Trilinear Higgs self-couplings in $S(3)$SM

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Abstract

We carry out a detailed analysis of a minimal $S(3)$-invariant extension of the Standard Model, with an extended $S(3)$-Higgs sector. Within this extended $S(3)$-Standard Model, we study the trilinear Higgs self-couplings and its dependence on the details of the model, even when the lightest Higgs boson mass is taken to be a fixed parameter. We study quantitatively the trilinear Higgs self-couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space.

A precise measurement of the trilinear Higgs self-coupling will also make it possible to test this extended $S(3)$-Standard Model which has a different trilinear Higgs couplings as compared to the Standard Model. We present analytical expressions for the trilinear Higgs couplings.

Introduction

In the SM, only one $SU(2)_L$ doublet Higgs field is included, which, upon acquiring a vacuum expectation value, breaks the $SU(2)_L \times U(1)_Y$ symmetry.

Although their existence is a fundamental piece of the theory and the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, this may not be the final form of the theory.

In the SM each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory.

In this direction interesting work has been done with the addition of discrete symmetries to the SM.

It is noticeable that many interesting features of masses and mixing of the SM can be understood using a minimal discrete group, namely the permutational group $S(3)$.

[Derman(1979), Derman and Tsao(1979), Pakvasa and Sugawara(1978)]
[Pakvasa and Sugawara(1979), Mondragon and Rodriguez-Jauregui(1998)]
[Mondragon and Rodriguez-Jauregui(1999)]
[Mondragon and Rodriguez-Jauregui(2000), Harrison and Scott(2003)]
[Kubo et al.(2004)Kubo, Okada, and Sakamaki]
[Caravaglios and Morisi(2005), Araki et al.(2006)Araki, Kubo, and Paschos]
[Kubo et al.(2005)]
[Koide(2006), Grimus and Lavoura(2005), Teshima(2006)]
[Kimura(2005), Koide(2007), Mohapatra et al.(2006)Mohapatra, Nasri, and Yu]
[Kaneko et al.(2007)Kaneko, Sawanaka, Shingai, Tanimoto, and Yoshioka]
[Beltran et al.(2009)Beltran, Mondragon, and Rodriguez-Jauregui, Morisi and Peinado(2010)].

Introduction

Prior to the introduction of the Higgs boson, the SM is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. After the introduction of the Higgs boson in the theory, this field may be treated as an $S(3)$ singlet H_S, but then, only one fermion in each family can acquire mass.

Introduction

To give mass to all fermions and, at the same time, preserve the $S(3)$ flavour symmetry of the theory, an extended flavoured Higgs sector is required with three Higgs $SU(2)$ doublets, one in a singlet and the other two in a doublet irreducible representation of $S(3)$

[Kubo et al.(2003)Kubo, Mondragon, Mondragon, and Rodriguez-Jauregui]
[Felix et al.(2007)Felix, Mondragon, Mondragon, and Peinado]
[Mondragon et al.(2007)Mondragon, Mondragon, and Peinado]

Introduction

We carry out a detailed analysis of a minimal $S(3)$-invariant extension of the Standard Model, with an extended $S(3)$-Higgs sector. Within this, we study the trilinear Higgs self-couplings and its dependence on the details of the model, even when the lightest Higgs boson mass is taken to be a fixed parameter.

We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space.

A precise measurement of the trilinear Higgs self-coupling will also make it possible to test this extended $S(3)$-Standard Model which has a different trilinear Higgs couplings as compared to the Standard Model.

Higgs Boson in the Standard Model

Higgs Boson in the Standard Model

In the Standard Model, one $SU(2)$ doublet Higgs Field is included for the symmetry breaking of the $SU(2) \times U(1)$ gauge groups.

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \qquad \text{where} \qquad \Phi(X) = \begin{pmatrix} \phi(X)^+ \\ \phi(X)^0 \end{pmatrix}$$

The parameter λ must be positive to produce a stable vacuum.
The parameter μ can be either sign.
In fact if the sign of the quadratic term is negative namely $\mu^2 > 0$, at the origin the potential has a maximum, hence, the stable vacuum state corresponds to a non-zero value of the Φ field.

- The states satisfying
  $|\phi(X)^{+}|^2 + |\phi(X)^{0}|^2 = \mu^2/2\lambda = v^2/2$
  are degenerate minima of the potential.
- We can choose the vacuum expectation value in the $< \phi^0 >= v/\sqrt{2}$ direction.
- There is one important prediction of this model, one scalar particle appears in the physical spectrum which is called the Higgs boson.
- The mass of the Higgs boson is given by $m_h = \sqrt{2\lambda}v$, the W and Z masses are $m_W = (g/2)\, v$, $m_Z = \left(\sqrt{g^2 + {g'}^2}/2\right) v$.
- Through the Yukawa couplings, the Higgs gives mass to the quarks and leptons $m_f = Y_f v/\sqrt{2}$.

- Prior to the introduction of the Higgs boson, and mass terms the Lagrangian of the Standard Model is chiral and invariant with respect to any permutations of the left and right quark and lepton fields $\leftrightarrow S(3)$ flavour symmetry.
- If we assume that the $S(3)$ permutational symmetry is not broken, the Higgs in the S. M. is an $S(3)$ singlet and only one fermion can acquire mass.
- Altough the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, we believe that this is not the final form of the theory but rather an effective description of a more fundamental theory.

The $S(3)$ flavour symmetry

The ingredients of the extension of the Standard Model are the following:

- To extend the flavour and family concepts to the Higgs sector
- To asociate each family to an irreducible representation of the flavour group
- To construct a Lagrangian invariant under the action of the $SU(3)_c \times SU(2) \times U(1) \times S(3)^f$ group

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$S(3)$ irreducible representations

The group $S(3)$ has two one dimensional irreducible representations (singlets) and a two dimensional irreducible representation (doublet)

- One dimensional representations: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- Bi - dimensional: $\mathbf{2}$ doblet

Direct product of an $S(3)$ irreducible representations

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \qquad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \qquad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s,$$

$$\mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \qquad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}}$$

Direct product of two $S(3)$ doblets

$\mathbf{p_D} \otimes \mathbf{q}_D = r_s \oplus r_A \oplus \mathbf{r}_D$

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \qquad \text{and} \qquad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

it has two singlets, r_s and r_A, just one doublet r_D^T

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2}$$ which is invariant,

$$r_A = p_{D1}q_{D2} - p_{D2}q_{D1}$$ it is not invariant

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

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The Higgs sector is modified

$$\Phi \to H = (\Phi_a, \Phi_b, \Phi_c)^T$$

H is a reducible representation to $\mathbf{1_s} \oplus \mathbf{2}$ of $S(3)$

$$H_s = \frac{1}{\sqrt{3}}\Big(\Phi_a + \Phi_b + \Phi_c\Big), \qquad \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_a - \Phi_b) \\ \frac{1}{\sqrt{6}}(\Phi_a + \Phi_b - 2\Phi_c) \end{pmatrix}$$

The quark, lepton and Higgs fields are given by

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \qquad H$$

All the fields have three species (flavours) and belong to a representation reducible to $\mathbf{1} \oplus \mathbf{2}$ de $S(3)$

The $S(3)$ extended Higgs doublet model

The Lagrangian $\mathcal{L}_\Phi$ of the Higgs sector is given by

$$\mathcal{L}_\Phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S),$$

where D_μ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S(3)$.

The $S(3)$ extended Higgs doublet model

The analysis of the stability properties of the potential V is of great relevance to study the phenomenological implications of this model.

The $S(3)$ extended Higgs doublet model

There are many different ways of writing the Higgs potential for this model, but for the purpose of this work the best basis is

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix},$$

$$H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

The $S(3)$ extended Higgs doublet model

The numbering of the real scalar ϕ fields is chosen for convenience in writing the mass matrices for the scalar particles and the subscript S is the flavour index for the Higgs field singlet.

The $S(3)$ extended Higgs doublet model

$$
\begin{aligned}
V = {} & \mu_1^2\left(x_1+x_2\right)+\mu_0^2 x_3+a x_3^2+b\left(x_1+x_2\right) x_3+c\left(x_1+x_2\right)^2 \\
& -4 d x_7^2+2 e\left[\left(x_1-x_2\right) x_6+2 x_4 x_5\right]+f\left(x_5^2+x_6^2+x_8^2+x_9^2\right) \\
& +g\left[\left(x_1+x_2\right)^2+4 x_4^2\right]+2 h\left(x_5^2+x_6^2-x_8^2-x_9^2\right) .
\end{aligned}
$$

where

- the $\mu_{0,1}^2$ parameters have dimensions of mass squared,
- the $a, \cdots, h$ parameters are dimensionless.

The $S(3)$ extended Higgs doublet model

The invariants x_i, the potential V depends on the fields ϕ_i through x_i, considering our assignment as

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R}\left(H_1^\dagger H_2\right), & x_7 &= \mathcal{I}\left(H_1^\dagger H_2\right), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R}\left(H_1^\dagger H_S\right), & x_8 &= \mathcal{I}\left(H_1^\dagger H_S\right), \\ x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R}\left(H_2^\dagger H_S\right), & x_9 &= \mathcal{I}\left(H_2^\dagger H_S\right). \end{aligned} \quad (1)$$

The $S(3)$ extended Higgs doublet model

If the Higgs potential $S(3)$ invariant is bounded from below, being a quartic polynomial function it will certainly have a global minimum somewhere. We can two types of minima: the "trivial" one, for which the Higgs acquires zero vevs, and the usual one, where electroweak symmetry breaking occurs, away from the origin, for

$$\phi_7 = v_1\,,\ \phi_8 = v_2\,,\ \phi_9 = v_3\,,\ \phi_i = 0,\ \ i \neq 7, 8, 9\,,$$

defined as the normal minimum, with VEVs which do not have any complex relative phase.

The $S(3)$ extended Higgs doublet model

From the relations among the ϕ_i's and the x_l's, we obtain

- $\langle x_l \rangle = v_l^2/2$ for $l = \overline{1,3}$,
- $\langle x_4 \rangle = v_1 v_2/2$,
- $\langle x_5 \rangle = v_1 v_3/2$,
- $\langle x_6 \rangle = v_2 v_3/2$, and
- $\langle x_8 \rangle = \langle x_7 \rangle = \langle x_9 \rangle = 0$.

The $S(3)$ extended Higgs doublet model

Once the scalars receive vacuum expectation values, the replacement

$$\phi_i \to \left(h_i^+, v_i + \eta_i + i\chi_i\right)^T \tag{2}$$

for $i = 1, 2, 3$.

After diagonalizing the mass matrices the masses of the physical scalars and pseudoscalars are obtained. In our analysis we are not taking into account the parameter space with negative eigenvalues solutions for the squared masses of the physical Higgs fields.

Of the original twelve scalar degrees of freedom,

- three Goldstone bosons ($G^{\pm}$ and G) are absorbed by $W^{\pm}$ and Z.

The remaining nine physical Higgs particles are

- three $CP-$even scalar (h and H_1, H_2, with $m_h \geq m_{H_1} \geq m_{H_2}$),
- two CP-odd scalar (A_1, and A_2, with $m_{A_1} \leq m_{A_2}$), and
- two charged Higgs pair ($H^{\pm}_{1,2}$, mass degenerate).

To generate the correct $W^{\pm}$ and Z^0 masses, with the assignments $v_1^2 + v_2^2 + v_3^2 = v^2$ has to hold, where $v = 246$ GeV, and

$$v_1 = \sqrt{3} v_2$$

The squared-mass parameters μ_0^2 and μ_1^2 can be eliminated by minimizing the scalar potential,

$$\frac{\partial V}{\partial v_i} = 0 \qquad \text{if and only if} \qquad \frac{\partial V}{\partial x_j}\frac{\partial x_j}{\partial v_i} = 0, \tag{3}$$

where $i = \overline{1,3}$, and $j = \overline{1,9}$, and we get

$$\mu_1^2 = -\left(v_3\right)^2(b+f+2h) - 8\left(v_2\right)^2(c+g) - 6ev_2v_3. \tag{4}$$

$$\mu_0^2 = -2a\left(v_3\right)^2 - 4\left(v_2\right)^2(b+f+2h) - \frac{8e\left(v_2\right)^3}{v_3}. \tag{5}$$

From eq.(3) and using the resulting minimization conditions to eliminate μ_0^2 and μ_1^2, one obtains the elements

$$\mathcal{M}_{ij}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j},$$

of the tree-level mass squared matirx.

Defining the physical mass eigenstates m_h^2, $m_{H_1}^2$, and $m_{H_2}^2$, the masses are found from the diagonalization process

$$\mathcal{M}^2_{diag} = R^T \mathcal{M}^2_{even} R = \text{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2),$$

with $m_h^2 > m_{H_1}^2 > m_{H_2}^2$, can be parametrized as

$$R = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot Q \,, \qquad (6)$$

with

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_S & \sin\theta_S \\ 0 & -\sin\theta_S & \cos\theta_S \end{pmatrix} \tag{7}$$

and

$$\mathcal{M}^2_{even} = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{pmatrix} \tag{8}$$

where

$$\tan\theta_S = \frac{2s_{13}}{\sqrt{3}(m^2_{H_1} - s_{33})}. \quad (9)$$

The trilinear self-couplings of the neutral Higgs bosons are defined as

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}, \tag{10}$$

which are most easily obtained from the corresponding derivatives of V with respect to the fields $\{\eta_i\}$ with $i = 1, 2, 3$.

[Osland et al.(2008)Osland, Pandita, and Selbuz]
[Carena et al.(2003)Carena, Ellis, Mrenna, Pilaftsis, and Wagner]
[Carena et al.(2000)Carena, Ellis, Pilaftsis, and Wagner]

We can then write the trilinear couplings in terms of the derivatives of the potential V with respect to η_i and the elements of the rotation matrix R as

$$\lambda_{ijk} = N \sum_{lmn} R_{il} R_{jm} R_{kn} \frac{\partial^3 V}{\partial \eta_l \partial \eta_m \partial \eta_n}, \tag{11}$$

where the indices l, m, n refer to the weak field basis, and $l \leq m \leq n = 1, 2, 3$, N is a factor of $n!$ for n identical fields.

Let to be

$$\mathcal{A}_{l,m,n} = \frac{\partial^3 V}{\partial \eta_l \partial \eta_m \partial \eta_n},$$

Then

$$
\begin{aligned}
\mathcal{A}_{1,1,1} &= 6\sqrt{6}(c+g)v_2 \\
\mathcal{A}_{1,1,2} &= \sqrt{2}(2(c+g)v_2 + 3ev_3) \\
\mathcal{A}_{1,1,3} &= \sqrt{2}(3ev_2 + (b+f+2h)v_3) \\
\mathcal{A}_{1,2,2} &= 2\sqrt{6}(c+g)v_2 \\
\mathcal{A}_{1,2,3} &= 3\sqrt{6}ev_2 \\
\mathcal{A}_{1,3,3} &= \sqrt{6}(b+f+2h)v_2 \\
\mathcal{A}_{2,2,2} &= 3\sqrt{2}(2(c+g)v_2 - ev_3) \\
\mathcal{A}_{2,2,3} &= \sqrt{2}((b+f+2h)v_3 - 3ev_2) \\
\mathcal{A}_{2,3,3} &= \sqrt{2}(b+f+2h)v_2 \\
\mathcal{A}_{3,3,3} &= 6\sqrt{2}av_3
\end{aligned}
$$

Then, here we have ten trilinear Higgs scalar self-couplings and substituting for the elements of the rotation matrix, Eq. (6), one obtains

$$\begin{aligned}
\lambda_{1,1,1} &= 6v\left(\lambda_1 s_{\omega_3} + \lambda_2 c_{\omega_3}\right), & \lambda_{2,2,2} &= 6v\left(\lambda_3 s_{\omega_3} + \lambda_4 c_{\omega_3}\right), \\
\lambda_{3,3,3} &= 6v\left(\lambda_5 s_{\omega_3} + \lambda_6 c_{\omega_3}\right), & \lambda_{1,1,2} &= 2v\left(\lambda_7 s_{\omega_3} + \lambda_8 c_{\omega_3}\right), \\
\lambda_{1,1,3} &= 2v\left(\lambda_9 s_{\omega_3} + \lambda_{10} c_{\omega_3}\right), & \lambda_{1,2,2} &= 2v\left(\lambda_{11} s_{\omega_3} + \lambda_{12} c_{\omega_3}\right), \\
\lambda_{1,2,3} &= v\left(\lambda_{13} s_{\omega_3} + \lambda_{14} c_{\omega_3}\right), & \lambda_{1,3,3} &= 2v\left(\lambda_{15} s_{\omega_3} + \lambda_{16} c_{\omega_3}\right), \\
\lambda_{2,2,3} &= 2v\left(\lambda_{17} s_{\omega_3} + \lambda_{18} c_{\omega_3}\right), & \lambda_{2,3,3} &= 2v\left(\lambda_{19} s_{\omega_3} + \lambda_{20} c_{\omega_3}\right).
\end{aligned} \tag{12}$$

We here have self-couplings to three Higgs bosons, one obtains by hhh

$$\begin{aligned}\lambda_{h^0h^0h^0} \quad &= \tfrac{\sqrt{3}}{4}v\left[2(18as_{\theta_S}^3 + (b+f+2h)(3c_{\theta_S}^2+1)s_{\theta_S}\right. \\ &\left. -9ec_{\theta_S}^3 + 3ec_{\theta_S}\right) c_{\omega_3} \\ &+(3(b+f+2h)(c_{\theta_S}-1)s_{\theta_S}^2 + 2(c+g)) \\ &(9c_{\theta_S}^3 - 3c_{\theta_S}^2 + c_{\theta_S} - 3) \\ &-3e(3c_{\theta_S}(c_{\theta_S}+1)-1)s_{\theta_S})s_{\omega_3}\left.\right], \end{aligned} \tag{13}$$

where

$$v^2 = v_1^2 + v_2^2 + v_3^2$$

$$v_1 = \sqrt{3}v_2\,, \quad \text{and}\,, \quad \tan\omega_3 = \frac{2v_2}{v_3}$$

In the general case, we find it convenient to study the dimensionless ratios of the couplings

$$\chi_1 = \frac{\lambda_{h^0 h^0 h^0}}{\lambda^{SM}_{HHH}}.$$

where,

$$\lambda^{SM}_{HHH} = \frac{3M_H^2}{v},$$

with $M_H = 125$ GeV and $v = 246$ GeV.

We fixed values of the Higgs boson masses:

- $$M_{A_1^0} = 550 \text{ GeV}, \quad M_{A_2^0} = 270 \text{ GeV}$$
- $$M_{H_1^\pm} = 260 \text{ GeV}, \quad M_{H_2^\pm} = 270 \text{ GeV}$$
- $$M_{h_0^0} = 540 \text{ GeV}, \quad M_{H_1^0} = 530 \text{ GeV}$$
- $$M_{H_2^0} = 125 \text{ GeV}$$

The figures are plotted for parameters approach with

- $a = 8.05133$,
- $b = 7.64346$,
- $c = 3.01554$,
- $d = 0.7788041$,
- $e = -0.199190$,
- $f = -4.6013417$,
- $g = -2.00223$,
- $h = -1.51583$,
- $v = 246$ GeV

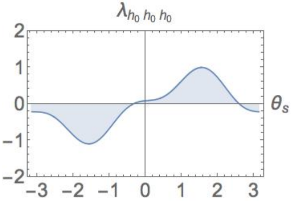

Figure: Trilinear Higgs coupling ratio χ_1. The ratios are plotted for the case of the CP-conserving limit. χ_1 for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

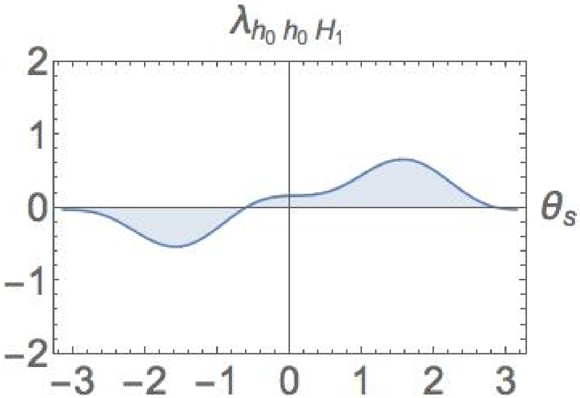

Figure: $\chi_2 = \lambda_{112}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

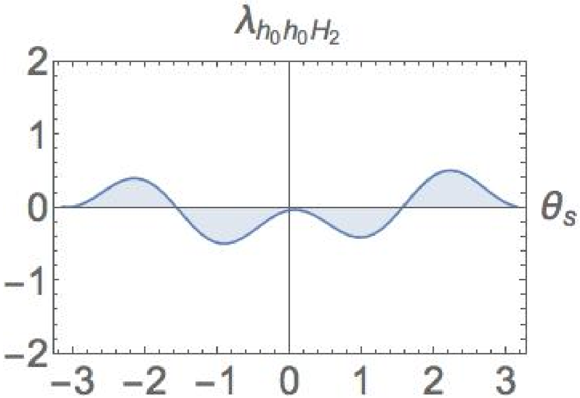

Figure: $\chi_3 = \lambda_{113}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

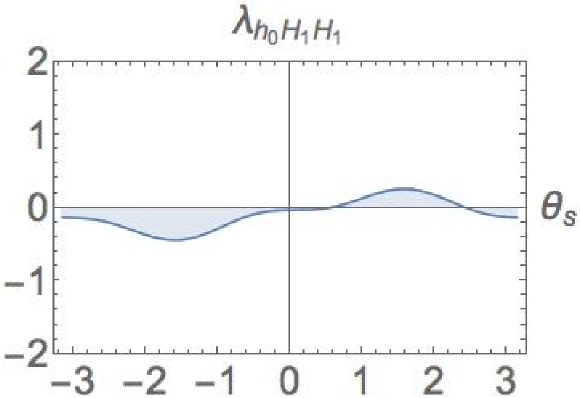

Figure: $\chi_4 = \lambda_{122}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

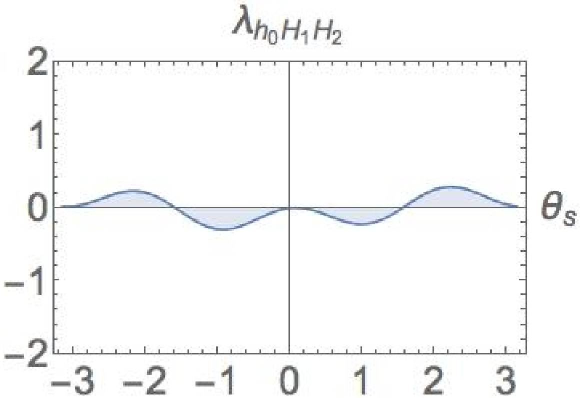

Figure: $\chi_5 = \lambda_{123}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

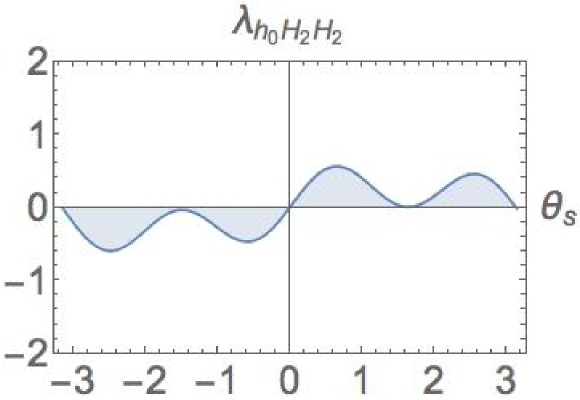

Figure: $\chi_6 = \lambda_{133}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

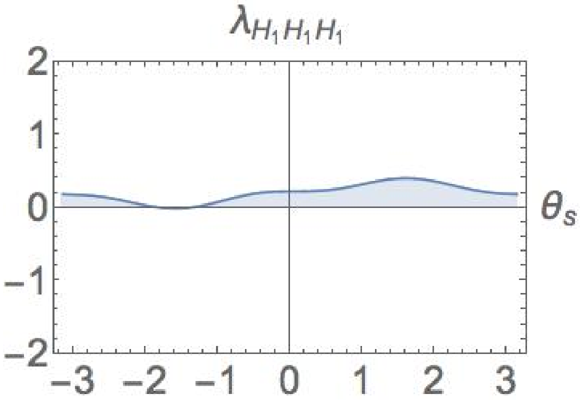

Figure: $\chi_7 = \lambda_{222}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

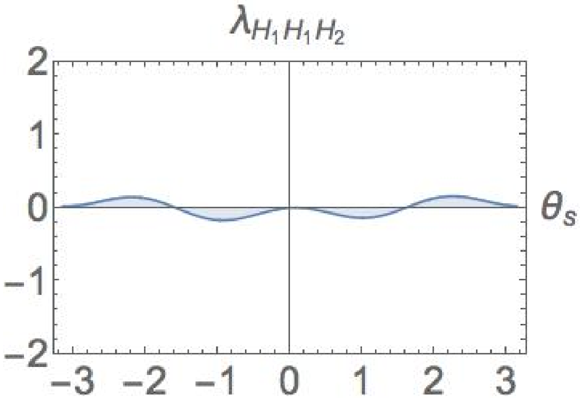

Figure: $\chi_8 = \lambda_{223}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

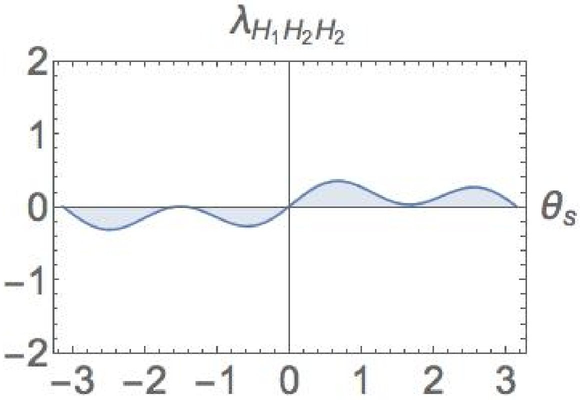

Figure: $\chi_9 = \lambda_{233}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

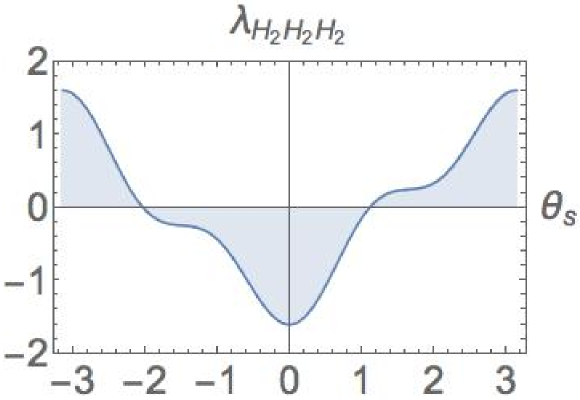

Figure: $\chi_{10} = \lambda_{333}/\lambda^{SM}$ for $-\pi \leq \theta_S \leq \pi$, and $\omega_3 = \pi/4$.

The trilinear coupling ratio χ_i, $i = 1, 2, 3$, defined at the tree level plotted as function of ω_3 ($-\pi$ to π).

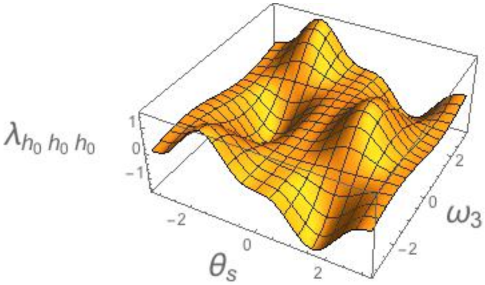

Figure: χ_1 for $-\pi/2 \leq \omega_3 \leq \pi/2$ and 200 GeV $\leq M_h \leq 800$ GeV

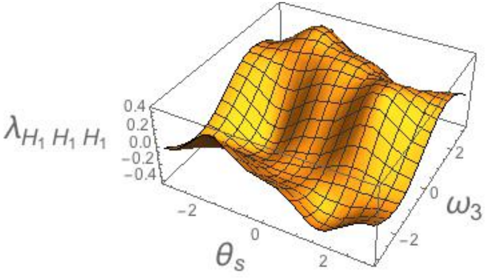

Figure: $\chi_2 = \lambda_{112}/\lambda^{SM}$ for $-\pi/2 \leq \omega_3 \leq \pi/2$ and 200 GeV $\leq M_h \leq 800$ GeV

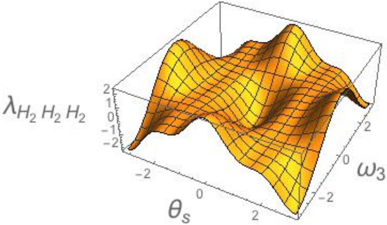

Figure: $\chi_3 = \lambda_{113}/\lambda^{SM}$ for $-\pi/2 \leq \omega_3 \leq \pi/2$ and 200 GeV $\leq M_h \leq 800$ GeV

Summary and Conclusions

We studied only the scalar sector assuming the pseudoscalars to be too heavy to be relevant. In this work we have analyzed the complete scalar sector of an S(3) avour model. We deal with three CP-even, two CP-odd and two sets of charged scalar particles. In this work we have improved our potential minimization technique which enabled us to explore a larger region of the allowed parameter space. In this work we have studied in detail the trilinear couplings of the lightest Higgs boson of this model.
Within the allowed domain of the parameter space of model, the trilinear coupling can have a very strong dependence on the neutral Higgs boson mixing angle ω_3.

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