# Partially Natural Two Higgs Doublet Models

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# **References**

- 1. P. Draper, H.E. Haber and J. Ruderman, "Partially Natural Two Higgs Doublet Models," preprint in preparation.
- P.M. Ferreira, H.E. Haber and J.P. Silva, "Generalized CP symmetries and special regions of parameter space in the two-Higgs-doublet model," Phys. Rev. D 79, 116004 (2009) [arXiv:0902.1537 [hep-ph]].
- S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," Phys. Rev. D 72, 035004 (2005) [Erratum: ibid. 72, 099902 (2005)] [hep-ph/0504050].

Related work with a slightly different motivation and emphasis:

A. Pierce and J. Thaler, "Natural Dark Matter from an Unnatural Higgs Boson and New Colored Particles at the TeV Scale," JHEP **0708**, 026 (2007) [hep-ph/0703056].

#### Motivation for this work

Why is the scale of electroweak symmetry breaking (EWSB) v = 246 GeV, when its natural value is at some large cutoff scale  $\Lambda \gg v$ ?

1. The standard paradigm is "naturalness" of the electroweak symmetry breaking mechanism. Realizations include TeV-scale supersymmetry, composite Higgs models, ...

2. Given the absence of experimental evidence for physics beyond the Standard Model (BSM), there is an alternative approach where the mass scale of EWSB is finely tuned due to environmental selection in a multiverse.

In the Standard Model (SM), one tuning is required to achieve a Higgs mass at 125 GeV. If additional Higgs states are present in some generic BSM theory in which the EWSB scale is set by a single fine-tuning, then one would expect these non-minimal Higgs boson masses to reside near  $\Lambda$  in the absence of a natural explanation of the EWSB scale. On the other hand, suppose additional Higgs scalars are discovered with masses below 1 TeV. What is one to conclude?

1. The simplest explanation is that the "natural" dynamics responsible for the EWSB scale also provides for non-minimal Higgs states with masses of  $\mathcal{O}(v)$ . The MSSM provides an example of this.

2. In contrast, the non-minimal Higgs scalars are unlikely to be light because of selection effects, as suggested by the decoupling limit of many BSM theories.

We wish to explore a third possibility in which one fine-tuning is required to obtain the EWSB scale and the Higgs mass of 125 GeV. But, additional Higgs scalars are also light due to an approximate symmetry that links their mass scale to the scale of EWSB. We call a two Higgs doublet model (2HDM) of this type *partially natural*, in which one fine-tuning is sufficient to obtain the entire Higgs spectrum with masses of  $\mathcal{O}(v)$ .

# <u>Outline</u>

- The general 2HDM and its fine-tuning conditions
- Symmetries of the 2HDM scalar potential
  - Exceptional region of parameter space (with one fine-tuning condition)
- Extending the Yukawa sector
  - The need for mirror fermions
  - Softly-broken symmetries
  - The 2HDM vacuum state and the corresponding scalar spectrum
  - Phenomenological constraints and implications
- Final comments
  - Supersymmetric extensions
  - Challenges for split SUSY with a pair of light  $Y = \pm 1$  Higgs doublets

## The Two-Higgs Doublet Model (2HDM)

The scalar fields of the 2HDM are complex SU(2) doublet, hypercharge-one fields,  $\Phi_1$  and  $\Phi_2$ , where the corresponding vacuum expectation values (vevs) are  $\langle \Phi_i \rangle = |v_i| e^{i\xi_i} / \sqrt{2}$ , and  $v^2 \equiv |v_1|^2 + |v_2|^2 = (246 \text{ GeV})^2$ . The most general renormalizable SU(2)×U(1) scalar potential is given by

$$\begin{split} \mathcal{V} &= m_{11}^2 \Phi_1^{\dagger} \Phi_1 + m_{22}^2 \Phi_2^{\dagger} \Phi_2 - \left( m_{12}^2 \Phi_1^{\dagger} \Phi_2 + \text{h.c.} \right) + \frac{1}{2} \lambda_1 \left( \Phi_1^{\dagger} \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^{\dagger} \Phi_2 \right)^2 \\ &+ \lambda_3 \Phi_1^{\dagger} \Phi_1 \Phi_2^{\dagger} \Phi_2 + \lambda_4 \Phi_1^{\dagger} \Phi_2 \Phi_2^{\dagger} \Phi_1 + \left[ \frac{1}{2} \lambda_5 \left( \Phi_1^{\dagger} \Phi_2 \right)^2 + \left[ \lambda_6 \Phi_1^{\dagger} \Phi_1 + \lambda_7 \Phi_2^{\dagger} \Phi_2 \right] \Phi_1^{\dagger} \Phi_2 + \text{h.c.} \right], \end{split}$$

where  $m_{12}^2$  and  $\lambda_5$ ,  $\lambda_6$  and  $\lambda_7$  are potentially complex; all other scalar potential parameters are real.

After spontaneous symmetry breaking of  $SU(2) \times U(1)$  to  $U(1)_{EM}$ , there are five physical Higgs states—three neutral scalars and a charged Higgs pair. One of these scalars is identified with the observed Higgs boson with  $m_h = 125$  GeV. In the SM, one fine-tuning is required to achieve the observed Higgs mass. If the masses of all the other physical scalars are also of order the electroweak scale, how many additional fine-tunings of the squared-mass parameters are required?

In the most general 2HDM, the fields  $\Phi_1$  and  $\Phi_2$  are indistinguishable. Thus, it is always possible to define two orthonormal linear combinations of the two doublet fields without modifying any prediction of the model.

### The Higgs basis

It is convenient to define new Higgs doublet fields:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v},$$

where  $\langle H_1^0 \rangle = v/\sqrt{2}$  and  $\langle H_2^0 \rangle = 0$  and v = 246 GeV. This is the *Higgs basis*.

In the Higgs basis, the scalar potential is:

$$\begin{split} \mathcal{V} &= Y_1 H_1^{\dagger} H_1 + Y_2 H_2^{\dagger} H_2 + \left( Y_3 H_1^{\dagger} H_2 + \text{h.c.} \right) + \frac{1}{2} Z_1 \left( H_1^{\dagger} H_1 \right)^2 + \frac{1}{2} Z_2 \left( H_2^{\dagger} H_2 \right)^2 \\ &+ Z_3 H_1^{\dagger} H_1 H_2^{\dagger} H_2 + Z_4 H_1^{\dagger} H_2 H_2^{\dagger} H_1 + \left[ \frac{1}{2} Z_5 \left( H_1^{\dagger} H_2 \right)^2 + [Z_6 H_1^{\dagger} H_1 + Z_7 H_2^{\dagger} H_2] H_1^{\dagger} H_2 + \text{h.c.} \right], \end{split}$$

where  $Y_1$ ,  $Y_2$  and  $Z_1, \ldots, Z_4$  are real and  $Y_3$ ,  $Z_5$ ,  $Z_6$  and  $Z_7$  are potentially complex. Assuming that  $|\lambda_i|/(4\pi) \leq O(1)$  (so that the scalar potential satisfies unitarity constraints at tree-level), it follows that the  $Z_i$  cannot become arbitrarily large.

After minimizing the scalar potential,

$$Y_1 = -\frac{1}{2}Z_1v^2$$
,  $Y_3 = -\frac{1}{2}Z_6v^2$ .

One fine tuning of the parameter  $Y_1$  is required to obtain v = 246 GeV. The scalar potential minimum condition guarantees that  $Y_3 \sim \mathcal{O}(v^2)$ . But, to obtain all Higgs masses of  $\mathcal{O}(v)$ , a second fine tuning  $Y_2 \sim \mathcal{O}(v^2)$  is required.

#### The physical Higgs masses

In the alignment limit where one of the neutral Higgs bosons h is SM-like, we have to good approximation  $m_h^2 \simeq \frac{1}{2}Z_1v^2$ , which yields  $Z_1 \simeq 0.26$ . This means that  $Y_1 \sim \mathcal{O}(v^2)$ , which constitutes the first fine tuning. Note that  $Y_3 \sim \mathcal{O}(Y_1)$  by virtue of the scalar potential minimum conditions.

The charged Higgs boson squared mass is  $m_{H^{\pm}}^2 = Y_2 + \frac{1}{2}Z_3v^2$ . Thus, a second fine tuning,  $Y_2 \sim \mathcal{O}(v^2)$  is requires to ensure that the charged Higgs mass is of order the electroweak scale.

The sum of the squared-masses of the three neutral Higgs bosons is equal to the trace of the corresponding squared-mass matrix  $\mathcal{M}_H^2$ ,

Tr 
$$\mathcal{M}_H^2 = 2Y_2 + (Z_1 + Z_3 + Z_4)v^2$$
.

Having performed the second fine tuning of  $Y_2$ , we are now assured that all scalar masses are of order the electroweak scale.

The scalar potential of the most general 2HDM is governed by 11 free parameters: 1 vev, 8 real parameters and two relative phases. It is possible to impose a discrete or continuous global symmetry on the Higgs potential [beyond the hypercharge  $U(1)_Y$ ] to reduce the number of 2HDM parameters.

symmetry	$m_{22}^2$	$m_{12}^2$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\lambda_6$	$\lambda_7$
$\mathbb{Z}_2$		0					0	0
$\Pi_2$	$m_{11}^2$	real	$\lambda_1$			real		$\lambda_6^*$
U(1)		0				0	0	0
SO(3)	$m_{11}^2$	0	$\lambda_1$		$\lambda_1 - \lambda_3$	0	0	0
CP1		real				real	real	real
CP2	$m_{11}^2$	0	$\lambda_1$					$-\lambda_6$
CP3	$m_{11}^2$	0	$\lambda_1$			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0

Classification of 2HDM scalar potential symmetries and their impact on the coefficients of the scalar potential in a basis where the symmetry is manifest [Ivanov; Ferreira, Haber and Silva].

#### Higgs family symmetries

$\mathbb{Z}_2$ :	$\Phi_1 \to \Phi_1,$	$\Phi_2 \rightarrow -\Phi_2$
$\Pi_2$ :	$\Phi_1 \longleftrightarrow \Phi_2$	
$U(1)_{PQ}$ [Peccei-Quinn]:	$\Phi_1 \to e^{-i\theta} \Phi_1,$	$\Phi_2 \to e^{i\theta} \Phi_2$
SO(3):	$\Phi_a  ightarrow U_{ab} \Phi_b$ ,	$U \in \mathrm{U}(2)/\mathrm{U}(1)_Y$

#### Generalized CP (GCP) transformations

CP1:  $\Phi_1 \to \Phi_1^*$ ,  $\Phi_2 \to \Phi_2^*$ CP2:  $\Phi_1 \to \Phi_2^*$ ,  $\Phi_2 \to -\Phi_1^*$ CP3:  $\Phi_1 \to \Phi_1^* c_{\theta} + \Phi_2^* s_{\theta}$ ,  $\Phi_2 \to -\Phi_1^* s_{\theta} + \Phi_2^* c_{\theta}$ , for  $0 < \theta < \frac{1}{2}\pi$ 

where  $c_{\theta} \equiv \cos \theta$  and  $s_{\theta} \equiv \sin \theta$ . Some observations of note:

- 1.  $\Pi_2$  symmetry is equivalent to  $\mathbb{Z}_2$  symmetry in a different basis.
- 2. Applying  $\mathbb{Z}_2$  and  $\Pi_2$  simultaneously  $\iff$  CP2 in a different basis.
- 3. Applying U(1)<sub>PQ</sub> and  $\Pi_2$  simultaneously  $\iff$  CP3 in a different basis.

#### Exceptional region of the parameter space (ERPS)

An exceptional region of the 2HDM parameter space (first identified by Davidson and Haber) consists of:

ERPS:  $m_{22}^2 = m_{11}^2$ ,  $m_{12}^2 = 0$ ,  $\lambda_1 = \lambda_2$ ,  $\lambda_7 = -\lambda_6$ 

The corresponding conditions in the Higgs basis are,

$$Y_2 = Y_1, \qquad Y_3 = Z_6 = Z_7 = 0, \qquad Z_1 = Z_2.$$

Indeed, in the ERPS one of the two fine-tuning conditions is removed.

The ERPS includes SO(3), CP3 (equivalent to  $U(1)_{PQ} \otimes \Pi_2$  in another basis), and CP2 (equivalent to  $\mathbb{Z}_2 \otimes \Pi_2$  in another basis). To avoid an extra massless Goldstone boson, one must softly-break the SO(3) and CP3 symmetries.

However, none of the ERPS models can be extended to the Yukawa interactions without generating a massless quark or some other phenomenologically untenable feature [P.M. Ferreira and J.P. Silva, Eur. Phys. J. C **69**, 45 (2010)].

# **2HDM with mirror fermions**

To try to save the  $\mathbb{Z}_2 \otimes \Pi_2$  discrete symmetry model we introduce mirror fermions. Focus first on the top sector. SM fermions are denoted by lower case letters (e.g. left-handed doublet fields q and right-handed singlet fields u and d); mirror fermions by upper case letters.

We take the top sector to transform under the discrete symmetries as follows,

$$\Pi_2: q \Leftrightarrow q, \quad u \Leftrightarrow U, \qquad \overline{U} \Leftrightarrow \overline{U},$$
$$\mathbb{Z}_2: q \Leftrightarrow q, \quad u \Leftrightarrow -u, \quad U \Leftrightarrow U, \quad \overline{U} \Leftrightarrow \overline{U}.$$

where  $\overline{U}$  is in the representation conjugate to U (to avoid anomalies).

The Yukawa couplings consistent with the  $\mathbb{Z}_2 \otimes \Pi_2$  discrete symmetry are

$$\mathscr{L}_{\text{Yuk}} \supset y_t \left( q \Phi_2 u + q \Phi_1 U \right) + \text{h.c.}$$

The model is not phenomenologically viable due to experimental limits on mirror fermion masses. Thus, we introduce a vectorlike mass,

$$\mathscr{L}_{\text{mass}} \supset M_U U \overline{U} + \text{h.c.}$$

which preserves the  $\mathbb{Z}_2$  but explicitly breaks the  $\Pi_2$  discrete symmetry. This symmetry breaking is soft, so that  $m_{22}^2 - m_{11}^2$  is protected from quadratic sensitivity to the cutoff scale  $\Lambda$ .

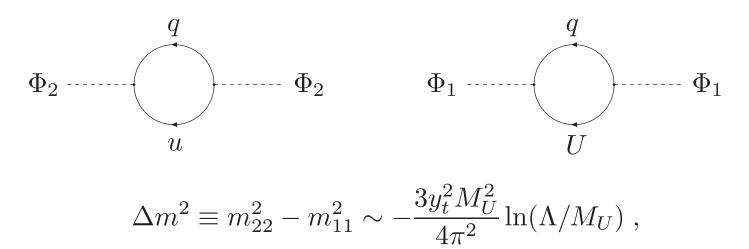
The other SM fermions can also be included by introducing the appropriate mirrors such that

$$\Pi_2: d \Leftrightarrow D, \quad e \Leftrightarrow E, \quad \overline{D} \Leftrightarrow \overline{D}, \quad \overline{E} \Leftrightarrow \overline{E}$$
$$\mathbb{Z}_2: d \Leftrightarrow -d, \quad e \Leftrightarrow -e, \quad D \Leftrightarrow D, \quad E \Leftrightarrow E.$$

The corresponding Yukawa couplings and vectorlike fermion masses are

$$\mathscr{L} \supset y_b \left( q \Phi_2^* d + q \Phi_1^* D \right) + y_\tau \left( \ell \Phi_2^* e + \ell \Phi_1^* E \right) + M_D D \overline{D} + M_E E \overline{E} \,.$$

Effects of the softly-broken  $\Pi_2$  discrete symmetry



neglecting finite thresholds proportional to  $M_U^2$ . Since  $\mathbb{Z}_2$  is unbroken (or at worst spontaneously broken if  $v_2 \neq 0$ ),  $m_{12}^2$  is not generated in this approximation. Assuming that  $\ln(\Lambda/M_U)$  is not much larger than  $\mathcal{O}(1)$ , we see that there is no second fine tuning if  $\Delta m^2 \leq \mathcal{O}(v^2)$ , or roughly

$$M_U \lesssim \frac{\pi v^2}{m_t},$$

which is satisfied for  $M_U$  less than a few TeV. Note that the other mirror masses are far less constrained since the corresponding SM fermion masses are significantly less than  $m_t$ .

Integrating out the mirror fermions below the scale  $M_U$ , one generates a splitting between  $\lambda_1$  and  $\lambda_2$ . Above the scale M, the diagrams



contribute equally to  $\lambda_2 (\Phi_2^{\dagger} \Phi_2)^2$  and  $\lambda_1 (\Phi_1^{\dagger} \Phi_1)^2$ , respectively. Below the scale  $M_U$ , the diagrams with internal U lines decouple, which then yields

$$\Delta \lambda \equiv |\lambda_1 - \lambda_2| \sim \frac{3y_t^4}{4\pi^2} \log(M_U/m_t) \sim 0.1 \,,$$

for  $M_U \sim 1$  TeV. Note:  $\lambda_6$  and  $\lambda_7$  are not generated due to the unbroken  $\mathbb{Z}_2$ .

Henceforth, we write  $m_{11}^2$  and  $m_{22}^2$  (at the scale  $m_t$ ) in terms of

$$m^2 \equiv \frac{1}{2}(m_{11}^2 + m_{22}^2), \qquad \Delta m^2 \equiv m_{22}^2 - m_{11}^2.$$

We denote  $\tan \beta \equiv v_2/v_1$  and we neglect the effects of  $\Delta \lambda$  which are small.

# Local minima of the 2HDM scalar potential

We define  $\lambda \equiv \lambda_1 = \lambda_2$  and

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \qquad R = \frac{\lambda_{345}}{\lambda},$$

and demand that  $\lambda > 0$  and R > -1 to ensure that the vacuum is bounded from below. Solving for the potential minimum, there are two possible phases:

#### 1. The inert phase

Assuming that  $\Delta m^2 < -2m^2$ , the Higgs vacuum is

$$\left\langle \Phi_1^0 \right\rangle^2 = \frac{1}{2}v^2 = -\left(\frac{m^2 + \frac{1}{2}\Delta m^2}{\lambda}\right) \qquad \left\langle \Phi_2 \right\rangle = 0.$$

In this case,  $\mathbb{Z}_2$  is unbroken by the vacuum.

#### 2. The mixed phase\*

If both  $v_1 \neq 0$  and  $v_2 \neq 0$ , then the  $\mathbb{Z}_2$  is spontaneously broken. Minimizing the scalar potential yields

$$m^2 = -\frac{1}{4}\lambda(1+R)v^2$$
,  $\tan\beta = \sqrt{\frac{1-\epsilon}{1+\epsilon}}$ ,

where

$$\epsilon \equiv \frac{2\Delta m^2}{\lambda (1-R)v^2} \, .$$

The positivity of  $v_1^2$  and  $v_2^2$  and the curvature at the extremum requires

$$|R| < 1$$
,  $|\epsilon| < 1$ .

Given the constraint on R, the constraint on  $\epsilon$  can also be written

$$m^2 < 0$$
,  $\Delta m^2 < -2m^2 \left(\frac{1-R}{1+R}\right)$ 

<sup>\*</sup>There is a parameter regime in which both the inert phase and the mixed phase coexist. However, one can check that in this case, the mixed phase minimum is deeper than the inert phase minimum.

#### Scalar spectrum of the inert phase

The physical neutral Higgs bosons are eigenstates of CP.

$$\begin{split} m_h^2 &= \lambda v^2 \,, \\ m_H^2 &= -\frac{1}{2} \lambda v^2 (1-R) - \Delta m^2 \,, \\ m_A^2 &= m_H^2 - \lambda_5 v^2 \,, \\ m_{H^{\pm}}^2 &= m_H^2 - \frac{1}{2} (\lambda_4 + \lambda_5) v^2 \,, \end{split}$$

where the couplings of h are precisely those of the SM Higgs boson.

Since we are interested in the case where all Higgs boson masses are of  $\mathcal{O}(v)$ , we restrict  $\Delta m^2 \sim \mathcal{O}(v^2)$  as previously stated. Of course, if  $M_U \gg v$ , then we can make  $-\Delta m^2$  arbitrarily large (which is an allowed regime of the inert phase), in which case H, A and  $H^{\pm}$  become arbitrarily heavy.

#### Scalar spectrum of the mixed phase

In the convention where the ratio of the vevs is real, it follows from the scalar potential minimum conditions that  $\lambda_5 \leq 0$ . The Higgs mass spectrum is:

$$m_{h,H}^2 = \frac{1}{2}\lambda v^2 (1 \mp \sqrt{R^2 + (1 - R^2)\epsilon^2}),$$
  

$$m_A^2 = -\lambda_5 v^2,$$
  

$$m_{H^{\pm}}^2 = -\frac{1}{2}(\lambda_4 + \lambda_5)v^2.$$

Requiring h to be SM-like, it follows that  $|\cos(\beta - \alpha)| \ll 1$  [the so-called alignment limit], assuming that<sup>†</sup>

$$-1 < R < -\frac{\epsilon^2}{1-\epsilon^2},$$

and  $\alpha-\beta$  is the CP-even mixing angle in the Higgs basis, with

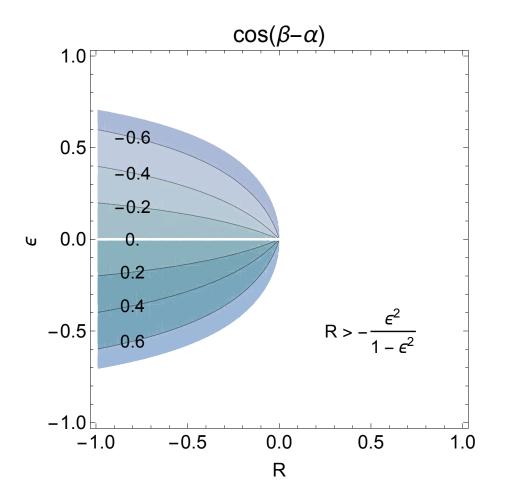
$$\sin(\beta - \alpha)\cos(\beta - \alpha) = \frac{\epsilon(\epsilon^2 - 1)(1 - R)}{2\sqrt{R^2 + \epsilon^2(1 - R^2)}}.$$

<sup>†</sup>Otherwise, H is SM-like and  $|\sin(\beta - \alpha)| \ll 1$ .

When  $\epsilon$  and  $|\cos(\beta - \alpha)|$  are small [in a convention where  $\sin(\beta - \alpha) \ge 0$ ], then

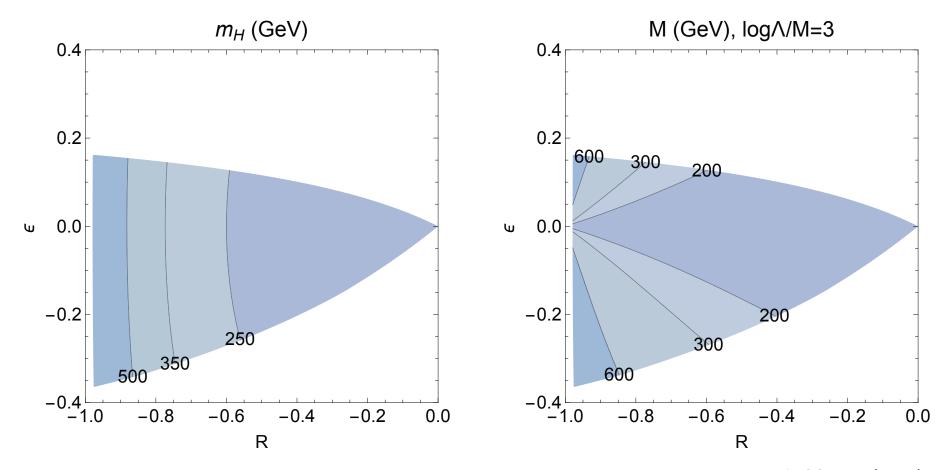
$$\cos(\beta - \alpha) \simeq -\frac{\epsilon(1-R)}{2|R|}.$$

In particular, the alignment limit favors small  $|\epsilon|$ , which yields  $\tan \beta \sim \mathcal{O}(1)$ .



It is convenient to rewrite  $m_H$  in terms of  $m_h$ ,

$$m_H^2 = m_h^2 \left( \frac{1 + \sqrt{R^2 + (1 - R^2)\epsilon^2}}{1 - \sqrt{R^2 + (1 - R^2)\epsilon^2}} \right) .$$



The shaded regions are consistent with the Higgs couping fits taken from N. Craig et al., JHEP 1506, 137 (2015).

# **Phenomenological constraints and implications**

- Below the scale of  $M_U$ , the effective theory is that of a Type-I 2HDM.
- In the inert phase, the lightest scalar in the  $\Phi_2$  doublet is a stable dark matter candidate. There is no mixing of U with SM quarks due to the exact discrete  $\mathbb{Z}_2$  symmetry. But  $U \rightarrow q\Phi_2$  is a possible decay, which can be discovered in the  $t\bar{t}$  + missing energy channel. Current LHC limits yield  $m_U \gtrsim 500$  GeV for  $m_H \lesssim 150$  GeV.
- In the mixed phase, the discrete  $\mathbb{Z}_2$  symmetry is broken and U can mix with the top quark. In this case  $U \to Wb$ , Zt and ht are possible decays. LHC experimental limits require  $m_U \gtrsim 700$  GeV if no other decay modes are present. If tH and  $bH^+$  are kinematically allowed, they will dominate and the experimental limits must be reconsidered.
- In the regime of the mixed phase where the non-minimal Higgs states have masses below 1 TeV,  $\tan \beta$  is moderate, of order a few. This is a very difficult regime for the LHC. Perhaps  $H \rightarrow hh$  and the production of  $t\bar{t}H$ ,  $t\bar{t}A$  and  $t\bar{b}H^-$  provide the best opportunities for discovery.

#### Final Comments—Supersymmetric Extensions

1. Given the non-observation of supersymmetry (SUSY), some theorists have considered SUSY models with a SUSY-breaking scale  $M_{\rm SUSY} \gg 1$  TeV. Since SUSY models contain (at least) two Higgs doublets, one can now pose our Higgs sector fine-tuning question in a SUSY context.

- In the SUSY Higgs sector,  $H_d$  is a hypercharge -1 Higgs doublet and  $H_u$  is a hypercharge +1 Higgs doublet. The superpotential is holomorphic in the corresponding Higgs superfields, and this restricts the model building.
- To obtain a SUSY extension of our the partially natural 2HDM, one must add two mirror Higgs doublet superfields  $H'_u$  and  $H'_d$ . The Higgs sector is now a 4HDM, with a superpotential whose quadratic terms are of the form

$$V \supset m_1^2(|H_d|^2 + |H_d'|^2) + m_2^2(|H_u|^2 + |H_u'|^2) - m_3^2(H_uH_d + H_u'H_d')$$

Two linear combinations of fields can be made light with one tuning.

2. Can one implement sufficient discrete symmetries (perhaps softly broken) on the minimal SUSY Higgs sector so that  $H_u$  and  $H_d$  are light fields? Since  $H_u$  and  $H_d$  have opposite hypercharge, the mirror symmetry that interchanges  $H_u$  and  $H_d$  is now a GCP symmetry.

All attempts to construct such a model have failed. It seems that one inevitably encounters hard symmetry breakings, and the desired symmetries cannot be protected at low energies.

3. Are these hard symmetry breakings necessarily fatal? Once we accept the possibility of hard breaking, we might as well discard the mirror fermions and examine this question in the context of the split MSSM. In some special cases, it may be possible to find scenarios where the corrections to  $m_{22}^2 - m_{11}^2$  remain small enough such that both Higgs doublets remain light compared to the mass scale that governs the squarks. In such scenarios the higgsino states are expected to be as light as the non-minimal Higgs states, and the staus are also expected to be light, at most an order of magnitude heavier.