

# LIGHT FIELDS AND FLAT DIRECTIONS FROM NONLINEAR SIGMA MODELS IN SUPERGRAVITY

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(In collaboration with Simeon Hellerman & Tsutomu Yanagida)

[arXiv:1411.3720](https://arxiv.org/abs/1411.3720)



VANDERBILT  
UNIVERSITY

Susy 2015  
Lake Tahoe, California  
August 23—29, 2015

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  - What about other light fields?
- **Inflation** may be at a *high scale* (*chaotic inflation*): how are higher dimensional operators *suppressed*?
  - There are also fundamental questions, like the *origin* of the inflaton potential and the *large field values*

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A NONLINEAR SIGMA MODEL  
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For our earlier work on sigma models for [charge quantization](#) and  
quantum number relations in nonlinear sigma models

see [arXiv:1309.0692](#), [1312.6889](#)



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- The structure of the supergravity potential is why it is difficult to construct models of chaotic inflation: the potential has an **exponential** factor of  $e^{K/M_p^2}$ 
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- The solution from KYY [1] is to use a **shift symmetry**:

$$\Phi \rightarrow \Phi + iCM_p \quad \Rightarrow \quad K(\Phi + \Phi^*)$$

- Then the exponential factor does not contain the imaginary part of the field and the potential is **exactly flat**



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- Kugo-Yanagida (KY [4]): we need to **break** any **U(1)** factors



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- We use the non-compact model  $U(3)/SU(2) \times U(1) \cong SU(3)/SU(2)$   
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  - The unbroken subgroup is gauged as the **electroweak** group of the SM
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- There is an **equivalence** between this model and the previous Kähler potential [for  $SU(3)/SU(2) \times U(1)$ ], and an explicit connection between the different ways of understanding NLSMs in supergravity

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  - This allows one to connect to a *linear* sigma model

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  - This is a *compact* NLSM with **radius quantized in units of  $M_p$**
- Can there be a relation between these different ways of coupling a NLSM to supergravity?



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$$K = f_\phi^2 \left[ \log \left( 1 + \frac{\phi\phi^*}{f_\phi^2} \right) + \frac{1}{f_\phi} (Z + Z^*) \right] + \frac{a^2}{2} f_\phi^2 \left[ \log \left( 1 + \frac{\phi\phi^*}{f_\phi^2} \right) + \frac{1}{f_\phi} (Z + Z^*) \right]^2 + \frac{b}{2} (Z + Z^*)^2 + XX^*.$$



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- One branch explains the **large field values**, while the other explains the **shift symmetry and extra field**

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- We have a **general model-building framework which simultaneously gives light fields and chaotic inflation**
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- The *structure* of **NLSMs and supergravity** was crucial and with no additional ad hoc ingredients
- Along the way we have understood and connected different proposals for coupling to supergravity: extra fields/extended supergravity multiplet, broken  $U(1)$ s, quasi-NGBs
- Finally, we have **conjectured a link to Witten-Bagger models and an origin for large field values** (string theory realization?)





### Our work:

S. Hellerman, J. Kehayias, and T.T. Yanagida, “Charge quantization in the  $CP(1)$  non-linear  $\sigma$ -model,” Phys. Lett. B728, 358–362 (2014), [arXiv:1309.0692](#) [hep-th];

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[1] M. Kawasaki, M. Yamaguchi, and T. Yanagida, “Natural chaotic inflation in supergravity,” Phys. Rev. Lett. 85, 3572–3575 (2000), [arXiv:hep-ph/0004243](#) [hep-ph]; “Natural chaotic inflation in supergravity and leptogenesis,” Phys. Rev. D63, 103514 (2001), [arXiv:hep-ph/0011104](#) [hep-ph].

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- This is a function of  $e^{2\kappa(Z+Z^\dagger)} + e^{\kappa(Z+Z^\dagger)} (|\phi_1|^2 + |\phi_2|^2)$   
or (after a field redefinition)  $x = e^{2\kappa(Z+Z^\dagger)} \left( 1 + \phi'_i \phi'^{\dagger}_i \right)$

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- Once we define  $y \equiv \log x = \log \left( 1 + \phi'_i \phi_i'^{\dagger} \right) + 2\kappa (z + z^{\dagger})$  we see we reproduce the form of the Kähler potential in KS [3] for a  $\mathbb{C}\mathbb{P}^2 \cong SU(3)/SU(2) \times U(1)$  model

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- This is exactly the condition given in Kugo-Yanagida [4]



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- In this model this is not a problem, it is simply a **scaling** in  $\xi$  or  $\phi$