RPV from Discrete R Symmetries

(and Recent Nucleon Decay Searches in Super-K)

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SUSY 2015

Lake Tahoe – August 25, 2015

Primarily based on:

– **M.-C. Chen, M. Ratz and V. Takhistov** **[NPB 819 (2015) 322; hep–ph/1410.3474]**

Overview

- Problems of the MSSM
- “Fixing” the MSSM → **discrete symmetries**
- Properties of discrete symmetries
- Equivalent symmetries
- Algorithm for finding maximal discrete symmetry
- Survey of discrete symmetries for R-parity violating and conserving MSSM
- What else can we do with these symmetries … ?

+ (if time permits) a word about recent nucleon decay searches at Super-Kamiokande

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- SUSY well motivated

  → top down / "formally" *(Haag's theorem, local SUSY → gravity, strings)*

  → bottom up / "phenomenologically" *(hierarchy problem, coupling unification, DM, etc.)*

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- Minimal SUSY SM extension *(MSSM)* is attractive, with renormalizable superpotential

μ-term

Yukawa terms

$$\mathscr{W}_{\rm ren} = \mu H_u H_d + Y^u_{fg} Q_f \overline{U}_g H_u + Y^d_{fg} Q_f \overline{D}_g H_d + Y^e_{fg} L_f \overline{E}_g H_d$$
$$+ \kappa^f L_f H_u + \lambda^{fgh} L_f L_g \overline{E}_h + \lambda'^{fgh} L_f Q_g \overline{D}_h + \lambda''^{fgh} \overline{U}_f \overline{D}_g \overline{D}_h$$

baryon & lepton number violating *(RPV)* terms

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baryon & lepton number violating *(RPV)* terms

- Problems of the MSSM:

  → rapid proton decay *(RPV terms)* and μ problem

Fixing the MSSM: Discrete Symmetries – I

- Phenomenological constraints:

μ-term *(EWSB)*: $\mu \sim 10^{-16} M_{Pl}$

Neutrino mass: $\kappa^i \leq 10^{-21} M_{Pl}$

Proton stability $\lambda' \lambda'' \leq 10^{-27}$

$\lambda' \lambda_3 \leq 10^{-10}$

$\lambda_1 \leq 10^{-8},\ \lambda_2 \leq 10^{-8}$

+ higher order RPV terms

$$\begin{aligned}
\mathcal{O}_1 &= [Q\,Q\,Q\,L]_F\,, & \mathcal{O}_2 &= [\overline{U}\,\overline{U}\,\overline{D}\,\overline{E}]_F\,,\\
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→ discrete $\mathbb{Z}_2$ symmetry, kills $L\,H_u$ term and dim-4 RPV terms

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combine with R-parity ($\mathbb{Z}_2$)
→ form proton hexality ($\mathbb{Z}_6$)

[Babu & Gogoladze & Wang (2002); Dreiner & Luhn & Thormier (2006)]

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(also found w/o GS but with adding extra fields [Kurosawa, Maru, Yanagida (2001)])

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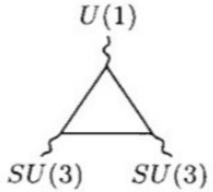
- **Anomalies** *(failure of maintaining the symmetry at quantum level)*

"triangle diagram"



OR

non-invariant path integral measure (Fujikawa method)

QM measure *vs.* classical measure

$$\mathcal{D}\psi'\mathcal{D}\overline{\psi'} = (\det J)^{-1}\left(\det\overline{J}\right)^{-1}\mathcal{D}\psi\mathcal{D}\overline{\psi}.$$

$$\equiv \exp\left(i\int d^4x\,\alpha(x)\mathcal{A}(x)\right)$$

anom. coeff. for this diagram is 0 in SM

$$\mathcal{A}(x) = \frac{1}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}F_a^{\mu\nu}(x)F_b^{\rho\sigma}(x)\mathrm{tr}[tT^aT^b]$$

anomaly coefficient

→ *usually set anomaly coefficient to 0 (SM)*

Green-Schwarz Anomaly Cancellation

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  $\rightarrow$ use path integral to get coefficients $A_{G-G-\mathbb{Z}_N^R}$

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- **Green-Schwarz (GS) mechanism**: axion a coupled to field strengths

$$\mathcal{L} \supset -\frac{a}{8} F^a \tilde{F}^a$$

can absorb path integral shift from anomaly

$$\Delta\mathcal{L}_{\text{anomaly}} = \sum_G \frac{\alpha}{32\pi^2} F^a \tilde{F}^a A_{G-G-\mathbb{Z}_N}$$

into Lagrangian shift after axion transforms under the symmetry $a \to a + \frac{i}{2}\Delta_{\text{GS}}$

$$\pi N \Delta_{\text{GS}} \equiv A_{G-G-\mathbb{Z}_N} \mod N \quad \forall G$$

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- **Anomaly universality**: GS can cancel anomalies separately, but coupling unification and/or GUT requires universality

$$A_{G_i-G_i-\mathbb{Z}_N^R} = \rho \mod N$$

[Chen, Ratz, Staud, Vaudrevange (2012)]

($\rho = 0$ without GS)

R-parity Violation & Survey of Discrete Symmetries

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  → R-parity Violation *(RPV)* *(lots of literature, rich pheno.)*

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Surveying Discrete Symmetries

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- **<u>We will provide</u>**:

  – general criteria for identical symmetries → identify redundancies

  – novel algorithm for identifying maximal discrete symmetry

  – models consistent with Pati—Salam group, some favoring Dirac neutrino mass

  – minimal solutions of both RPV and R-parity conserving surveys

  – a counter-example to statement in the literature regarding L-viol. symmetries

Equivalent Symmetries

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- **Criteria of equivalence for symmetry of order *N***:

  – **Common divisors:** if symmetry order N and all the charges have a common divisor M, $\mathbb{Z}_N^R$ is equivalent to $\mathbb{Z}_{N/M}^R$ with its charges divided by M

  – **Non-trivial centers:** in the presence of an $SU(N)$ gauge factor, acting with the center of $SU(N)$, the $Z_{SU(M)} \simeq \mathbb{Z}_M$, is an equivalent symmetry

  – **Hypercharge-shifts:** can add multiples of the hypercharge to respective field charges to obtain equivalent symmetries

  – **Coprime factors:** multiplying all charges by a factor co-prime with the symmetry order N leads to the same symmetry

less general criteria can be found in **[Petersen, Ratz, Schieren (2009)]**

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- With charges as variables, using *Smith form* can "diagonalize" constraint matrix

$$A \cdot q \;=\; 0 \mod N \quad \longrightarrow \quad U \cdot A \cdot V \;=\; D$$

U, V – unimodular matrices

where $D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_{n_q} \\ \hline & 0_{n_q \times (n_c - n_q)} & \end{pmatrix}$ and $q \;=\; V \cdot \begin{pmatrix} k_1 \frac{d_{n_q}}{d_1} \\ \vdots \\ k_{n_q} \frac{d_{n_q}}{d_{n_q}} \end{pmatrix} \mod N$

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applying the method

- imposing SU(5) + Weinberg op.
  → maximal order is 24

- imposing SO(10) + Weinberg op.
  → maximal order is 4

(agreement with literature)

GUT Compatibility

- **GUT compatibility**:

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– partial unification of Pati—Salam $G_{\rm PS} = {\rm SU}(4) \times {\rm SU}(2)_{\rm L} \times {\rm SU}(2)_{\rm R}$ is allowed

(not considered in previous surveys)

– don't enforce anomaly universality for Pati—Salam, since no single unifying gauge group → PS doesn't predict coupling unification in general

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  – many RPV symmetries consistent with Pati—Salam

  – found models (eg. $\mathbb{Z}_{12}^{R}$) "effectively" R-parity conserving and even order, but without R-parity

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  + others *(see paper for further details)*

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- Can identify some interesting features just from operators:

  Example: for B-viol. found

$$\left.\begin{array}{c}\text{assume Pati-Salam compatibility}\\ \text{allow } \overline{U}\,\overline{D}\,\overline{D}\\ \text{forbid } L\,H_u\end{array}\right\} \curvearrowright \text{Forbid Weinberg operator } L\,H_u\,L\,H_u$$

  → B-viol. models consistent with pheno + Pati—Salam prefer Dirac neutrino masses

L-violating Symmetries

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- **"non-perturbative"** $\mathbb{Z}_3^R$

  – non-universal anomalies *(can prove no solution with uni. anom.)*

field	Q	$\overline{U}$	$\overline{D}$	L	$\overline{E}$	H_u	H_d	θ
$\mathbb{Z}_3^R$	1	1	1	1	1	0	0	1

  – no B and L-viol. @ renormalizable level, only "non-perturbatively"

  → assuming R symmetry breaking is of order $m_{3/2}$

$$\mathscr{W}_{\text{eff}} \supset \frac{m_{3/2}}{M_{\text{P}}}\,L\,L\,\overline{E} + \frac{m_{3/2}}{M_{\text{P}}}\,Q\,L\,\overline{D} + \frac{m_{3/2}}{M_{\text{P}}}\,\overline{U}\,\overline{D}\,\overline{D}$$

  – LH_u is suppressed by $m_{3/2}^2/M_P$, but the μ term is of order $m_{3/2}$

L-violating Symmetries

- L-violating symmetries *(L-viol. at renorm. level)* are disfavored

  → get $L\,H_u$ of μ-term size [Acharya, Kane *et. al.* (2014)]

- Avoid conclusion if require μ and L-viol. terms to arise after R symmetry breaking
- **“non-perturbative” $\mathbb{Z}_3^R$**
  - non-universal anomalies *(can prove no solution with uni. anom.)*

field	Q	$\overline{U}$	$\overline{D}$	L	$\overline{E}$	H_u	H_d	θ
$\mathbb{Z}_3^R$	1	1	1	1	1	0	0	1

  - no B and L-viol. @ renormalizable level, only “non-perturbatively”

  → assuming R symmetry breaking is of order $m_{3/2}$

$$\mathscr{W}_{\mathrm{eff}} \supset \frac{m_{3/2}}{M_{\mathrm{P}}}\,L\,L\,\overline{E} + \frac{m_{3/2}}{M_{\mathrm{P}}}\,Q\,L\,\overline{D} + \frac{m_{3/2}}{M_{\mathrm{P}}}\,\overline{U}\,\overline{D}\,\overline{D}$$

  - LH_u is suppressed by $m_{3/2}^2/M_P$, but the μ term is of order $m_{3/2}$

  → <u>counter-example to statement</u>: in L-viol. RPV they are <u>always</u> of same size

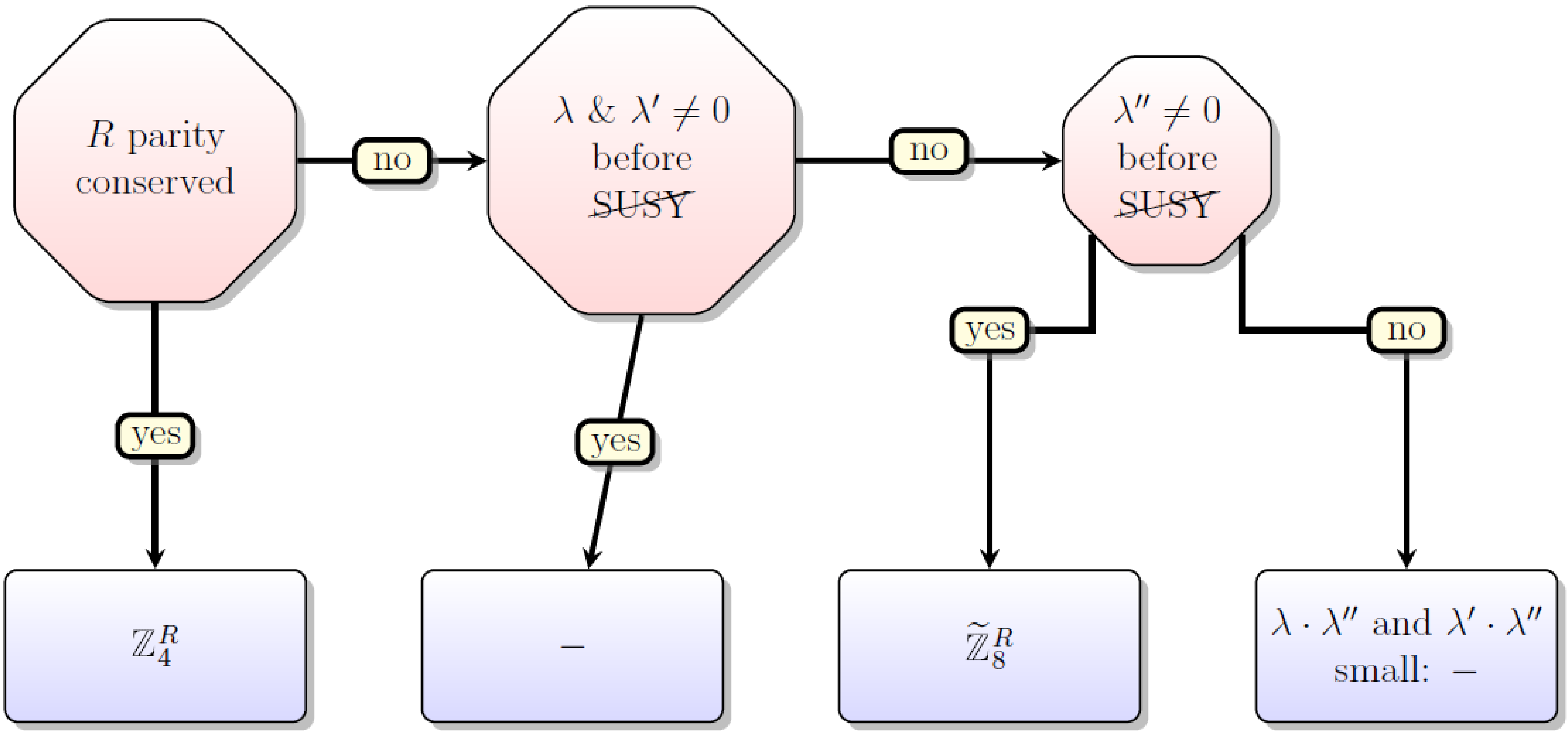

Figure 1: Summary of our results. We present the simplest discrete R symmetries with universal anomalies and the specified properties. The symbol "–" indicates the absence of a solution.

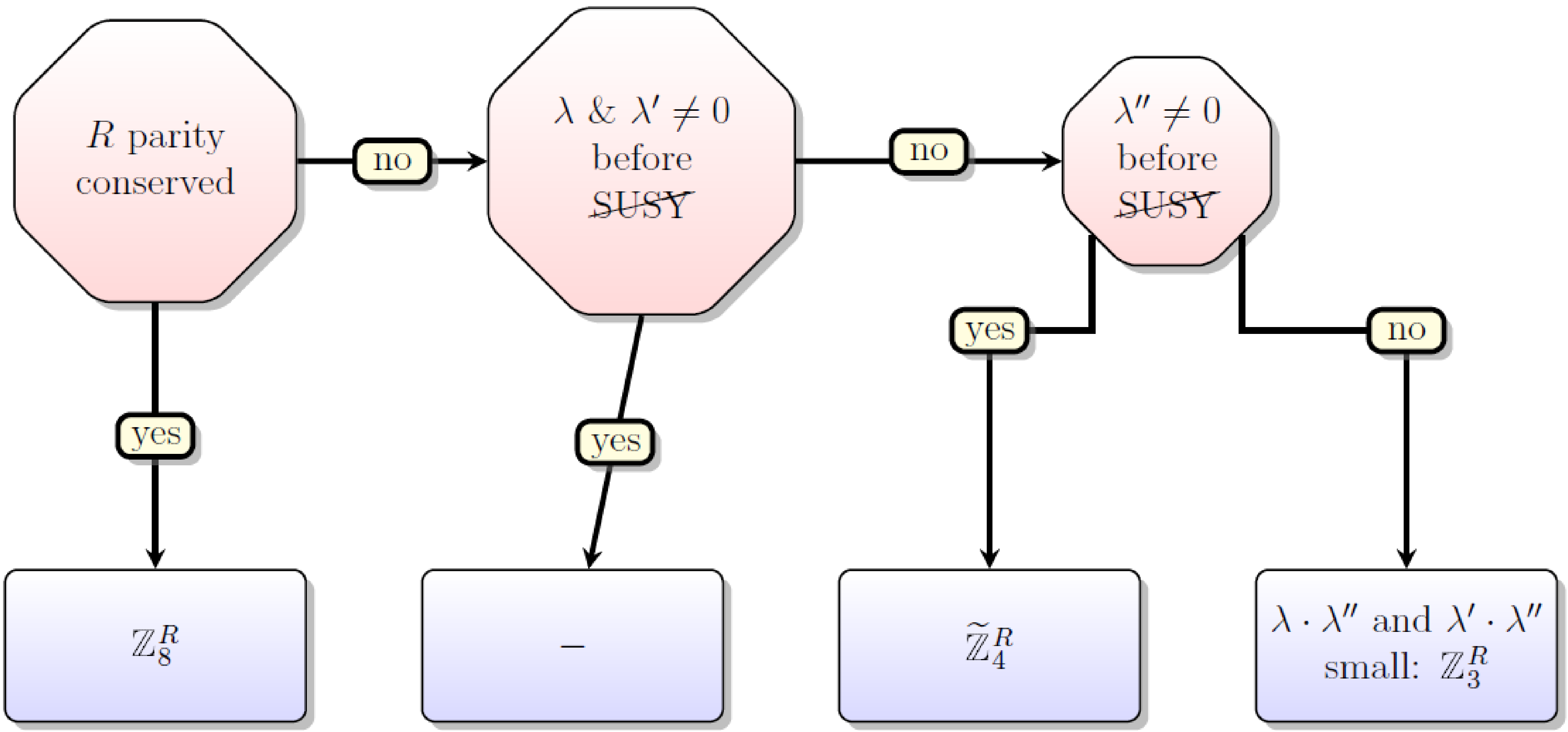

Figure 2: Summary of our results. We present the simplest discrete R symmetries with non–universal anomalies and the specified properties. The symbol "–" indicates the absence of a solution.

What else can we do with them?

- With flavor symmetry:

  – anomalous U(1) always present after compactification

  – long history of using it as flavor symmetry **[Binetruy, Ramond (1994)]**

  (introduce flavon(s), U(1) *charges determine how many flavons couple to different Yukawas, after flavon gets vev different Yukawa "textures" emerge)*

  – combine baryon triality $\mathbb{Z}_3$ and U(1) flavor: $U(1)_R \to \mathbb{Z}_3$ **[Dreiner, Luhn, Murayama, Thormier (2008)]**

  – combine nice $\mathbb{Z}_4^R$ and U(1) flavor: $U(1)_R \to \mathbb{Z}_4^R$ **[Dreiner, Opferkuch, Luhn (2014)]**

  – use $\mathbb{Z}_N^R$ directly as flavor symmetry **[Babu, Gogoladze, Wang (2002)]**

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  – use $\mathbb{Z}_N^R$ directly as flavor symmetry [Babu, Gogoladze, Wang (2002)]

- With baryogenesis:

  – decay of heavy baryon scalar which dominates before nucleosynthesis is a source of both cold DM and baryon asymmetry, model given by $\mathbb{Z}_9 \times \mathbb{Z}_2$

  [Kitano, Murayama, Ratz (2002)]

(… some work in progress)

Seen how constraining proton decay to model building is, so

… a brief word about

recent nucleon decay searches at Super-K

Extending the SM: Grand Unification

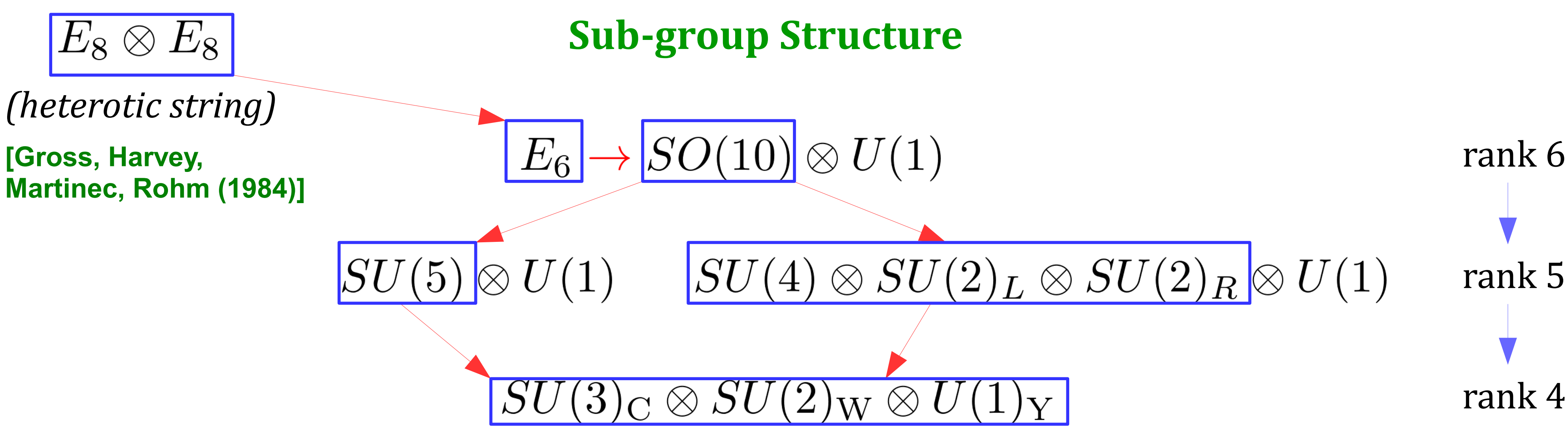
- Can unify 3 seeming unrelated forces of the SM into 1 simple Lie group

$$SU(3)_C \otimes SU(2)_W \otimes U(1)_Y \to G$$

- **Many hints**: gauge coupling “unification” at high energies, charge is quantized, quark and lepton mixing patterns seem to have some structure, etc. *(inflation?)*
- **Most promising candidates** *(anomaly free, rank ≥ 4, contain SM as subgroup)*:

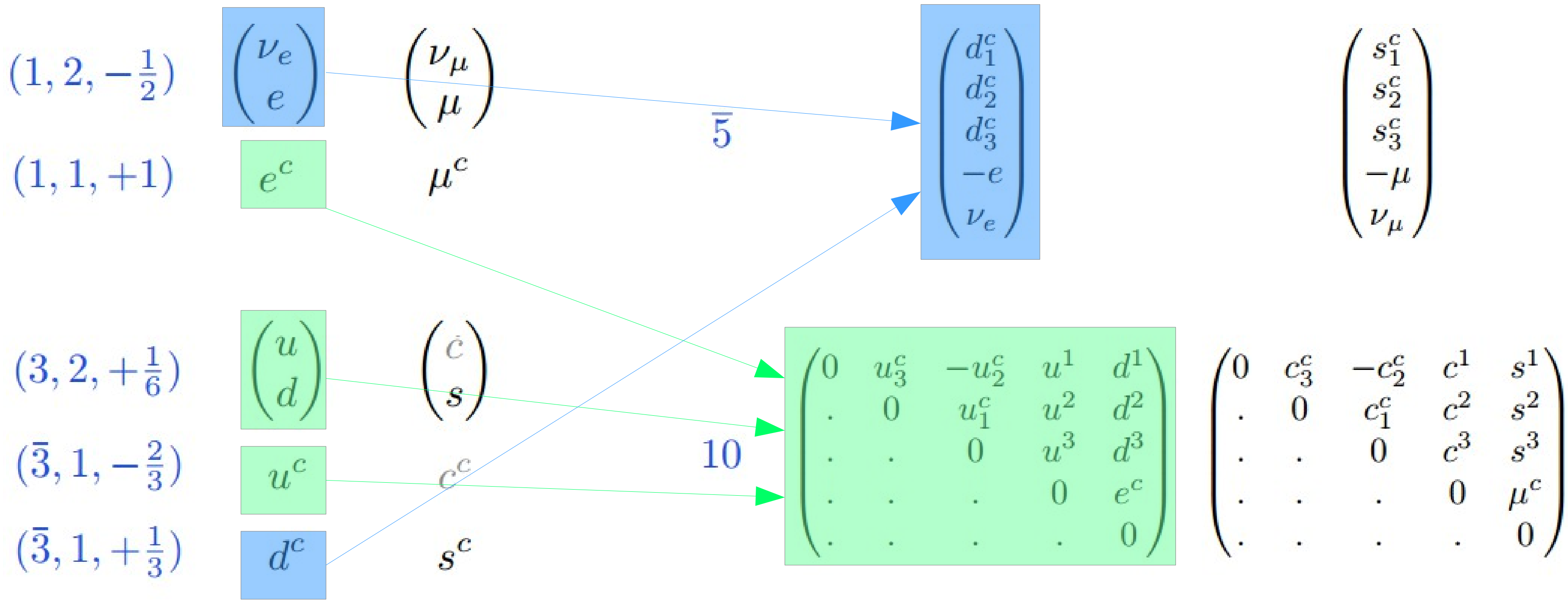
G simple: $SU(5), SO(10), E_6$ [Georgi, Glashow (1974); Fritzsch, Minkowski (1974); Georgi (1974)]

G product group: $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ [Pati, Salam (1974)]



Extending the SM: Grand Unification

- Particle content of the SM fits into $\overline{\mathbf{5}}, \mathbf{10}$ of $SU(5)$



- Gauge and Higgs sectors fit into $\mathbf{24}, \overline{\mathbf{5}}$ of $SU(5)$ *(new g. bosons X, Y and H. triplet T)*
- Even better with $SO(10)$ $\mathbf{16}$ of $SO(10)$ contains all SM particles $+$ rh. ν
- **Many great features**

...but GUT scale ~10^(16) GeV, colliders can't reach ***...but proton decay can!***

Proton Decay: non–SUSY GUTs

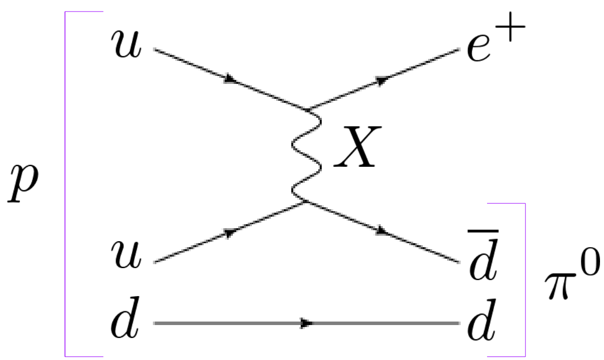
- In non-SUSY models, such as minimal $SU(5)$, proton decay originates from dim-6 operators: $\frac{QQQL}{\Lambda^2}, \frac{\overline{U}\,\overline{U}\,\overline{E}\,\overline{D}}{\Lambda^2}, \frac{\overline{U}\,\overline{E}\,QQ}{\Lambda^2}, \frac{\overline{D}\,\overline{U}\,QL}{\Lambda^2}$

- **The typical dominant non-SUSY decay channel** is $p \to e^+\pi^0$

  → mediated by GUT gauge bosons X and Y, can also be by GUT color triplet Higgs T

  (limit on this leads to the doublet-triplet splitting problem)

sample diagram



$$\tau = \frac{1}{\Gamma} \approx \left[\frac{M_X^2}{\alpha^2}\right]^2 m_p^5$$

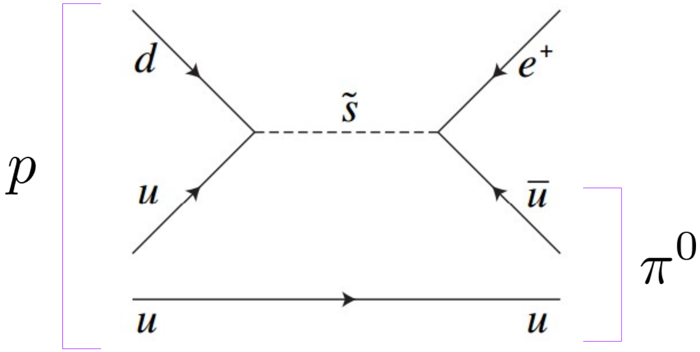
- For minimal *SU(5)*, predicted lifetime is $\tau > 10^{29\pm2}$ years

  → ruled out by experiment with $\tau > 10^{34}$ years **[Nishino *et. al.* (Super-K) (2012)]**

Proton Decay: SUSY GUTs

- SUSY pushes up unification scale → $\tau(p \to e^+\pi^0) > 10^{35-38}$ years
- Sparticles present → new decays start to dominate
  - dimension–4, originating from operators $LQ\overline{D}, \overline{U}\,\overline{D}\,\overline{D}$

    → $\tau \sim 1s$ if squark mass ~ TeV, forbidden by *R* parity

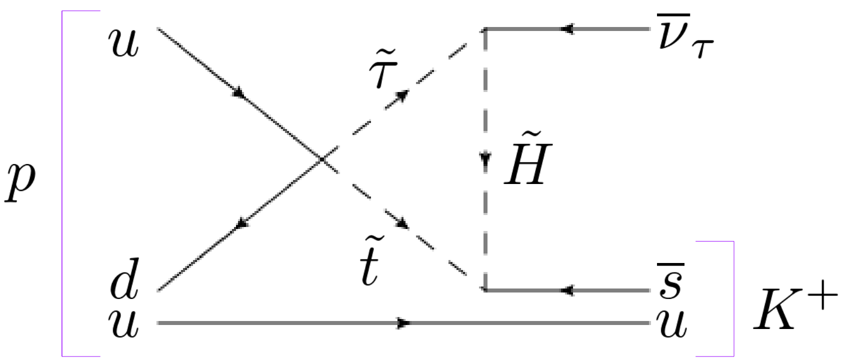


$$\tau = \frac{1}{\Gamma} \approx \left[\frac{M_{\tilde{s}}^2}{\alpha^2}\right]^2 m_p^5$$

  - dimension–5, originating from $\frac{QQQL}{\Lambda}$, **typical dominant SUSY mode** $p \to K^+\,\overline{\nu}$

with predicted lifetime of around

$\tau > 10^{29-35}$ years



Searching for Proton Decay: Water Cherenkov Detectors

- To see if proton lives longer than 10^{31} years, can either look at 1 proton for

  10^{32} years ... **OR ...** look at 10^{33} protons *(~10 kiloton)* for 1 month

  → large underground detectors → water Cherenkov detectors, cheap + large

- **(1979) Irvine–Michigan–Brookhaven** *(IMB)*

  – no proton found, limit $\tau(p \to e^{+}\pi^{0}) > 10^{32}$ years (1990)

  – saw SN1987A neutrinos and atmospheric neutrino "anomaly" *(later oscillations)*

- **(1980~) Kamiokande**

  – saw SN1987A neutrinos and atmospheric

  neutrino "anomaly", solar neutrinos

  Koshiba's Nobel Prize (2002)

IMB experiment

Searching for Proton Decay: Water Cherenkov Detectors

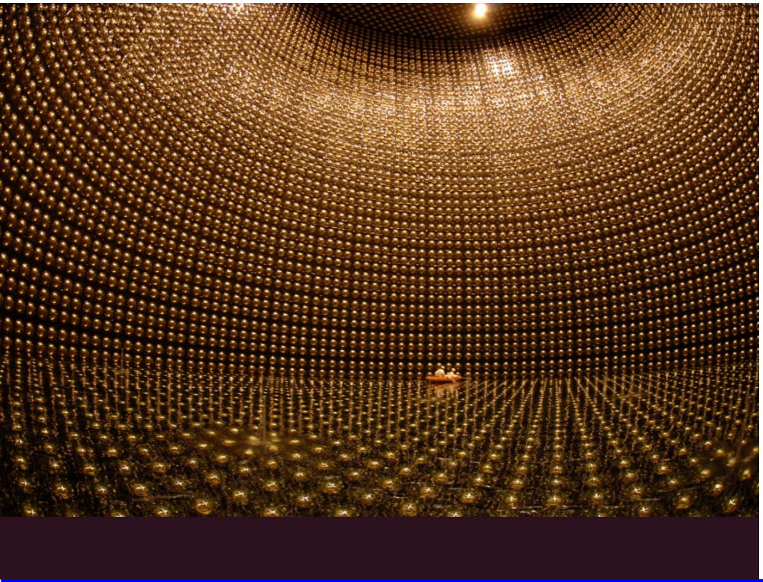
- **(1996 -) Super–Kamiokande** *(SK)*
  - largest water-C. detector, successor of Kamiokande
  - discovery of neutrino oscillations (1998)

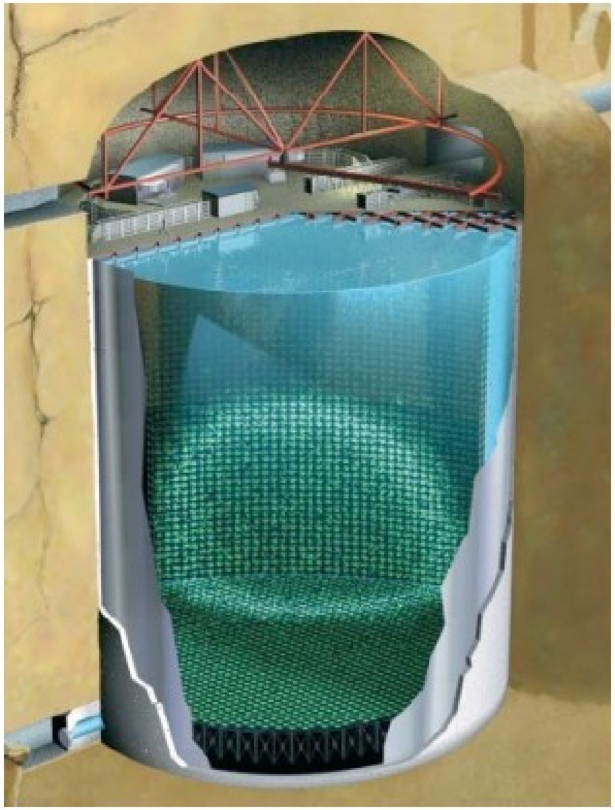
  → neutrinos have mass
  - lifetime of proton

$$\tau(p \to e^+\pi^0) > 10^{34} \quad \text{years}$$

from Super-K Webpage / Ed Kearns, NEPPRS 09

SK experiment





Super-Kamiokande

22.5 kton fiducial volume
$7.5\times10^{33}\,p + 6\times10^{33}\,n$

SK-I: 1996 - 2001
11146 50-cm inner PMTs , 40% coverage
1885 20-cm outer PMTs

SK-II: Jan 2003 - Oct 2005
Recovery from accident
5182 50-cm inner PMTs
Acrylic + FRP protective
Outer detector fully restored

SK-III: May 2006 - August 2008
Restored 40% coverage
Outer detector segmented (top | barrel | bottom)

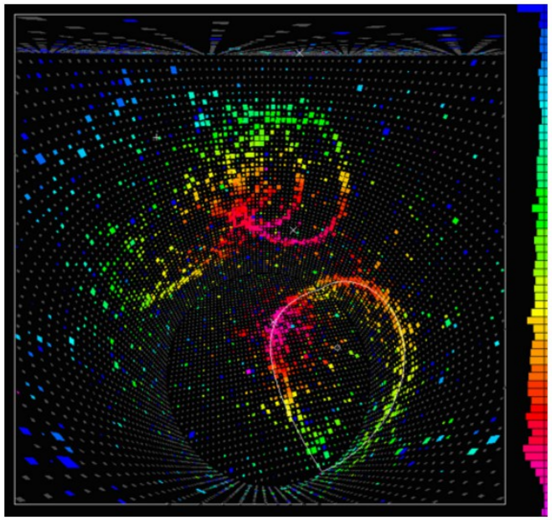
SK-IV: September 2008 -
SK-IV Replace all electronics – 2008
T2K beam – late 2009
Add gadolinium - 201?

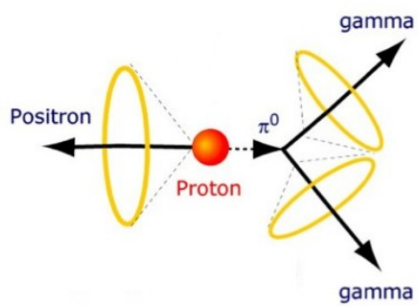
Proton Decay at Super–Kamiokande

"Golden channel": $p \to \pi^0 e^+$, $\pi^0 \to 2\gamma$

$p_\pi = p_e$ = 459 MeV

$p_{\gamma/\pi R}$ = 68 MeV



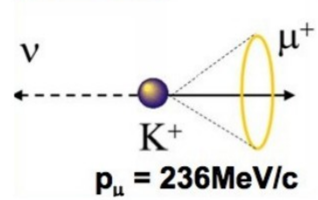


"Silver channel": $p \to K^+ \nu$

p_K = 340 MeV

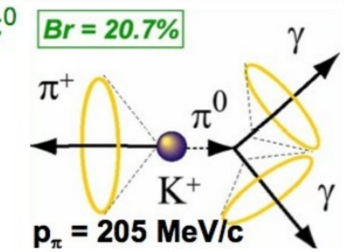
Kaons don't shine !

$K^+ \to \mu^+ + \bar{\nu}$ *Br = 63.5%*



- single cone

$K^+ \to \pi^+ + \pi^0$ *Br = 20.7%*

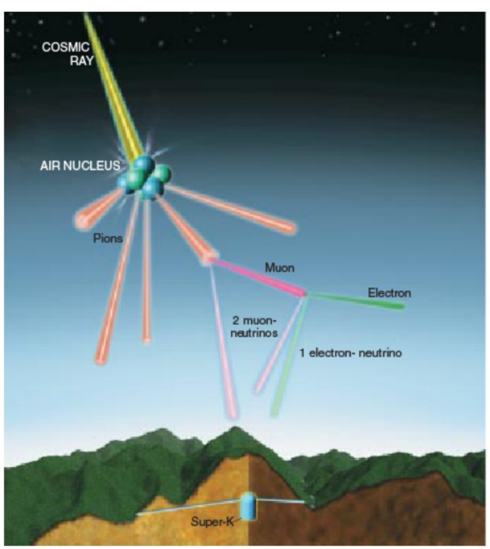


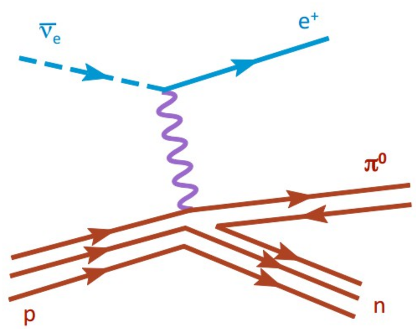
- 2 EM cones
- little opposite-side activity

About one order of magnitude less sensitive than $p \to \pi^0 e^+$

from Malinsky, Brussels '13

Background: Atmospheric Neutrinos





from Ed Kearns, NEPPRS '09

Setting a limit

a simple calculation of the rate if we measured something:

$$\frac{\tau}{B} = \frac{\lambda\varepsilon}{n-b}$$

n = number of observed events
b = expected number of background events
λ = exposure = $N_{proton} \cdot \Delta t$
ε = efficiency

$$\frac{\tau}{B} = \frac{\lambda\varepsilon}{S_{90}}$$

⇐ a simple calculation of a 90% CL limit, but...
does not take into account n=0 properly (see F&C)
and does not take into account systematic uncertainty

$$S_{90} = \frac{\int_0^{S_{90}} P_{poiss.}(n, x+b)\,dx}{\int_0^{\infty} P_{poiss.}(n, x+b)\,dx}$$

$$\tau / B > 8.9 \times 10^{33} \text{ years}$$

treatment of limit using Bayes theorem to incorporate systematic uncertainty:

$$P(\Gamma \mid n) = \iiint \frac{e^{-\Gamma\lambda\varepsilon + b}(\Gamma\lambda\varepsilon + b)^n}{n!} P(\Gamma)\, P(\lambda)\, P(\varepsilon)\, P(b)\, d\Gamma\, d\lambda\, d\varepsilon\, db$$

$$\tau / B(e^+\pi^0) > 8.2 \times 10^{33} \text{ years}$$

Story is Actually More Complex ...

- Proton decay hasn't been found in ~30 years, why continue looking?
- **Many models** beyond simple *SU(5)*, proton decay can rule some of them out ...

[Bueno (2007)]

Model	Ref.	Modes	τ_N (years)
Minimal $SU(5)$	Georgi, Glashow [2]	$p \to e^+\pi^0$	$10^{30} - 10^{31}$
Minimal SUSY $SU(5)$	Dimopoulos, Georgi [11], Sakai [12] Lifetime Calculations: Hisano, Murayama, Yanagida [13]	$p \to \bar{\nu}K^+$ $n \to \bar{\nu}K^0$	$10^{28} - 10^{32}$
SUGRA $SU(5)$	Nath, Arnowitt [14, 15]	$p \to \bar{\nu}K^+$	$10^{32} - 10^{34}$
SUSY $SO(10)$ with anomalous flavor $U(1)$	Shafi, Tavartkiladze [16]	$p \to \bar{\nu}K^+$ $n \to \bar{\nu}K^0$ $p \to \mu^+K^0$	$10^{32} - 10^{35}$
SUSY $SO(10)$ MSSM (std. $d=5$)	Lucas, Raby [17], Pati [18]	$p \to \bar{\nu}K^+$ $n \to \bar{\nu}K^0$	$10^{33} - 10^{34}$ $10^{32} - 10^{33}$
SUSY $SO(10)$ ESSM (std. $d=5$)	Pati [18]	$p \to \bar{\nu}K^+$	$10^{33} - 10^{34}$ $\lesssim 10^{35}$
SUSY $SO(10)/G(224)$ MSSM or ESSM (new $d=5$)	Babu, Pati, Wilczek [19, 20, 21], Pati [18]	$p \to \bar{\nu}K^+$ $p \to \mu^+K^0$ $B \sim (1-50)\%$	$\lesssim 2 \cdot 10^{34}$
SUSY $SU(5)$ or $SO(10)$ MSSM ($d=6$)	Pati [18]	$p \to e^+\pi^0$	$\sim 10^{34.9\pm1}$
Flipped $SU(5)$ in CMSSM	Ellis, Nanopoulos and Wlaker[22]	$p \to e/\mu^+\pi^0$	$10^{35} - 10^{36}$
Split $SU(5)$ SUSY	Arkani-Hamed, *et. al.* [23]	$p \to e^+\pi^0$	$10^{35} - 10^{37}$
$SU(5)$ in 5 dimensions	Hebecker, March-Russell[24]	$p \to \mu^+K^0$ $p \to e^+\pi^0$	$10^{34} - 10^{35}$
$SU(5)$ in 5 dimensions option II	Alciati *et.al.*[25]	$p \to \bar{\nu}K^+$	$10^{36} - 10^{39}$
GUT-like models from Type IIA string with D6-branes	Klebanov, Witten[26]	$p \to e^+\pi^0$	$\sim 10^{36}$

TABLE I: Summary of the expected nucleon lifetime in different theoretical models.

..... **many predicted modes**

$p \to \mu^-\pi^+K^+$	$B+L$
$n \to \bar{n}$	$\Delta B = 2$, TeV < scale < GUT
$pp \to K^+K^+$	$\lambda''_{uds} < 10^{-8}$
$p \to e^-\pi^+\pi^+\nu\,\nu$	6 dimensions
$n \to \nu\,\nu\,\nu$	invisible
$p \to e^+\,\gamma$	radiative

+ *others*

from Ed Kearns, NEPPRS '09

- Many predictions in the $\tau \approx 10^{34-36}$ year range

... are we on the verge of discovery *(Super–K, or near future Hyper–K)* ?

Some Novel SK Searches

- Minimal *SU(5)* and SUSY *SU(5)* ~ *ruled out by experiment*
- Can consider larger group, eg. *SO(10)* → *Pati-Salam*
- In certain variations trilepton modes can be significant

  → maybe also useful for baryogenesis **[Gu and Sarkar (2012)]:**

  → predicted lifetimes $\tau \approx 10^{31-33}$ years **[Pati (1984), Gu and Sarkar (2012)]:**

$p \to e^+ \nu\nu$

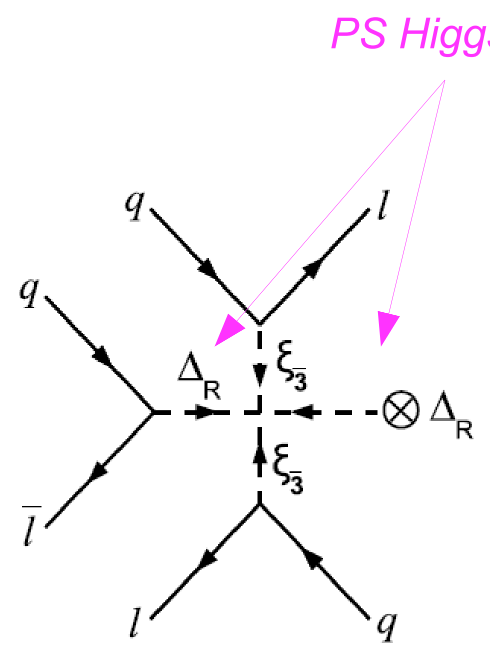
$p \to \mu^+ \nu\nu$

- First 3-body decay search in SK
- In SK can't see neutrinos, only spectra from e+, μ+

  (can use μ → eνν *spectra to describe the above)*

  Chen, Takhistov [PRD (2014)]

[Pati (1984)]:



- Spectral fit analysis

  → set limit ~ 10^32 years (>1 order improvement)

  Takhistov *et. al.* (Super-K Collab.) [PRL (2014)]

Some Novel SK Searches

Other modes which can be similarly analyzed *(spectral fit to momenta)*:

- $p \to e^+ X$ $p \to \mu^+ X$ inclusive decays *(X is invisible particle)*
- $n \to \gamma\nu$ radiative mode **[Nath and Perez (2007)]:**
- $np \to e^+\nu$ $np \to \mu^+\nu$ $np \to \tau^+\nu$ dinucleon decays, which can arise in

models with extended Higgs sector, may be connected to baryogenesis **[Arnellos Marciano (1982), Arnold, Fornal, Wise (2013); Bryman (2014)]:**

- **Results for SK search of nucleon decay modes with charged lepton + inv.:**

Takhistov *et. al.* (Super-K Collab.) (accepted to PRL)

Mode	SK I-IV Sensitivity (years)	SK I-IV Limit (years)	PDG Limit (years)
$p \to e^+ X$	$7.9 \cdot 10^{32}$	$7.9 \cdot 10^{32}$	–
$p \to \mu^+ X$	$7.7 \cdot 10^{32}$	$4.1 \cdot 10^{32}$	–
$n \to \nu\gamma$	$5.8 \cdot 10^{32}$	$5.5 \cdot 10^{32}$	$2.8 \cdot 10^{31}$
$np \to e^+\nu$	$9.9 \cdot 10^{31}$	$2.6 \cdot 10^{32}$	$2.8 \cdot 10^{30}$
$np \to \mu^+\nu$	$1.1 \cdot 10^{32}$	$2.2 \cdot 10^{32}$	$1.6 \cdot 10^{30}$
$np \to \tau^+\nu$	$1.1 \cdot 10^{31}$	$2.9 \cdot 10^{31}$	–

Preliminary

not in PDG, first ever search

Other Recent SK Searches

$pp \to \pi^+\pi^+$ [Gustafson *et. al.* (2015)]:

$n \to \nu\pi^0$ [Abe *et. al.* (2014)]:

$p \to \nu\pi^+$ [Abe *et. al.* (2014)]:

$pp \to K^+K^+$ [Litos *et. al.* (2014)]:

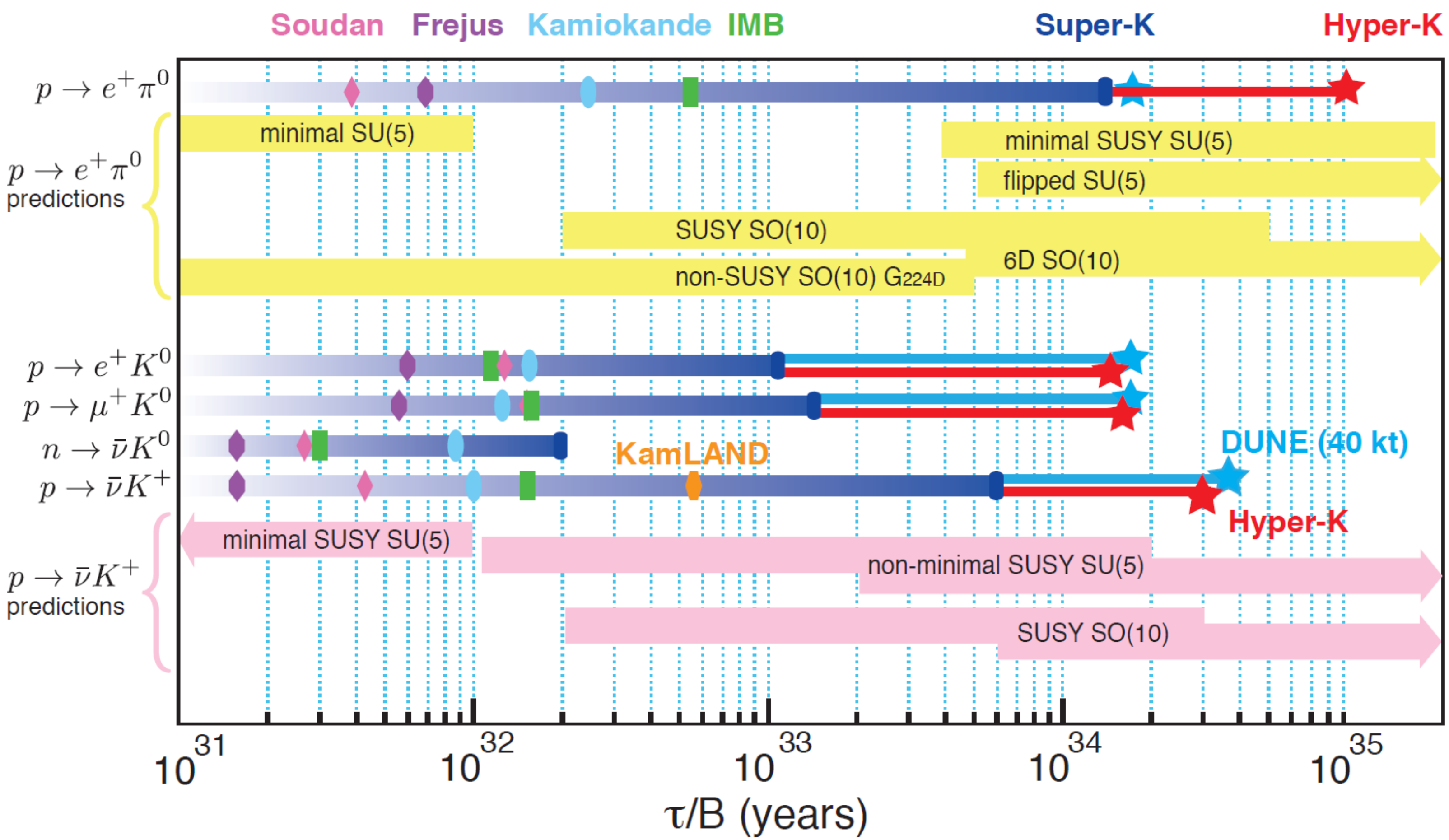
....

Future

– HyperK is bigger version of SuperK (20 x SK size)

– aside mass hierarchy and CP violation also improved proton decay search ....

(1st proto-collaboration meeting June 2015)



from Ed Kearns

Thank You!