

# Advances in Smooth Heterotic String Theory

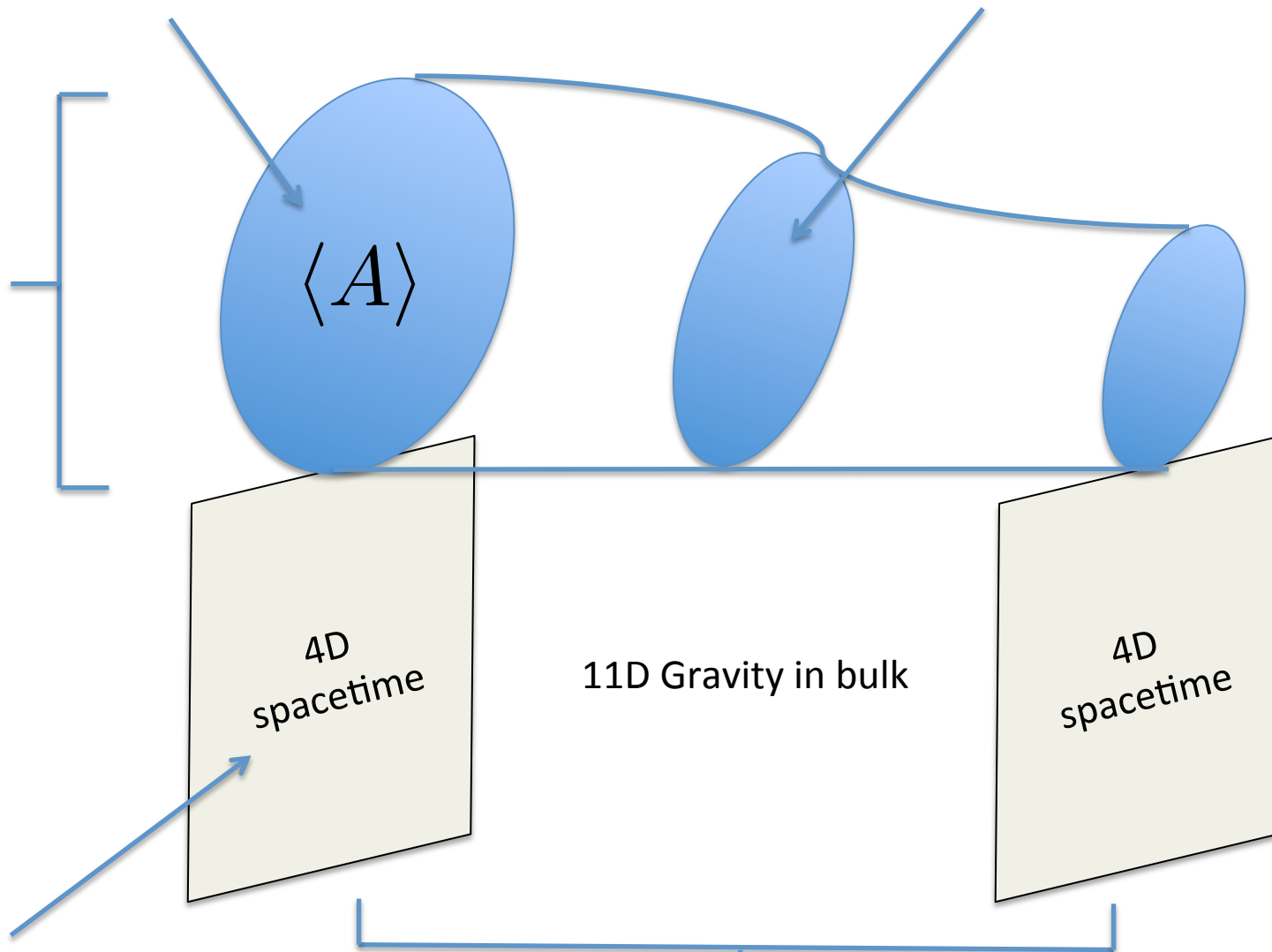
James Gray – Virginia Tech



Gauge fields “Higgs” E8 down to a GUT group.  
This is then broken by Wilson lines to SM.

6D manifold of SU(3)  
structure

Size  
determines  
gauge  
couplings



E8 Super Yang-Mills broken  
down to (extensions of)  
the MSSM

Size determines Newton's constant

- For a perturbative N=1 SUSY vacuum, six manifold must admit SU(3) structure with:

$$\mathcal{W}_1 = \mathcal{W}_2 = 0 \quad \mathcal{W}_4 = \frac{1}{2}\mathcal{W}_5 = d\phi$$

$$\mathcal{W}_3 = \text{anything}$$

Strominger, Hull '86

Lopes et al hep-th/0211118

- More general cases with no perturbative SUSY vacuum are known but I will ignore these today.  
(Lukas et al: hep-th/1005.5302, Gray et al: hep-th/1205.6208, Angus et al: to appear.)
- **Calabi-Yau case**: huge number of explicit examples to work with (**algebraic geometry can be used**).
- **Non-Calabi-Yau case**: very few interesting examples.

# Model Building

- **Calabi-Yau case:** Huge number of models with exact MSSM charged spectrum known.

Single models:

Bouchard and Donagi: hep-th/0512149  
and Bouchard, Cvetic and Donagi: hep-th/0602096  
Braun, He, Ovrut and Pantev: hep-th/0501070  
Anderson, Gray, He and Lukas: hep-th/0911.1569  
Braun, Candelas, Davies and Donagi: hep-th/1112.1097

Data set of  
100's of models:

Anderson, Gray, Lukas and Palti: arXiv: 1106.4804  
arXiv:1202.1757

- We can compute superpotential couplings and some other phenomenological details as well.  
**Missing:** matter field  $K$ , reliable susy breaking vacua.

- **Non-Calabi-Yau case:** No exact standard models are known.

Becker, Becker, Fu, Tseng and Yau: hep-th/0604137

Fu and Yau: hep-th/0604063

Goldstein and Prokushkin: hep-th/0212307

Klaput, Lukas and Matti: arXiv:1107.3573

Chatzistavrakidis and Zoupanos: arXiv:0905.2398

Chatzistavrakidis, Manousselis and Zoupanos: arXiv:  
0811.2182

Some work  
towards this goal:

The problem is a paucity of examples due to the fact that we can't directly use algebraic geometry in this case.

In fact some of the above cases are not Strominger system examples...

# Moduli Stabilization

- We have to remove the uncharged massless scalar fields, moduli, which appear in the four dimensional theory.

This remains the weakest point of heterotic string phenomenology.

- **Calabi-Yau case:** Requires an interplay of perturbative and non-perturbative effects, especially to stabilize overall volume – no convincing stable vacuum yet. Pieces of the 4d theory still being understood.

- **Non-Calabi-Yau case:** More promising, especially with regard to overall volume. Superpotential gains extra terms for example:

$$W \propto \int_X (H + idJ) \wedge \Omega = \int_X (H + i\mathcal{W}_3) \wedge \Omega$$

- Hard to stabilize the moduli such that internal volumes are large enough to give the correct gravitational/gauge couplings so far (excepted from paucity of examples):

This year only!:

Lukas, Lalak and Svanes: arXiv:1504.06978

(links to:

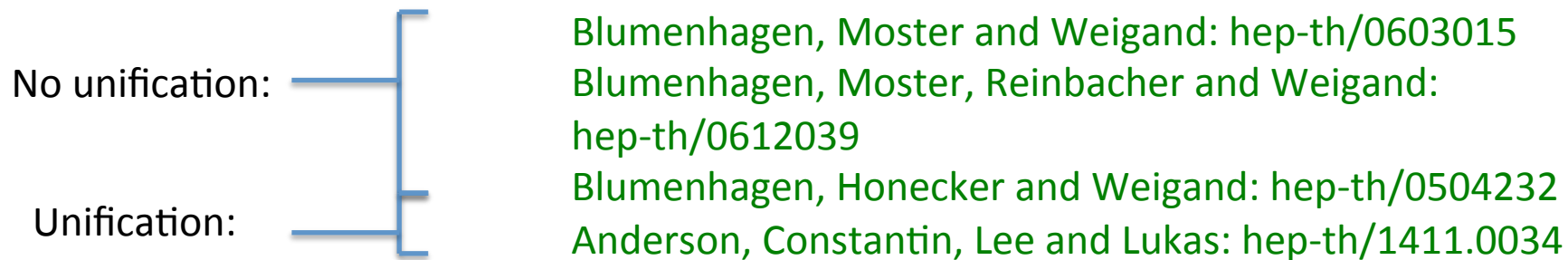
Klaput, Lukas, Matti and Svanes: arXiv: 1210.5933)

# Two Recent Pieces of Work



# Hypercharge flux in heterotic

- Instead of breaking the GUT group to the standard model with a Wilson line – use non-vanishing field strength!



- Group theory:

$$\begin{aligned} E_8 &\supset SU(3) \times SU(2) \times SU(6) \\ &\supset SU(3) \times SU(2) \times S(U(n_1) \times \dots \times U(n_m)) \end{aligned}$$

- Commutant of an  $S(U(n_1) \times \dots \times U(n_m))$  structure group is (low energy gauge group):

$$SU(3) \times SU(2) \times U(1)^{m-1}$$

- Generically the U(1)s will be Green-Schwarz massive.
- Can you:
  - Keep only one massless and have it be hypercharge
  - Keep gauge unification (say to within 5%)
  - Get the charged matter spectrum of the MSSM
- From **group theory** alone – **yes!**
- In actual **Calabi-Yau** reductions – **no!** (charged exotics)

# Moduli and Spectra of Non-Calabi-Yau cases

- Calabi-Yau case: Light states (moduli and matter) naively given in terms of quasi-topological properties:

$$H^1(\mathcal{TX}) , H^1(\mathcal{TX}^\vee) , H^1(\mathcal{V}) \dots$$

- Can we obtain as similar a result as possible for Non-Calabi-Yau cases?

Anderson, Gray and Sharpe: [arXiv:1402.1532](https://arxiv.org/abs/1402.1532)

De la Ossa and Svanes: [arXiv:1402.1725](https://arxiv.org/abs/1402.1725)

- “Massless” degrees of freedom can be found by perturbing equations of motion so...

- The most general  $\mathcal{N} = 1$  heterotic compactification with maximally symmetric 4d space:

- Complex manifold

$$F_{ab} = F_{\bar{a}\bar{b}} = 0 \quad H = i/2(\bar{\partial} - \partial)J$$

$$dH = -\frac{1}{30}\alpha' \text{tr} F \wedge F + \alpha' \text{tr} R \wedge R$$

$$g^{a\bar{b}} F_{a\bar{b}} = 0 \quad H_{\bar{b}c\bar{a}} g^{\bar{b}c} = -6\bar{\partial}_{\bar{a}}\phi$$

Gillard, Papadopoulos and Tsimpis

hep-th/0304126

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- Perturb all of the fields:

$$\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J} \quad A = A^{(0)} + \delta A$$

$$J = J^{(0)} + \delta J$$

$$H = H^{(0)} + \delta H^{\text{closed}} - \frac{1}{30} \alpha' \delta\omega_3^{\text{YM}} + \alpha' \delta\omega_3^{\text{L}}$$

- And look at what the first order perturbation to the supersymmetry relations looks like...

Restrict attention to manifolds obeying the  $\partial\bar{\partial}$ -lemma

**Lemma:** Let  $X$  be a compact Kähler manifold. For  $A$  a  $d$ -closed  $(p, q)$  form, the following statements are equivalent.

$$\begin{aligned} A = \bar{\partial}C &\Leftrightarrow A = \partial C' \Leftrightarrow A = dC'' \\ &\Leftrightarrow A = \partial\bar{\partial}\tilde{C} \Leftrightarrow A = \partial\hat{C} + \bar{\partial}\check{C} \end{aligned}$$

For some  $C, C', C'', \tilde{C}$  and  $\check{C}$ .

- Perturb all of the equations to get a mess.
- Then repack in terms of something which is easier to comprehend...

$$H^1(\mathcal{H}) = \left\{ \begin{array}{l} \ker \left( \ker \{ H^1(TX) \xrightarrow{[F],[R]} H^2(\text{End}_0(V)) \oplus H^2(\text{End}_0(TX)) \} \xrightarrow{M} H^2(TX^\vee) \right) \\ \oplus \\ \ker \left( H^1(\text{End}_0(V)) \xrightarrow{-\frac{4}{30}\alpha'[F]} H^2(TX^\vee) \right) \oplus \ker \left( H^1(\text{End}_0(TX)) \xrightarrow{4\alpha'[R]} H^2(TX^\vee) \right) \\ \oplus \\ H^1(TX^\vee) . \end{array} \right.$$

- This is a subspace of

$$\begin{aligned} & H^1(\mathcal{TX}^\vee) \oplus H^1(\mathcal{TX}) \oplus H^1(\text{End}_0(\mathcal{V})) \\ & \oplus H^1(\text{End}_0(\mathcal{TX})) \end{aligned}$$

defined by maps determined by the supergravity data  
(**matter is included!**).

- All maps are well defined, as are associated extensions.
- **This precisely matches the supergravity computation.**



# Conclusions

- **Calabi-Yau Case:**
  - Huge number of examples/amount of calculation control.
  - Model building reasonably far along.
  - Moduli Stabilization/SUSY breaking still a problem.
- **Non-Calabi-Yau Case:**
  - More promising from point of view of moduli stabilization.
  - Paucity of examples is really hindering progress.