# $Z'$ limits and naturalness in $U(1)$ extended models

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# Outline

# Motivation

- MSSM tree-level upper bound on Higgs mass $m_{h_1}^2 \leq M_Z^2 \cos^2 2\beta \lesssim (91 \text{ GeV})^2$
- $m_{h_1} \approx 125$ GeV $\Rightarrow$ large higher order corrections

$$m_{h_1}^2 \approx M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2}\right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2} + \dots$$

- $\Rightarrow$ also large corrections to prediction for $M_Z$ at SUSY scale $M_S$:

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{\overbrace{m_{H_d}^2 - m_{H_u}^2}^{\text{RGE effects}} \tan^2\beta}{\tan^2\beta - 1} + \delta_{1-\text{loop}},$$

$$\delta_{1-\text{loop}} = \frac{3}{8\pi^2} \frac{m_t^2}{v^2 \cos 2\beta} \left[ m_{\tilde{t}_1}^2 \left( \ln \frac{m_{\tilde{t}_1}^2}{M_S^2} - 1 \right) + m_{\tilde{t}_2}^2 \left( \ln \frac{m_{\tilde{t}_2}^2}{M_S^2} - 1 \right) \right] + \dots$$

- $\Rightarrow$ naturalness problem in the MSSM (Little Hierarchy Problem)
- Related is the "$\mu-$problem": $M_Z^2 = -\mu^2 + \dots \Rightarrow \mu \sim$ soft parameters?

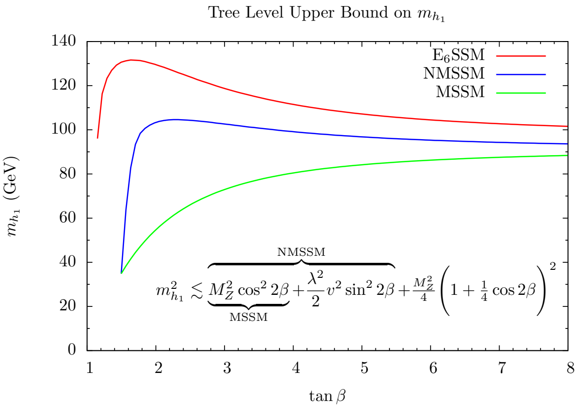
# $U(1)$ Extended Models: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$

- Raise upper bound on $m_{h_1} \longrightarrow$
  - additional $F$-terms (extra matter)
  - additional $D$-terms (from $U(1)'$)
- Include SM singlet with superpotential coupling $\lambda \hat{S} \left( \hat{H}_d \cdot \hat{H}_u \right)$
- $\Rightarrow$ dynamically generate $\mu$ term

$$V \supset \lambda S \left( H_d \cdot H_u \right) \to \lambda \langle S \rangle \left( H_d \cdot H_u \right),$$

i.e. $\mu_{\text{eff}} = \lambda \langle S \rangle$

- Break $U(1)'$ $\Rightarrow$ massive $Z'$ boson
- Need to take care of e.g. anomaly cancellation




$$W_{USSM} = y_{ij}^U \hat{u}_i^c \hat{H}_u \cdot \hat{Q}_j + y_{ij}^D \hat{d}_i^c \hat{Q}_j \cdot \hat{H}_d + y_{ij}^E \hat{e}_i^c \hat{L}_j \cdot \hat{H}_d + \lambda \hat{S} \hat{H}_d \cdot \hat{H}_u$$

# $E_6$ Inspired Models

- Class of low-energy effective models based on breakdown of $E_6$:

$$\begin{aligned} E_6 &\longrightarrow SO(10) \times U(1)_\psi \\ &\longrightarrow SU(5) \times U(1)_\psi \times U(1)_\chi \\ &\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \\ &\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \end{aligned}$$

$\Rightarrow$ low-energy $U(1)' = U(1)_\chi \cos\theta_{E_6} + U(1)_\psi \sin\theta_{E_6}$

- Mixing angle $\theta_{E_6} \in [0, \pi)$ parametrizes choice of $U(1)'$
- Note $\theta_{E_6} = 0$ (i.e. $U(1)_\chi$) allows elementary $\mu$-term
  - we do not consider this here
- Matter content fills $3 \times \mathbf{27}-$plets, $+SU(2)$ $\mathbf{2}$, $\bar{\mathbf{2}}$ from incomplete $\mathbf{27}'$, $\overline{\mathbf{27}'}$ (gauge unification)
  $\Rightarrow$ anomaly free

# $E_6$ Inspired Models (cont.)

- 4 models compared:

$$Q(\theta_{E_6}) = Q^\chi \cos\theta_{E_6} + Q^\psi \sin\theta_{E_6}$$

$U(1)_N$: $Q_N = Q(\theta_{E_6} = \arctan\sqrt{15})$ (≡ E$_6$SSM [1])
$U(1)_\psi$: $Q_\psi = Q(\theta_{E_6} = \pi/2)$
$U(1)_\eta$: $Q_\eta = -Q(\theta_{E_6} = \pi - \arctan\sqrt{5/3})$
$U(1)_I$: $Q_I = -Q(\theta_{E_6} = \arctan\sqrt{3/5})$

- Low-energy matter content from **27**-plet (integrate out $\hat{N}^c$):

$$(\hat{Q}_i,\, \hat{u}^c_i,\, \hat{d}^c_i,\, \hat{L}_i,\, \hat{e}^c_i) + (\hat{D}_i,\, \hat{\overline{D}}_i) + (\hat{S}_i) + (\hat{H}^u_i) + (\hat{H}^d_i)$$

- Higgs doublets $\hat{H}^d_3 \equiv \hat{H}_d$, $\hat{H}^u_3 \equiv \hat{H}_u$ and singlet $\hat{S}_3 \equiv \hat{S}$ get VEVs (⇒ EWSB and break $U(1)'$)
- Superpotential bilinear $\mu'$ allowed for $\hat{H}'$, $\hat{\overline{H}}'$ from $\mathbf{27}'$, $\overline{\mathbf{27}}'$, but not involved in EWSB (⇒ set to $\mu' = 5$ TeV)

| | $2\sqrt{6}Q^\psi_i$ | $2\sqrt{10}Q^\chi_i$ |
|---|---|---|
| $\hat{Q}$ | 1 | $-1$ |
| $\hat{u}^c$ | 1 | $-1$ |
| $\hat{d}^c$ | 1 | 3 |
| $\hat{L}$ | 1 | 3 |
| $\hat{e}^c$ | 1 | $-1$ |
| $\hat{S}$ | 4 | 0 |
| $\hat{H}_u$ | $-2$ | 2 |
| $\hat{H}_d$ | $-2$ | $-2$ |
| $\hat{D}$ | $-2$ | 2 |
| $\hat{\overline{D}}$ | $-2$ | $-2$ |
| $\hat{H}'$ | 1 | 3 |
| $\hat{\overline{H}}'$ | $-1$ | $-3$ |

$$W_{E_6} \approx y_\tau \hat{L}_3 \cdot \hat{H}_d \hat{e}^c_3 + y_b \hat{Q}_3 \cdot \hat{H}_d \hat{d}^c_3 + y_t \hat{H}_u \cdot \hat{Q}_3 \hat{u}^c_3 + \lambda_i \hat{S} \hat{H}^d_i \cdot \hat{H}^u_i + \kappa_i \hat{S} \hat{D}_i \hat{\overline{D}}_i + \mu' \hat{H}' \cdot \hat{\overline{H}'}$$

[1] S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. **D** 73, 035009 (2006) (arXiv:hep-ph/0510419)

# Impact of $M_{Z'}$

- Increased Higgs mass upper bound $\Rightarrow$ less need for heavy superpartners
- But extra $D$-terms also appear in tree-level EWSB condition:

$$\underbrace{c\left(\tan\beta;\theta_{E_6}\right)}_{O(1)}\frac{M_Z^2}{2} \approx -\frac{\lambda^2 s^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} + d\left(\tan\beta;\theta_{E_6}\right)\frac{M_{Z'}^2}{2},$$

$$d\left(\tan\beta;\theta_{E_6}\right) = \frac{Q_{H_d} - Q_{H_u}\tan^2\beta}{Q_S\left(\tan^2\beta - 1\right)}$$

- Large $M_{Z'}$ $\Rightarrow$ large *tree-level* fine tuning (to get correct $v$)
- Sufficiently large $M_{Z'}$ $\Rightarrow$ dominant source of tuning
- For given $M_{Z'}$, coefficient $d$ sets size of contribution
  - i.e. expect models with small $d$ to suffer less tuning
- Also, EWSB $\Rightarrow M_S \sim M_{Z'}$ for $M_{Z'} \gg M_Z \Rightarrow$ heavy SUSY spectrum

# Calculating Fine Tuning

- Ellis-Barbieri-Giudice measure:

$$\Delta = \max_{\{p\}} \Delta_p, \qquad \Delta_p \equiv \left| \frac{\partial \ln M_Z^2(M_S)}{\partial \ln p} \right|, \quad \left( M_S = \sqrt{m_{\tilde{t}_1}^{\overline{\mathrm{DR}}} m_{\tilde{t}_2}^{\overline{\mathrm{DR}}}} \right)$$

  - widely used, easy to apply, derivatives not dissimilar to those in Bayesian analysis . . .

- $\{p\} =$ "fundamental" model parameters at input scale $M_X$
  - e.g. $m_{H_u}^2(M_X)$, $A_t(M_X)$, $A_\lambda(M_X)$, $M_3(M_X)$, . . . for unconstrained models studied here

- For low $M_X = 20$ TeV, approximate SUSY scale parameters ($t = \ln \frac{M_S}{M_X}$, $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4}$)

$$p(M_S) \approx p(M_X) + \frac{t}{(4\pi^2)^2} \left( \beta_p^{(1)}(M_X) + \frac{1}{(4\pi)^2} \beta_p^{(2)}(M_X) \right) + \frac{t^2}{2\,(4\pi)^4} \sum_{\{q\}} \beta_q^{(1)}(M_X) \left. \frac{\partial \beta_p^{(1)}}{\partial q} \right|_{M_X}$$

$\Rightarrow$ analytic expressions for $\Delta_p$, much faster for large scans

# Calculating Fine Tuning (cont.)

- For $U(1)$ models, $\Delta_p$ calculated for

  $$p = \{\lambda, A_\lambda, m^2_{H_d}, m^2_{H_u}, m^2_S, m^2_{Q_3}, m^2_{u^c_3}, A_t, M_1, M_2, M_3, M'_1\}$$

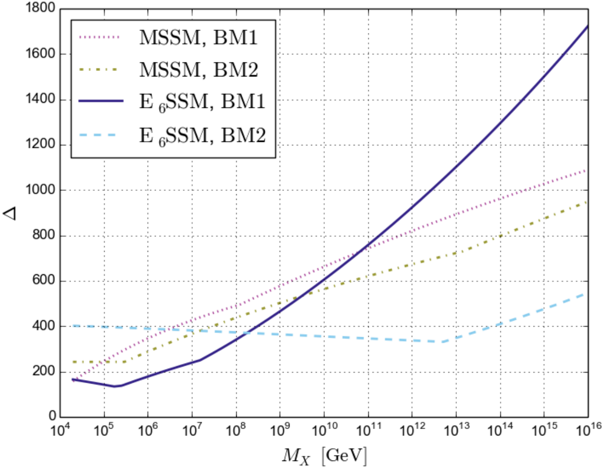
- Compare $\Delta$ for $M_{Z'} = 2.5$ TeV and $M_{Z'} = 4.5$ TeV
- Include only 1-loop $t$, $\tilde{t}$ contributions to $V_{\text{eff}}$
- Use 2-loop RGEs generated by SARAH + FlexibleSUSY (see talk by P. Athron)
- Compare to MSSM using modified SOFTSUSY 3.3.10
  - MSSM fine tuning calculated with $\mu, B \in [-1, 1]$ TeV and $M_X = 20$ TeV
- Neglect kinetic mixing
  - small in models considered
  - not negligible in general (e.g. $U(1)_{B-L}$)

Parameters scanned:

| |
|---|
| $2 \leq \tan\beta \leq 50$ for $U(1)_N$,<br>$\tan\beta = 10$ for all others |
| $-3 \leq \lambda \leq 3$ |
| $-10 \text{ TeV} \leq A_\lambda \leq 10 \text{ TeV}$ |
| $200 \text{ GeV} \leq m_{Q_3} \leq 2000 \text{ GeV}$ |
| $200 \text{ GeV} \leq m_{u_3} \leq 2000 \text{ GeV}$ |
| $-10 \text{ TeV} \leq A_t \leq 10 \text{ TeV}$ |
| $M_2 = 100 \text{ GeV}, 1050 \text{ GeV}, 2000 \text{ GeV}$ |

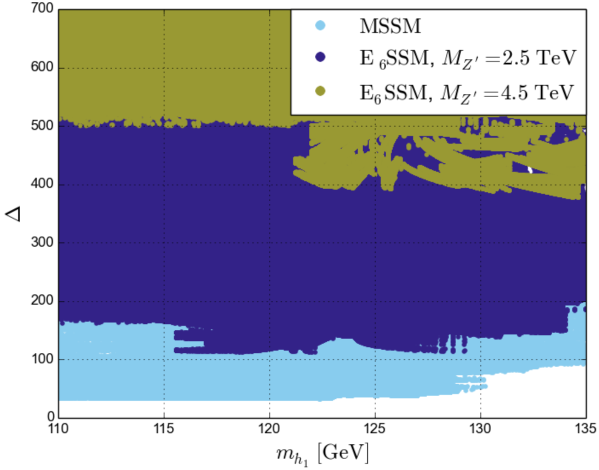
- Output $m^2_{H_d}$, $m^2_{H_u}$, $m^2_S$ from EWSB
- Other soft scalar masses $= 5$ TeV, $M_1 = M'_1 = 300$ GeV, $M_3 = 2$ TeV

# Impact of $M_X$ on Fine Tuning




- Tuning very dependent on $M_X$
  - $M_X = 20$ TeV v.s. $M_X = 10^{16}$ GeV
  - large $M_X \Rightarrow$ tuning mainly from RG contributions, e.g. from $M_3$ in $E_6$SSM BM1
- For given $M_X$, BC at high-scale also has large impact
  - compare $E_6$SSM BM1 (unconstrained) to BM2 ($\sim$ c$E_6$SSM)
  - i.e. assumptions about SUSY breaking impact tuning
- $Z'$ tuning not avoided by changing scale, BCs (tree-level effect)
- See impact of $Z'$ limits, look at low scales e.g. $M_X = 20$ TeV
  - minimize tuning from RG running

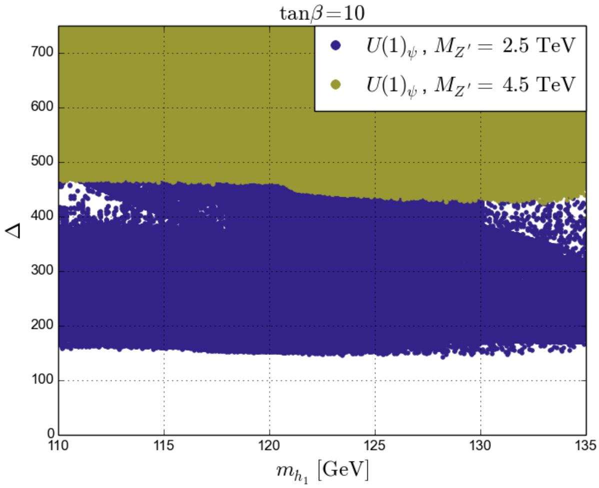
# $U(1)_N$: $Q_N = Q(\theta_{E_6} = \arctan\sqrt{15})$, $d(\tan\beta = 10) \approx 0.40$




- Low $M_X$ ⇒ reduced tuning due to e.g. stops, minimum MSSM tuning is $\approx 38$
- $M_{Z'}$ tuning not suppressed by low $M_X$
- $M_{Z'} \approx 2.5$ TeV ⇒ minimum tuning already $\approx 121$
  - ≈ equivalent to MSSM with $\geq 700$ GeV $\tilde{\chi}^{\pm}$ (⇒ lower bound on $\mu$)
- Compare constrained case [2]: $\Delta_{\min}^{\mathrm{cE_6SSM}} < \Delta_{\min}^{\mathrm{cMSSM}}$
- N.B. only 1-loop $t$, $\tilde{t}$ + leading 2-loop Higgs mass calculation ⇒ large Higgs mass uncertainty

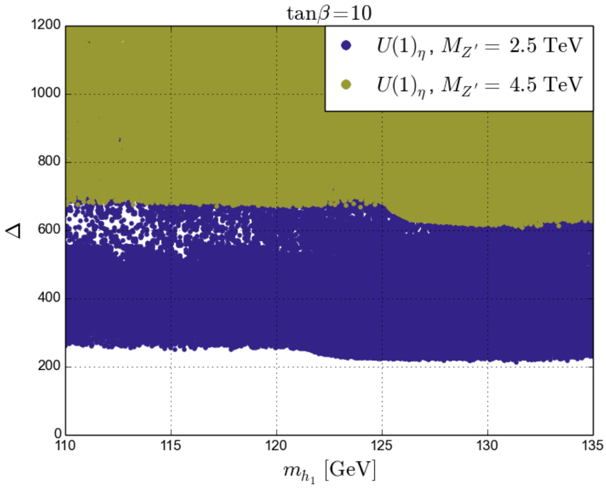

[2] P. Athron, M. Binjonaid, S. F. King, Phys Rev D **87**, 115023 (2013) (arXiv:1302.5291)

# $U(1)_\psi$: $Q_\psi = Q(\theta_{E_6} = \pi/2)$, $d(\tan\beta = 10) \approx 0.50$



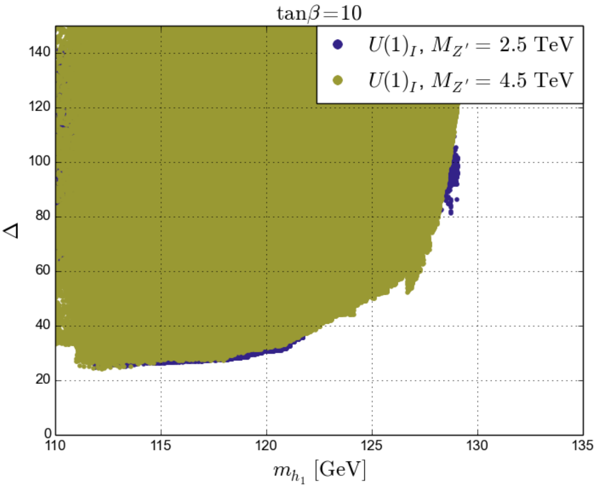

- As for $U(1)_N$, $M_{Z'}$ limit $\Rightarrow$ lower bound on tuning
- Increasing $M_{Z'}$ $\Rightarrow$ minimum tuning increases
- Lower bound for given $M_{Z'}$ is slightly larger compared to $U(1)_N$
- Due to slightly larger $D$-term contribution ($d \approx 0.4$ vs $d \approx 0.5$)
- Note additional scan done to fill in low-tuning regions

# $U(1)_\eta$: $Q_\eta = -Q(\theta_{E_6} = \pi - \arctan\sqrt{5/3})$, $d(\tan\beta = 10) \approx 0.81$




- $\Delta_{\text{min}}$ substantially larger for $M_{Z'} \approx 2.5, 4.5$ TeV compared to $U(1)_N$, $U(1)_\psi$
- Largest coefficient $d \approx 0.81 \Rightarrow$ largest tuning for given $M_{Z'}$
- Severity of tuning depends strongly on $\theta_{E_6}$ (i.e. charges)
- Current $Z'$ limits $\Rightarrow$ models with large $D$-terms already moderately tuned in this picture
- Models with small $d$?

## $U(1)_I$: $Q_I = -Q(\theta_{E_6} = \arctan\sqrt{3/5})$, $d(\tan\beta = 10) \approx -0.01$




- For $U(1)_I$, $Q_{H_u} = 0$ and so $d = -\frac{1}{\tan^2\beta - 1}$ is small

  $\Rightarrow$ fine tuning from $M_{Z'}$ substantially reduced

- Suppresses $D$-term contribution to Higgs mass, too:

$$m_{h_1}^2 \lesssim M_Z^2 \cos^2 2\beta + \frac{\lambda^2}{2} v^2 \sin^2 2\beta + g_1'^2 v^2 (Q_{H_d} \cos^2\beta + \underbrace{Q_{H_u} \sin^2\beta}_{=0})^2$$

- For current $Z'$ limits, tuning behaviour is MSSM-like
  - $M_{Z'}$ must be much larger before see any effect here

# Summary of $Z'$ Tuning Impact

- Tree-level tuning from $M_{Z'}$ present even for low $M_X$
- Increase $M_{Z'}$ $\Rightarrow$ increase minimum tuning that can be obtained
- $\Rightarrow$ $Z'$ search limits are important constraints on naturalness in these models
- But severity of fine tuning from $M_{Z'}$ depends strongly on charges
- Models with suppressed $D$-terms, e.g. $U(1)_I$, can still have low tuning even for $M_{Z'} \approx 4.5$ TeV
  - but smaller $D$-terms $\Rightarrow$ reduced Higgs mass too

| Model | $d(\tan\beta = 10)$ |
|---|---|
| $U(1)_I$ | $-0.01$ |
| $U(1)_N$ | $0.40$ |
| $U(1)_\psi$ | $0.50$ |
| $U(1)_\eta$ | $0.81$ |

$\downarrow$ Increasing $\Delta_{\text{min}}$

# Alternative $E_6$ Inspired Extensions

- For given $\theta_{E_6}$, reduce $D$-term contributions $\Rightarrow$ reduce $\Delta$?
- Alternative: modify field content for a given set of charges
  $\Rightarrow$ cancel $D$-term contribution to EWSB conditions
- E.g. $U(1)_N$ charges with extra SM singlets $S$, $\overline{S}$ from $\mathbf{27}'$, $\overline{\mathbf{27}'}$ with $Q_{\overline{S}} = -Q_S$
- $S$, $\overline{S}$ acquire VEVs $\langle S \rangle = s_1/\sqrt{2}$, $\langle \overline{S} \rangle = s_2/\sqrt{2}$ $\Rightarrow$ EWSB condition now reads

$$c(\tan\beta)\frac{M_Z^2}{2} \approx -\frac{\lambda^2 s_1^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + \underbrace{d(\tan\beta)}_{\text{as before}} \frac{M_{Z'}^2}{2} \cos 2\theta$$

- $\tan\theta = s_2/s_1$ $\Rightarrow$ for $s_1 \approx s_2$, (i.e. $\cos 2\theta \ll 1$), $D$-term contribution is much smaller
  $\Rightarrow$ potentially less tuning due to $Z'$ limits
- Also: include pure singlet $W \supset -\sigma\hat{\phi}\hat{S}\hat{\overline{S}}$ $\Rightarrow$ $M_{Z'} \sim M_S/\sigma$, i.e. can have lighter SUSY scale
- Currently studying phenomenology of constrained model (see also [3])

[3] P. Athron, M. Mühlleitner, R. Nevzorov, and A. G. Williams, J. High Energy Phys. **1501**, 153 (2015) (arXiv:1410.6288)

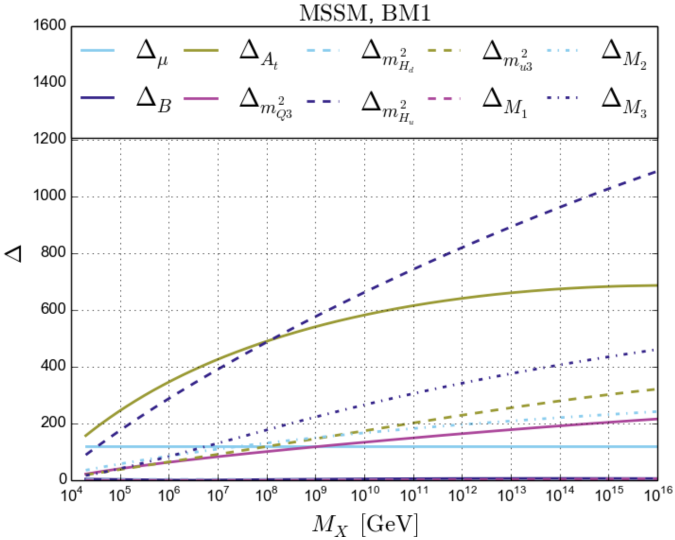
# Conclusion

- $Z'$ mass limits in $U(1)$ extensions $\Rightarrow$ constrain naturalness in these models
- Tuning due to e.g. RG effects is scale/boundary condition dependent
- $Z'$ tuning, on the other hand, enters at tree-level $\Rightarrow$ cannot be removed by choice of scale
- Tuning penalty from heavy $Z'$, but depends strongly on $U(1)'$ charges
- Appropriate choice of charges (e.g. $U(1)_I$) $\Rightarrow$ can still have low fine tuning for current limits
- Models with alternative field content (e.g. extra SM singlets)
  $\Rightarrow$ can reduce $D$-term contributions

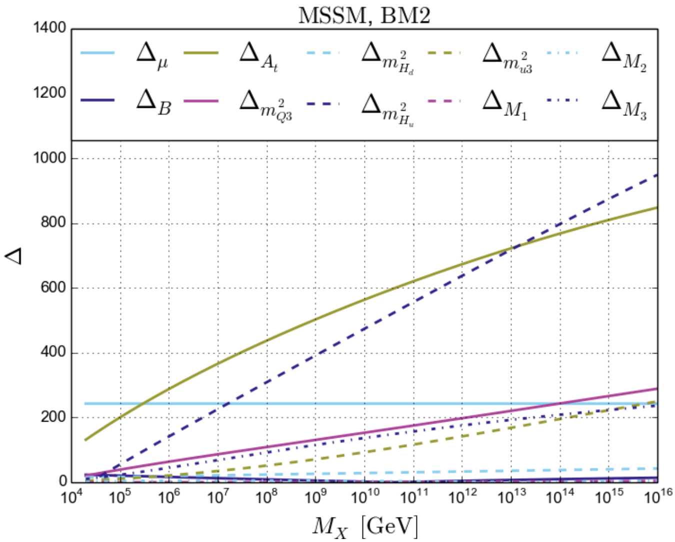
Thank you for listening!

Back-up slides

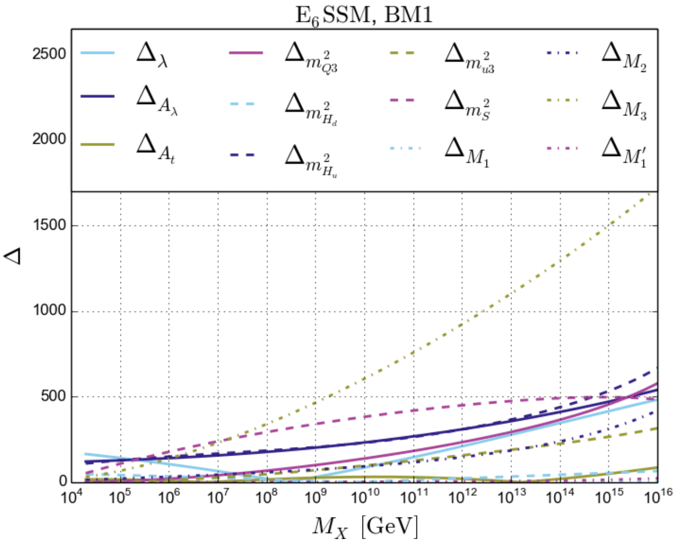
# MSSM BM1: Tuning Breakdown



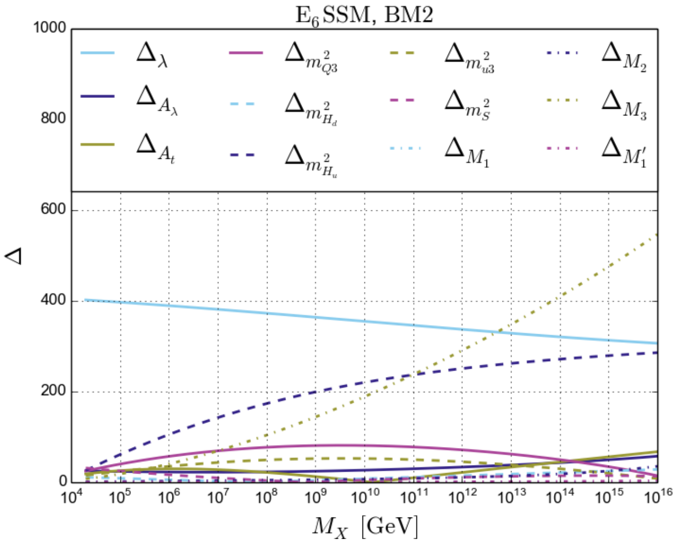
# MSSM BM2: Tuning Breakdown



# $E_6$SSM BM1: Tuning Breakdown



# $E_6$SSM BM2: Tuning Breakdown



# Low $M_X \Rightarrow$ Low $\tilde{t}$ Tuning