Gauge-Higgs Unification with the dynamical boundary conditions and it's $SU(5)$ application

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Purpose of our research

- For GHU model in extra dim. there are many possible choices for boundary conditions imposed on fields

- These boundary conditions are given by hand as definition of models

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- **Determine the boundary conditions from some dynamics**

$\longrightarrow$ Extend to GHU with dynamics of boundary conditions

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- Define the model by integrating all possible configurations of boundary conditions in the partition function of system

$\longrightarrow$ **Some class of boundary conditions practically don't contribute to partition function**

Gauge-Higgs Unificatioin

Gauge theory defined on compactified extra dimensions

In 5D case $A_M = (A_\mu, A_5)$

vacuum expectation value

Identify A_5 with Higgs field !

S.Funatsu,H.Hatanaka,Y.Hosotani,Y.Orikasa,T.Shimotani
Phys.Lett. B722 94-99

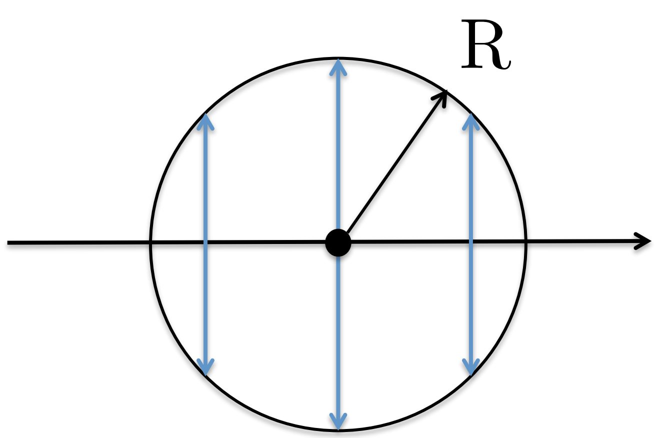
Good points

- VEV of A_5 is dynamically generated
- Finiteness of the VEV is guaranteed by gauge symmetry

Boundary conditions on S^1/Z_2 orbifold

Y.Kawamura, Prog. Theor. Phys. 103 613
L.J.Hall,Y.Nomura, Phys. Rev. D64 055003

S^1/Z_2 orbifold



Boundary conditions

$$\begin{pmatrix} A_\mu(x,-y) \\ A_5(x,-y) \end{pmatrix} = P_0 \begin{pmatrix} A_\mu(x,y) \\ -A_5(x,y) \end{pmatrix} P_0^\dagger$$

$$\begin{pmatrix} A_\mu(x,\pi R - y) \\ A_5(x,\pi R - y) \end{pmatrix} = P_1 \begin{pmatrix} A_\mu(x,\pi R + y) \\ -A_5(x,\pi R + y) \end{pmatrix} P_1^\dagger$$

- $P_0, \ P_1$ satisfy parity restriction $P_0^2 = P_1^2 = 1$
- P_0, P_1 are the elements of U(N)

$$P_0, P_1 \in U(N)$$

Arbitrariness problem of boundary conditions

· Gauge transformation $\Omega(x,y)$ affects $P_0,\ P_1$

$$P_0 \longrightarrow \Omega(x,-y)P_0\Omega^\dagger(x,y), \qquad P_1 \longrightarrow \Omega(x,\pi R-y)P_1\Omega^\dagger(x,\pi R+y)$$

· Symmetry of boundary conditions

$$\Omega(x,-y)P_0\Omega^\dagger(x,y) = P_0, \quad \Omega(x,\pi R-y)P_1\Omega^\dagger(x,\pi R+y) = P_1$$

Ex.) $SU(5)$ case

(i) $P_0 = diag\{+1,+1,+1,+1,+1\}, \quad P_1 = diag\{+1,+1,+1,+1,-1\}$

$$SU(5) \longrightarrow SU(4)\times U(1)$$

(ii) $P_0 = P_1 = diag\{-1,-1,-1,+1,+1\}$

$$SU(5) \to SU(3)\times SU(2)\times U(1)$$

There are many possible choices for boundary conditions

Arbitrariness problem !

Equivalence class

A_5 can acquire VEV from dynamics of Wilson line phases

$\longrightarrow$ leads to spontaneous symmetry breaking

Physical symmetry = Symmetry of boundary conditions + dynamics of Wilson line phases

The two boundary condition sets can be related

by dynamics of Wilson line phases $\longrightarrow$ the same physics !

Boundary conditions are classified into equivalence classes

Y.Hosotani, Annals. phys. 190 233-253

For $SU(N)$ case, boundary conditions are classified

into $(N+1)^2$ equivalence classes

N.Haba,Y.Hosotani,Y.Kawamura, Prog.Theor.Phys. 111 265- 289

Model

Partition function for $SU(N)$ gauge theory on $M^4 \times S^1/Z_2$

$$\mathrm{Z} = \int_C dP_0 \int_C dP_1 \int \mathcal{D}A_M \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi \Big|_{P_0,P_1} e^{iS(A_M,\psi,\phi,P_0,P_1)}$$

$$C = \{P_a \in U(N), \quad \rho_i = \pm 1\} \quad a = 1, 2$$

$dP_0,\ dP_1$: invariant measure for $U(N)$ group

$\int_C dP_0 \int_C dP_1$ = Sum of contribution of equivalence classes

$\longrightarrow$ Investigate each contribution of equivalence class in partition function

$\longrightarrow$ Compare the volumes among each equivalence class

Result

- There are the difference among each volume of equivalence class

- Only the equivalence classes which have the highest volume practically contributes to partition function

For $SU(5)$ case

$$P_0 = P_1 = diag\{+1, +1, +1, +1, +1\}$$
$$SU(5) \longrightarrow SU(5)$$

$$P_0 = diag\{+1, +1, +1, +1, +1\}, \quad P_1 = diag\{+1, +1, +1, +1, -1\}$$
$$SU(5) \longrightarrow SU(4) \times U(1)$$

$$P_0 = P_1 = diag\{-1, -1, -1, +1, +1\}$$
$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

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Nucl. Phys. B, 883, 45-58 (2014)

Result

For $SU(5)$ case

Only 4 equivalence classes contribute to the partition function

(i)
$$\begin{bmatrix} P_0 = \{+1,+1,+1,-1,-1\} \\ P_1 = \{+1,+1,+1,-1,-1\} \end{bmatrix}$$

(ii)
$$\begin{bmatrix} P_0 = \{+1,+1,+1,-1,-1\} \\ P_1 = \{-1,-1,-1,+1,+1\} \end{bmatrix}$$

(iii)
$$\begin{bmatrix} P_0 = \{-1,-1,-1,+1,+1\} \\ P_1 = \{+1,+1,+1,-1,-1\} \end{bmatrix}$$

(iv)
$$\begin{bmatrix} P_0 = \{-1,-1,-1,+1,+1\} \\ P_1 = \{-1,-1,-1,+1,+1\} \end{bmatrix}$$

· **These equivalence classes has** $SU(5) \to SU(3) \times SU(2) \times U(1)$ **symmetry breaking pattern with appropriate matter content**

Nucl. Phys. B, 883, 45-58 (2014)

Summary

- Gauge-Higgs Unification has the arbitrariness for the boundary condition and in present Gauge-Higgs study, these boundary conditions are given by hand

- We constructed Gauge-Higgs Unification model including the dynamics of boundary conditions

- We found Only restricted class of boundary conditions practically contribute to partition function by analyzing each volume of equivalence class

Thanks for your attention !

Reference

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