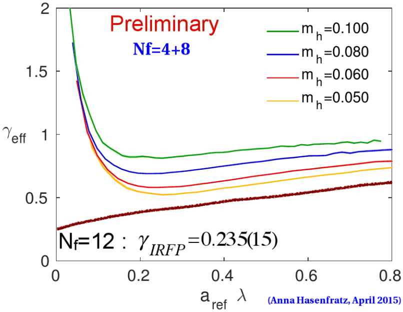
Chiral extrapolation becomes trickier

Mass anomalous dimension $\gamma_m = 3 - \mathrm{d}\left[\overline{\psi}\psi\right]$

Recently developed lattice methods predict scale-dependent γ_m

Large $\gamma_m \simeq 1$ over wide range of scales
helps new strong dynamics satisfy flavor constraints



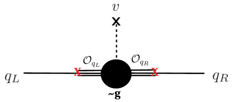


$N_F = 4+8$ applies emerging mixed-mass approach [arXiv:1411.3243]
that improves control over chiral extrapolation

Anomalous dimension $\gamma_3 = 4.5 - \mathrm{d}[\psi\psi\psi]$

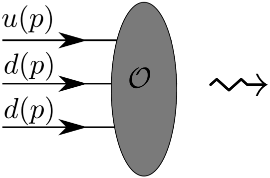
Fermionic $\mathcal{O} \sim \psi\psi\psi$ of new strong dynamics $\longrightarrow$ top partners (&c.)

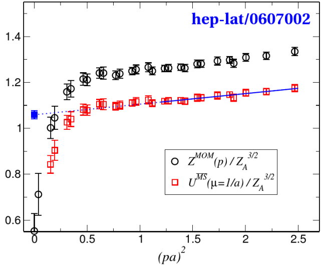
> Flavor from partial compositeness,
> linear coupling $\lambda_q q \mathcal{O}_q$ rather than $m_q \overline{q} q \overline{\psi} \psi$



Standard lattice QCD methods of non-perturbative renormalization

predict $Z_{\mathcal{O}}(\mu)$ from which $\gamma_{\mathcal{O}} = -\frac{d \log Z_{\mathcal{O}}(\mu)}{d \log \mu}$





Work underway to apply to near-conformal composite systems

Recapitulation and outlook

Lattice gauge theory

- Non-perturbative approach to study strong dynamics
- Many observables accessible for fixed UV completion
- Challenging extrapolation to chiral limit of near-conformal systems

Selected applications to composite Hggs

- Composite spectrum featuring light scalar
- Low-energy constants including smaller S parameter
- Anomalous dimensions featuring large γ_m

A few future directions

- d$[\psi\psi\psi]$ for direct connection to partial compositeness
- Emerging studies of mixed-mass systems
- Explorations of (pseudo)real reps [arXiv:1501.05665]
  for SU(N)/Sp(N) and SU(N)/SO(N) cosets

Thank you!

Thank you!

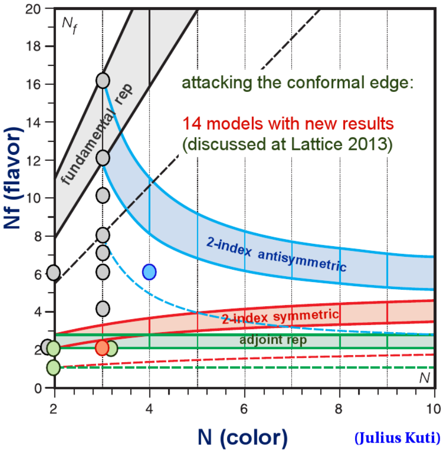
Input, plots, &c.

Kaustubh Agashe, Mike Buchoff, Anna Hasenfratz, Kieran Holland, Julius Kuti, Ethan Neil, Claudio Rebbi, Luca Vecchi, Pavlos Vranas, Evan Weinberg, Oliver Witzel, Ricky Wong, . . .

Funding and computing resources

Backup: Basic strategy for lattice studies beyond QCD

Systematically depart from solid ground of lattice QCD
($N = 3$ with $N_F = 2$ light flavors in fundamental rep)



—Add more light flavors
$\longrightarrow N_F = 8$ fundamental

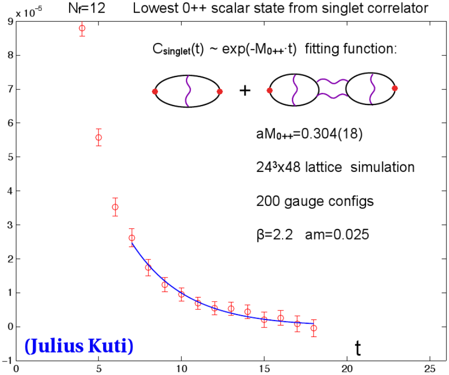
—Enlarge fermion rep
$\longrightarrow N_F = 2$ two-index symmetric

—Explore $N = 2$ and 4
$\longrightarrow$ (pseudo)real reps for cosets
SU(N)/Sp(N) and SU(N)/SO(N)

Backup: Technical lattice challenge for Higgs mass

Since only the new strong sector is included in the lattice calculation, the Higgs mixes with the vacuum

$\Longrightarrow$ Signal-to-noise problem $\longrightarrow$ large uncertainties in Higgs mass



Fermion propagator computation is relatively expensive

"Disconnected diagrams" need propagators at all L^4 sites

In practice compute simultaneously (stochastically)
$\longrightarrow$ results fairly noisy

Backup: Electroweak chiral lagrangian

At leading order $\mathcal{L}_\chi = \frac{F^2}{4}\text{Tr}\left[D_\mu U^\dagger D^\mu U\right] + \frac{F^2 B}{2}\text{Tr}\left[m\left(U + U^\dagger\right)\right]$

With $T \equiv U\tau_3 U^\dagger$ and $V_\mu \equiv (D_\mu U)\,U^\dagger$, next-to-leading order includes
oblique corrections $S \propto \alpha_1, \qquad T \propto \beta_1, \qquad U \propto \alpha_8$
triple gauge vertices and dominant contributions to WW scattering:

$$\mathcal{L}_1 = \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu}\text{Tr}\left(TW^{\mu\nu}\right) \qquad \mathcal{L}_2 = \frac{i\alpha_2}{2} g_1 B_{\mu\nu}\text{Tr}\left(T\left[V^\mu, V^\nu\right]\right)$$

$$\mathcal{L}_3 = i\alpha_3 g_2 \text{Tr}\left(W_{\mu\nu}\left[V^\mu, V^\nu\right]\right) \qquad \mathcal{L}_4 = \alpha_4\left\{\text{Tr}\left(V_\mu V_\nu\right)\right\}^2$$

$$\mathcal{L}_5 = \alpha_5\left\{\text{Tr}\left(V_\mu V^\mu\right)\right\}^2 \qquad \mathcal{L}_6 = \alpha_6 \text{Tr}\left(V_\mu V_\nu\right)\text{Tr}\left(TV^\mu\right)\text{Tr}\left(TV^\nu\right)$$

$$\mathcal{L}_7 = \alpha_7 \text{Tr}\left(V_\mu V^\mu\right)\text{Tr}\left(TV_\mu\right)\text{Tr}\left(TV^\nu\right) \qquad \mathcal{L}_8 = \frac{\alpha_8}{4} g_2^2\left\{\text{Tr}\left(TW_{\mu\nu}\right)\right\}^2$$

$$\mathcal{L}_9 = \frac{i\alpha_9}{2} g_2 \text{Tr}\left(TW_{\mu\nu}\right)\text{Tr}\left(T\left[V^\mu, V^\nu\right]\right) \qquad \mathcal{L}_{10} = \frac{\alpha_{10}}{2}\left\{\text{Tr}\left(TV_\mu\right)\text{Tr}\left(TV_\nu\right)\right\}^2$$

$$\mathcal{L}_{11} = \alpha_{11} g_2 \epsilon^{\mu\nu\rho\lambda}\text{Tr}\left(TV_\mu\right)\text{Tr}\left(V_\nu W_{\rho\lambda}\right) \qquad \mathcal{L}_1' = \frac{\beta_1}{4} g_2^2 F^2\left\{\text{Tr}\left(TV_\mu\right)\right\}^2$$

Backup: Vacuum polarization is just current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \text{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]; 0.71 [10f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Backup: Chiral perturbation theory for $\Pi_{V-A}(Q^2)$

$\Pi_{V-A}(Q^2)$ in hadronic χPT:

$$\Pi_{V-A}(M_{dd}^2, Q^2) = -F_P^2 - Q^2 \left[8L_{10}^r(\mu) + \frac{1}{24\pi^2}\left\{\log\left[\frac{M_{dd}^2}{\mu^2}\right] + \frac{1}{3} - H\left(\frac{4M_{dd}^2}{Q^2}\right)\right\}\right]$$

$$H(x) = (1+x)\left[\sqrt{1+x}\log\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1} + 2\right)\right]$$

Match with $S = -16\pi^2\alpha_1$ in electroweak chiral lagrangian:

$$S(\mu, M_{ds}) = \frac{1}{12\pi}\left[-192\pi^2\left(L_{10}^r(\mu) + \frac{1}{384\pi^2}\left\{\log\left[\frac{M_{ds}^2}{\mu^2}\right] + 1\right\}\right) + \log\left[\frac{\mu^2}{M_H}\right] - \frac{1}{6}\right].$$

Backup: More NLO chiral expansions

For general N_F, $A = 2 - N_F + 2N_F^2 + N_F^3$

$$M_P a_{PP} = -\frac{2mB}{16\pi F^2}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_{PP} - 2\frac{N_F - 1}{N_F^2} + \frac{A}{N_F^2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$M_P^2 = 2mB\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_M + \frac{1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$F_P = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_F - \frac{N_F}{2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

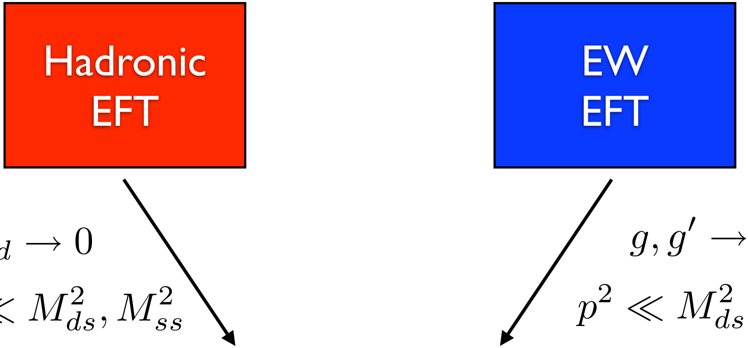
$$\langle\overline{\psi}\psi\rangle = \frac{F^2 2mB}{2m}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_C - \frac{N_F^2 - 1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

- LECs b are all linear combinations of low-energy constants L_i
- LECs' dependence on scale μ cancels the corresponding logs
- b_C includes "contact term" $m\Lambda^2 \sim m/a^2$
- NNLO M_P^2 coefficients enhanced by N_F^2

(arXiv:0910.5424)

Backup: EFT matching for WW scattering

—WW scattering guaranteed to contain information about EWSB
—Very direct probe (though **not** easiest) at LHC
—On the lattice, restricted to **low-energy** scattering



$m_d \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

$g, g' \to 0$

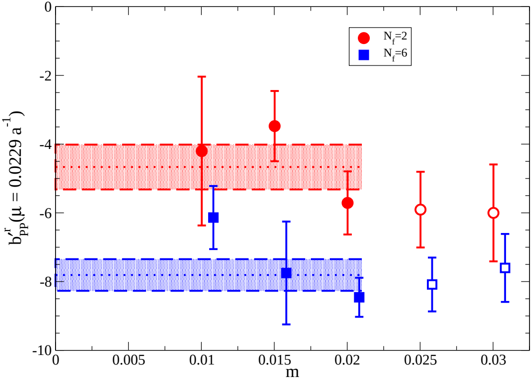
$p^2 \ll M_{ds}^2, M_{ss}^2$

$$\frac{f^2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$

Backup: Enhancement of WW scattering for $N_F = 6$

$$b'_{PP} = -256\pi^2 \left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$$

contains α_4 and α_5, but we aren't able to isolate them



$b'_{PP} = -4.67 \pm 0.65^{+1.08}_{-0.05}$ (2f); $b'_{PP} = -7.81 \pm 0.46^{+1.23}_{-0.56}$ (6f)

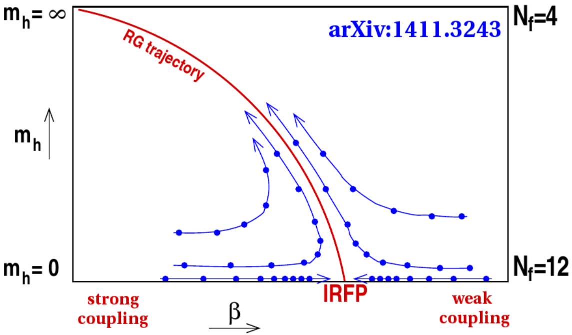
Larger $|b'_{PP}|$ for $N_F = 6$ corresponds to enhancement of WW scattering

Backup: Philosophy of mixed-mass approach

$N_F = N_\ell + N_h$ fermions, with light $m_\ell \to 0$ at fixed $m_h > 0$

Allows large N_F for approximate conformality
without introducing extra Goldstones

Reducing m_h extends the range of scales
over which theory is governed by conformal fixed point



Real-space RG flow lines
(from UV to IR)

γ_m above considered
strong-coupling side
$\longrightarrow$ "backward" flow