All rigid $N = 2$ supersymmetric backgrounds

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Based on [1505.03500] with Gianluca Inverso and Ivano Lodato

Motivation: Rigid SUSY on curved manifolds

Lots of work about exploiting SUSY on curved manifolds

- Wilson loop observables in $N = 4$ on S^4 [Pestun '07]
- Partition functions of $N = 2$ theories on S^3 to test various dualities [Kapustin, Willet, Yaakov '10]
- Computation of various indices for supersymmetric theories, etc. [Romelsberger '07] see also [Jafferis; Hama, Hosomichi, Lee; Imamura, Yokoyama; $\cdots$]

But how does one put a known supersymmetric field theory on a curved manifold in the first place?

Until recently, this was done piecemeal, theory-by-theory.

Festuccia and Seiberg gave a systematic scheme: derive rigid SUSY from SUGRA.

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Characterizing rigid manifolds with some SUSY

- 4D $N = 1$ theories with one or more supercharges
  [Cassani, Dumitrescu, Festuccia, Klare, Martelli, Seiberg, Tomasiello, Zaffaroni, $\cdots$]
- 3D papers are also exceedingly numerous...
  [Martelli, Passias, Sparks; Closset, Dumitrescu, Festuccia, Komargodski; $\cdots$]

Not a lot of work on 4D $N = 2$

- $N = 2$ theories have interesting features and more SUSY to exploit...
- Backgrounds with one supercharge... [Gupta, Murthy; Klare, Zaffaroni '13]
  but not the full eight. *Perhaps trivial?*
  It turns out full $N = 2$ richer than full $N = 1$!

We will address the following questions:

What are all curved backgrounds consistent with *full* rigid $N = 2$ SUSY?
What are all rigid actions for vector multiplets and hypermultiplets?

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Outline

1. Lessons from rigid supergravity and superspace
2. Supercoset spaces
3. Matter actions in rigid SUSY backgrounds

Lessons from rigid supergravity

Take a pause and recall the lesson of [Festuccia-Seiberg '11]:

A rigid SUSY matter action can be thought of as a coupled matter-SUGRA action with SUGRA fixed as background. "Auxiliary fields" of the supergravity multiplet play an essential role.

Finding a rigid SUSY means solving the SUGRA Killing spinor equation.

$$\delta\psi_{m\alpha}{}^{i} = 2\mathcal{D}_m\xi^i_\alpha(x) + \text{auxiliary fields} = 0.$$

Killing spinor $\xi^i_\alpha(x)$ parametrized by constants ϵ^i_α.

In Minkowski, $\xi^i_\alpha(x) = \epsilon^i_\alpha$.

In other backgrounds, $\xi^i_\alpha(x) = A(x)\epsilon^i_\alpha$

Different off-shell SUGRAs lead to different allowed backgrounds.

We need to address two questions in our case:

1. What is the most general off-shell SUGRA?
2. How do we solve the Killing spinor equation for maximal SUSY?

General off-shell $N = 2$ SUGRA

Conformal SUGRA: 24b+24f

$e_m{}^a \quad \psi_{m\alpha}{}^i \quad V_m{}^i{}_j \quad V_m \quad \Big\| \quad W_{ab}^- = T_{ab}^- \quad \chi_{\alpha i} \quad D$

$N = 2$ analogues of old and new minimal given by short compensators:

Vector multiplet: 8b + 8f

$X \quad \lambda_i \quad \Big\| \quad F_{mn} = (dA)_{mn} \quad Y_{ij}$

R-symmetry: $SU(2)$

Tensor multiplet: 8b + 8f

$L_{ij} \quad \lambda_i \quad \Big\| \quad H_{mnp} = (dB)_{mnp} \quad F$

R-symmetry: $SO(2) \times U(1)$

Use instead the longest possible compensator

General scalar multiplet: 128b + 128f

$\Omega \quad \lambda_i \quad \Big\| \quad S_{ij} \quad Y_{ab}^- \quad G_a \quad G_a{}^i{}_j \quad \cdots$

R-symmetry: $SU(2) \times U(1)$

General SUGRA to rigid SUSY

General SUGRA Killing spinor equation:

$$\delta_{\mathsf{Q}}\psi_{m\alpha}{}^{i} = 2\mathcal{D}_m\xi_{\alpha i} - i\bar{S}_{ij}(\sigma_m\bar{\xi}^j)_\alpha + i(Y^+_{mn} - W^-_{mn})(\sigma^n\bar{\xi}_i)_\alpha$$
$$+ 4iG^n(\sigma_{nm}\xi_i)_\alpha - 2G^{nj}{}_i(\sigma_n\bar{\sigma}_m\xi_j)_\alpha = 0\ .$$

Helpful to express this in superspace...

General SUGRA algebra (schematic form)

$$\{\mathcal{D}_\alpha{}^i, \mathcal{D}_\beta{}^j\} = \text{Lorentz and } R\text{-symmetry curvatures}\ ,$$
$$\{\mathcal{D}_\alpha{}^i, \bar{\mathcal{D}}_{\dot\beta j}\} = -2i\,\delta^i_j\mathcal{D}_{\alpha\dot\beta} + \text{Lorentz and } R\text{-symmetry curvatures}$$

curvatures involve: $S_{ij}, \quad Y^-_{ab}, \quad W^+_{ab}, \quad G_a, \quad G_a{}^{ij}$

A rigid SUSY must leave the curvatures invariant.

[Kuzenko, Novak, Tartaglino-Mazzucchelli '12, '14]

$$\delta_{\mathsf{Q}}S_{ij} = \xi^\alpha_k\mathcal{D}^k_\alpha S_{ij} = 0 \quad \Longrightarrow \quad \mathcal{D}^k_\alpha S_{ij} = 0 \quad \Longrightarrow \quad \{\mathcal{D}^k_\alpha, \mathcal{D}^l_\beta\}S_{ij} = 0$$

Integrability conditions imply that all curvatures are (covariantly) constant.

From constant curvatures to coset spaces

Riemann tensor is explicitly determined

$$R_{ab}{}^{cd} = S^{ij}\bar{S}_{ij}\delta_a{}^{[c}\delta_b{}^{d]} - \frac{1}{2}(\mathcal{Z}_{ab}\bar{\mathcal{Z}}^{cd} + \bar{\mathcal{Z}}_{ab}\mathcal{Z}^{cd}) + 8\,G^2\delta_a{}^{[c}\delta_b{}^{d]} - 16\,G_{[a}G^{[c}\delta_{b]}{}^{d]} + 4G^f_{ij}G^{ij}_f\delta_a{}^{[c}\delta_b{}^{d]} - 8G^{ij}_{[a}G^{[c}_{ij}\delta_{b]}{}^{d]}$$

Although all curvature tensors specified, we really want to know:

- What is the (global) structure of these spaces?
- How do we know the full set of Killing spinors actually exists?

We can easily resolve all these issues if we realize one important fact:

constant curvature tensors $\Longrightarrow$ (super) coset space

More accurately: for any superspace algebra with constant curvatures, we can construct a (global) super coset space with the same curvatures.

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Outline

(Super) coset spaces

A simple example of a (bosonic) coset space: $AdS_4 = SO(3,2)/SO(3,1)$

$$[\mathcal{D}_a, \mathcal{D}_b] = -\mu^2 M_{ab} \ , \qquad [M_{ab}, \mathcal{D}_c] = \eta_{c[a}\mathcal{D}_{b]}$$

Introduce matrix representations: $\mathcal{D}_a \to \hat{P}_a$ and $M_{ab} \to \hat{M}_{ab}$.

Recall basic steps for dealing with a coset G/H with $\mathfrak{g} = \{\hat{P}_a, \hat{M}_{ab}\}$ and $\mathfrak{h} = \{\hat{M}_{ab}\}$

1. Introduce coset element: $L = \exp(x^a \hat{P}_a)$.
2. Construct Cartan-Maurer form $L^{-1}\mathrm{d}L = \mathrm{d}x^m e_m{}^a(x)\hat{P}_a + \frac{1}{2}\mathrm{d}x^m \omega_m{}^{ab}\hat{M}_{ab}$
   Covariant derivatives $\mathcal{D}_a$ automatically inherit algebraic structure.
3. The isometries of G on the coset G/H can be written simply:

$$\delta = L^{-1}(\epsilon^a \hat{P}_a + \frac{1}{2}\lambda^{ab}\hat{M}_{ab})L = \xi^a(x)\hat{P}_a + \frac{1}{2}\xi^{ab}(x)\hat{M}_{ab}$$

   Local isometries encoded in $\xi^a(x) = A(x)^a{}_b\, \epsilon^b + B(x)^a{}_{bc}\, \lambda^{bc}$

Same approach holds for supercoset with $L = \exp(x^a \hat{P}_a + \theta_i \hat{Q}^i + \bar{\theta}^i \hat{\bar{Q}}_i)$.
Only $\theta = 0$ part is needed. see e.g. [Alonso-Alberca, Lozano-Tellechea, Ortin '02]

Classifying the allowed spaces

Background (constant) fields:

$$S^{ij} \;, \qquad \mathcal{Z}_{ab} = Y^-_{ab} - W^+_{ab} \;, \qquad G_a \;, \qquad G_a{}^i{}_j$$

- $\mathcal{Z}_{ab}$ is a complex field strength, $\mathrm{d}\mathcal{Z} = 0$.
  If the SUGRA algebra has (complex) central charge, $\mathcal{Z}_{ab}$ is its field strength.
- G_a may be thought of as dual three-form field strength H_{abc}.
- Isotriplets $G_a{}^{ij}$ and S^{ij} both break $SU(2)_R$ to $SO(2)_R$.
- $\mathcal{Z}_{ab}$ and S^{ij} are complex and break $U(1)_R$.

Three sets of solutions to integrability conditions for background fields:

1. S_{ij} alone is nonzero
2. $G_a{}^{ij}$ alone is nonzero and decomposes as $G_a{}^{ij} = g_a v^{ij}$
3. G_a and/or $\mathcal{Z}_{ab}$ are nonzero and obey $G^a \mathcal{Z}_{ab} = 0$

Menagerie of $N = 2$ backgrounds: The simplest cases

Active backgrounds	Geometry	Supergroup
S^{ij}	AdS$_4$	$OSp(4\|2)$
${G_a}^i{}_j$ timelike	$\mathbb{R} \times S^3$	$SU(2\|1) \times SU(2\|1)$
${G_a}^i{}_j$ null	plane wave	
${G_a}^i{}_j$ spacelike	AdS$_3 \times \mathbb{R}$	$SU(1,1\|1) \times SU(1,1\|1)$
G_a timelike	$\mathbb{R} \times S^3$	$SU(2\|2) \times SU(2)$
G_a null	plane wave	
G_a spacelike	AdS$_3 \times \mathbb{R}$	$SU(1,1\|2) \times SU(1,1)$
${\mathcal{Z}_a}^b$ elliptic	$\mathbb{R}^{1,1} \times S^2$	$D(2,1;\infty) \approx SU(2\|2)$
${\mathcal{Z}_a}^b$ hyperbolic	AdS$_2 \times \mathbb{R}^2$	$D(2,1;0) \approx SU(1,1\|2)$
${\mathcal{Z}_a}^b$ elliptic + hyperbolic	AdS$_2 \times S^2$	$D(2,1;\alpha)$
${\mathcal{Z}_a}^b$ parabolic	plane wave	

The two possible $\mathbb{R} \times S^3$

Round S^3 metric description as $S^1 \hookrightarrow S^3 \to S^2$.

$$ds^2 = -dt^2 + \frac{1}{16g^2}\left[d\theta^2 + \sin^2\theta\, d\phi^2 + (d\omega + \cos\theta\, d\phi)^2\right]$$

Arises from timelike background fields: $G_a = g_a$ or $G_a{}^{ij} = g_a v^{ij}$.

- Round $\mathbb{R} \times S^3$ has bosonic isometries $SO(4) \cong SU(2) \times SU(2)$.
- $G_a = g_a$ permits extension to supergroup $SU(2|2) \times SU(2)$. [Sen '90]
  $SU(2|2)_{(P_0)}$ gives SUSY along with spacetime $SU(2)$ and $SU(2)_R$

$$\{\mathcal{D}_\alpha{}^i, \bar{\mathcal{D}}_{\dot\beta j}\} = -2i\,\delta^i_j(\sigma^a)_{\alpha\dot\beta}\underbrace{(\mathcal{D}_a + \epsilon_a{}^{bcd} g_b M_{cd})}_{T_I,\,P_0} - 8\underbrace{g_{\alpha\dot\beta} I^i{}_j}_{SU(2)_R}$$

- $G_a{}^i{}_j = i g_a\,(\sigma_3)^i{}_j$ permits extension to $SU(2|1) \times SU(2|1)$

$$\{\mathcal{D}_\alpha{}^i, \bar{\mathcal{D}}_{\dot\beta j}\} = -2i\,\delta^i_j(\sigma^a)_{\alpha\dot\beta}\underbrace{\left(\mathcal{D}_a - (-1)^i \epsilon_a{}^{bcd} g_b M_{cd} + (-)^i g_a \mathbb{A}\right)}_{T_I^i,\,T_0^i}$$

  Two copies of (anti)commuting SUSY, with $T_I^i \in SU(2)$ and $T_0^i \in U(1)$
- General N-extended SUSY: $SU(2|p) \times SU(2|q)$ with $p + q = N$.

$AdS_2 \times S^2$ and $D(2,1;\alpha)$

Some historical observations

- A non-trivial spherically symmetric solution of $N=2$ gauged supergravity is $AdS_2 \times S^2$ of equal radii. The eight supercharges give $SU(1,1|2)$.

  This superalgebra describes the near horizon geometry of an extremal BPS Reissner-Nordstrom black hole.

  $\mathcal{Z}_{ab}$ is the graviphoton field strength.

- Can be generalized to different radii supergeometries with supergroup $D(2,1;\alpha)$. (These are not SUGRA solutions however.)

  [Bandos, Ivanov, Lukierski, Sorokin '02]

$D(2,1;\alpha)$ has bosonic body $SU(1,1) \times SU(2) \times SU(2)_R$ with $Q_{\tilde{a}\,\tilde{\alpha}\,i} \in (\mathbf{2},\mathbf{2},\mathbf{2})$

$$\{Q_{\tilde{a}\,\tilde{\alpha}\,i},\ Q_{\tilde{b}\,\tilde{\beta}\,j}\} = -\lambda_- \epsilon_{\tilde{\alpha}\tilde{\beta}}\epsilon_{ij} \underbrace{T_{\tilde{a}\tilde{b}}}_{AdS_2} - \lambda_+ \epsilon_{\tilde{a}\tilde{b}}\epsilon_{ij} \underbrace{T_{\tilde{\alpha}\tilde{\beta}}}_{S^2} + (\lambda_+ + \lambda_-)\epsilon_{\tilde{a}\tilde{b}}\epsilon_{\tilde{\alpha}\tilde{\beta}} \underbrace{I_{ij}}_{SU(2)_R}$$

The Euclidean version has been studied recently.

[Bawane, Bonelli, Ronzani, Tanzini '14; Sinamuli '14; Rodriguez-Gomez and Schmude '15]

Menagerie of $N = 2$ backgrounds: Mixed cases

New possibilities arise from turning on both G_a and $\mathcal{Z}_{ab}$

Active backgrounds	Geometry
G_a timelike	$\mathbb{R} \times S^3$
$\mathcal{Z}_a{}^b$ elliptic	$\mathbb{R} \times S^3$ squashed
G_a null	plane wave
$\mathcal{Z}_a{}^b$ elliptic	'lightlike' $S^3 \times \mathbb{R}$
$\mathcal{Z}_a{}^b$ parabolic	plane wave
G_a spacelike	$\text{AdS}_3 \times \mathbb{R}$
$\mathcal{Z}_a{}^b$ elliptic	
$0 < \|\mathcal{Z}\|^2 < 32\, G^2$	timelike stretched $\text{AdS}_3 \times \mathbb{R}$
$\|\mathcal{Z}\|^2 = 32\, G^2$	$\text{Heis}_3 \times \mathbb{R}$
$\|\mathcal{Z}\|^2 > 32\, G^2$	warped 'Lorentzian' $S^3 \times \mathbb{R}$
$\mathcal{Z}_a{}^b$ parabolic	null warped $\text{AdS}_3 \times \mathbb{R}$
$\mathcal{Z}_a{}^b$ hyperbolic	spacelike squashed $\text{AdS}_3 \times \mathbb{R}$

The squashed $\mathbb{R} \times S^3$

Squashing the S^3 is only possible for one of the supergroups

G_a	${G_a}^{ij}$
$SU(2\|2)_{(P_0)} \times SU(2)$	$SU(2\|1) \times SU(2\|1)$
$\downarrow \mathcal{Z}_{ab}$	$\downarrow$
$SU(2\|2)_{(P_0,U)}$	Not possible

Geometrically, we turn on $\mathcal{Z}_{ab}$ along S^3 and squash the S^1 fiber

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\upsilon}{16g^2}\big[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 + \upsilon(\mathrm{d}\omega + \cos\theta\,\mathrm{d}\phi)^2\big]$$

$$\upsilon \equiv \Big(1 + \frac{|\mathcal{Z}|^2}{32g^2}\Big)^{-1}, \qquad 0 \le \upsilon < 1$$

Can repeat for spacelike G_a to give squashings of $AdS_3 \times \mathbb{R}$.
$\mathbb{R}$ factor is spectator $\implies$ 3D $\mathcal{N} = 4$ SUSY on squashed AdS_3.

Euclidean backgrounds

The entire analysis can be repeated for Euclidean signature.

- But note: 4D $N = 2$ spinors can be chosen symplectic Majorana-Weyl. Left-handed and right-handed supercharges *completely independent* of each other. We can independently choose S_{ij} and $\widetilde{S}_{ij}$ as well as $\mathcal{Z}_{ab}$ and $\widetilde{\mathcal{Z}}_{ab}$.

Active backgrounds	Geometry
S^{ij} and $\widetilde{S}^{ij}$	S^4 and H^4
$G_a{}^i{}_j$	$H^3 \times \mathbb{R}$
G_a	$S^3 \times \mathbb{R}$
$\mathcal{Z} \cdot \widetilde{\mathcal{Z}} < 32 \, \lvert G \rvert^2$	Warped $S^3 \times \mathbb{R}$
$\mathcal{Z} \cdot \widetilde{\mathcal{Z}} = 32 \, \lvert G \rvert^2$	$\text{Heis}_3 \times \mathbb{R}$
$\mathcal{Z} \cdot \widetilde{\mathcal{Z}} > 32 \, \lvert G \rvert^2$	Warped Euclidean $AdS_3 \times \mathbb{R}$
$\mathcal{Z}_{ab}$ and $\widetilde{\mathcal{Z}}_{ab}$	$H^2 \times S^2$, $\mathbb{R}^2 \times S^2$ and $H^2 \times \mathbb{R}^2$
S^{ij}, $\mathcal{Z}_{ab}$	Flat space (deformed susy)

- Last case is flat space but includes full SUSY limit of Ω background.

see e.g. [Klare, Zaffaroni '13]

Outline

(Abelian) vector multiplets

Vector multiplet superfield: $\mathcal{X}^I \sim X^I + \theta^i \lambda_i^I + \theta_j \sigma^{ab} \theta^j F_{ab}^I + \theta_i \theta_j Y^{ijI}$

- Use rigid curved superspace with vector multiplet superfields $\mathcal{X}$.
  Actions still given by $-i \int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathcal{E} \, F(\mathcal{X}) + \text{h.c.}$
  F is still prepotential for rigid special Kähler geometry.
- If $U(1)_R$ is present ($G_{aij} \neq 0$), F must be superconformal.
- Background fields introduce new couplings in the action.
  - $\mathcal{Z}_{ab}$ gives new moment couplings like a background vector multiplet, e.g.

$$F_{ab}{}^I \Big(\frac{1}{4} \epsilon^{abcd} \mathcal{Z}_{cd} (F_I - \frac{1}{2}(F_{IJ} + \bar{F}_{IJ}) X^J) - \frac{1}{4} g_{IJ} X^J \mathcal{Z}^{ab} + \text{h.c.}\Big)$$

  - G_a gives composite $B \wedge F$ term via its dual two-form

$$4G^a (F_I \mathcal{D}_a \bar{X}^I + \bar{F}_I \mathcal{D}_a X^I) = 2i \, \epsilon^{mnpq} \, B_{mn} \, g_{IJ} \, \mathcal{D}_p X^I \, \mathcal{D}_q \bar{X}^J$$

    Already present in $N = 1$ case.

(Abelian) vector multiplets

SUSY transformations are deformed

$$\delta\lambda_{\alpha i}{}^I = (F_{ab}{}^I + \mathcal{Z}_{ab}X^I + \bar{\mathcal{Z}}_{ab}\bar{X}^I)(\sigma^{ab}\xi_i)_\alpha + (Y_{ij}{}^I + 2S_{ij}X^I)\xi_\alpha{}^j$$
$$- 2i\,\mathcal{D}_a X^I\,(\sigma^a\bar{\xi}_i)_\alpha + 4i\,G_{a\,ij}X^I\,(\sigma^a\bar{\xi}^j)_\alpha$$

This modifies the conditions for SUSY vacua

- $G_{a\,ij}X^I = 0$ — Sets X^I to zero
- $Y_{ij}{}^I = -2S_{ij}X^I = -2\bar{S}_{ij}\bar{X}^I$ — Fixes phase of X^I
- $F_{ab}{}^I = -\mathcal{Z}_{ab}X^I - \bar{\mathcal{Z}}_{ab}\bar{X}^I$ — Generalized attractor equation

Last result generalizes the standard BPS attractor equation $F_{ab}{}^I = W^+_{ab}X^I + \mathrm{h.c.}$

Straightforward to generalize to non-abelian vector multiplets and hypermultiplets.

- Basic properties hold (rigid special Kähler and hyperkähler) with extra features. For example, in AdS_4 (with S^{ij}), hypermultiplet target space must have extra $SO(2)_R$ isometry. [Butter, Kuzenko '11]

$N = 2^*$ action

Choose diagonal metric $g_{IJ} = \delta_{IJ}$ and adjoint hypermultiplet with scalars (A^I, B_I). In Minkowski background, mass term m softly breaks $N = 4$ to $N = 2^*$.

- In a general rigid background, the Lagrangian is

$$\mathcal{L} = -\mathcal{D}_m \bar{A}_I \mathcal{D}^m A^I - \mathcal{D}_m \bar{B}^I \mathcal{D}^m B_I - \mathcal{D}_m \bar{X}^I \mathcal{D}^m X^I - \frac{1}{8} F_{ab}{}^I F^{abI} + \frac{1}{2} F_{ab}{}^I (W^{ab+} X^I + W^{ab-} \bar{X}^I) + \mathcal{L}_{BF} + \mathcal{L}_{\text{pot}} + \text{fermions}$$

- The BF term involves couplings to the potentials for G_a and $G_a{}^{ij}$.

$$\mathcal{L}_{BF} = 2i\, \epsilon^{mnpq} B_{mn} \partial_p X^I \partial_q \bar{X}^I + 2\, \epsilon^{mnpq} B_{mn}{}^{12} (\partial_p A^I \partial_q \bar{A}_I + \partial_p B_I \partial_q \bar{B}^I) + 2\, \epsilon^{mnpq} B_{mn}{}^{11} \partial_p A^I \partial_q B_I + 2\, \epsilon^{mnpq} B_{mn}{}^{22} \partial_p \bar{A}_I \partial_q \bar{B}^I$$

- New contributions to scalar potential:

$$\mathcal{L}_{\text{pot}} = 2(|\mu|^2 - m^2)(A^I \bar{A}_I + B_I \bar{B}^I) + 2|\mu|^2 X^I \bar{X}^I + 2i\, \mu\, m\, (A^I B_I - \bar{A}_I \bar{B}^I) - \frac{1}{8} Z_{ab} \bar{Z}^{ab} \left(2 X^I \bar{X}^I + A^I \bar{A}_I + B_I \bar{B}^I\right) - \frac{1}{4} (W^+_{ab})^2 X^I X^I - \frac{1}{4} (W^-_{ab})^2 \bar{X}^I \bar{X}^I + 2 G_{a\, ij} G^{a\, ij} X^I \bar{X}^I + 4 G^2 (A^I \bar{A}_I + B_I \bar{B}^I)$$

Conclusions / Open questions

We have found all (global) rigid $N = 2$ spaces and constructed general rigid actions for vector and hypermultiplets. Some gaps / unanswered questions.

- We assumed global manifolds, but what about discrete quotients?
  e.g. The $\mathbb{R} \times S^3$: one can quotient along $U(1)$ fiber, giving a lens space $S^3/\mathbb{Z}_p$.
  Other cases?

- Is there a dynamical origin of all rigid supersymmetric backgrounds?
  Not for $4D$ supergravity + normal matter! [Hristov, Looyestijn, Vandoren '09]
  But maybe by compactifying higher dimensional theories.
  e.g. $D(2, 1; \alpha)$ from 6D theory vacuum $AdS_2 \times S^2 \times S^2$
  [Zarembo '10; Wulff '14]

- Many spaces include trivial $\mathbb{R}$ factors, so reduction to Euclidean or Lorentzian 3D $N = 4$ is clearly possible. Are there other 3D $N = 4$ spaces than these?

- The full supersymmetric configurations of vector and hypermultiplets are modified on rigid curved backgrounds. How much does this modify the analysis of quantum field theories on such curved backgrounds?

Thanks for your attention!