GLUON-GLUON FUSION

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UNIVERSITÀ DEGLI STUDI
DI MILANO

HXSWG general meeting

CERN, January 22, 2015

SUBGROUP ACTIVITIES

- BENCHMARKING EXERCISE ON THE TOTAL CROSS SECTION: STARTED OCT. 2014; DRAFT DOCUMENT JAN. 2015
- FIRST MEETING, WITH ALL RELEVANT PARTIES FOR THE TOTAL CROSS-SECTION BEYOND NNLO: NOVEMBER 18, 2014
- STUDIES OF JETS AND p_T DISTRIBUTION: ROADMAP JANUARY 2015
- STUDIES ON THEORETICAL UNCERTAINTIES: PLANNED

THIS TALK

BENCHMARKING THE TOTAL CROSS-SECTION BEYOND NNLO

- THE CURRENT RECOMMENDATION
- RESUMMATION AND ITS AMBIGUITIES
- APPROXIMATE AND PARTIAL $\mathrm{N^3LO}$ RESULTS

THE PIECES OF THE PUZZLE

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageAt8TeV#gluon_gluon_Fusion_Process

$$m_h = 125 \text{ GeV}; \qquad \sigma = 19.27 \text{ pb}^{+7.2}_{-7.8} \text{ (SCALE)} \ ^{+7.5}_{-6.9} \text{ (PDF+}\alpha_s\text{)}$$
$$\mu_r = \mu_{\rm F} = m_H$$
(NOTE: IF $m_H = 125.5 \to \sigma = 19.12$ pb)

(De Florian, Grazzini, 2012)

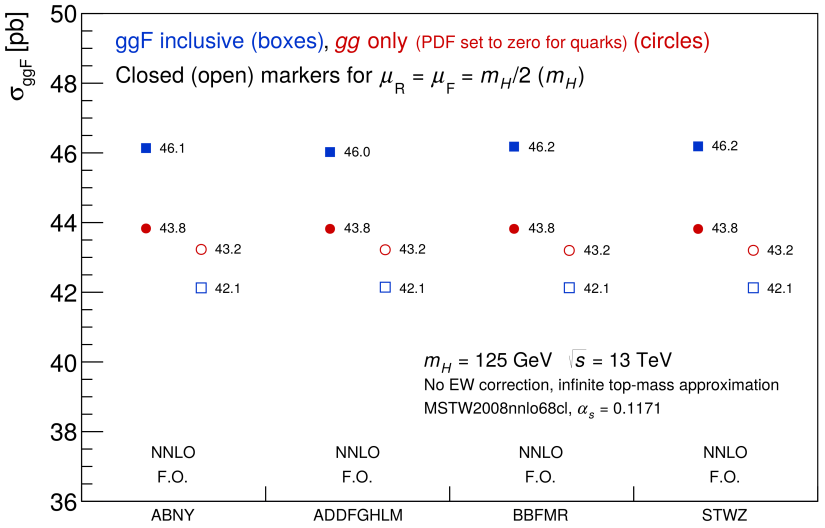
- NNLO+NNLL
- TOP MASS DEP, AND BOTTOM AND CHARM CONTRIBUTIONS AT NLO+NLL
  DECREASE $\sim 1.5\%$ DUE TO TOP+BOTTOM (MOSTLY BOTTOM)+
  FURTHER $\sim 1\%$ DUE TO CHARM
- NLO ELECTROWEAK CORRECTIONS (COMPLETE FACTORIZATION)
  INCREASE $\sim 5\%$ (WOULD CHANGE BY A FEW PERCENT WITH
  PARTIAL FACTORIZATION)
- COMPLEX-POLE (OFFP)
  FINITE-WIDTH BELOW $\sim 1\%$ FOR LIGHT HIGGS
- NO MIXED QCD-EW AND REAL EW RADIATION
  BELOW $\sim 1\%$

BENCHMARKING

THE GROUPS INVOLVED:

- ABNY Ahrens, Becher, Neubert, Yang → NNLO+NNLL RESUMMED (SCET)
- STWZ Stewart, Tackmann, Walsh, Zuberi → NNLO WITH EXP. CONSTANTS
- DFMMV de Florian, Mazzitelli, Moch, Vogt → APPROXIMATE N^3LO
- BBFMR Ball, Bonvini, SF, Marzani, Ridolfi → APPROXIMATE N^3LO+N^3LL
- ADDFGLHM Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Lazopoulos, Herzog, Mistlberger → PARTIAL N^3LO

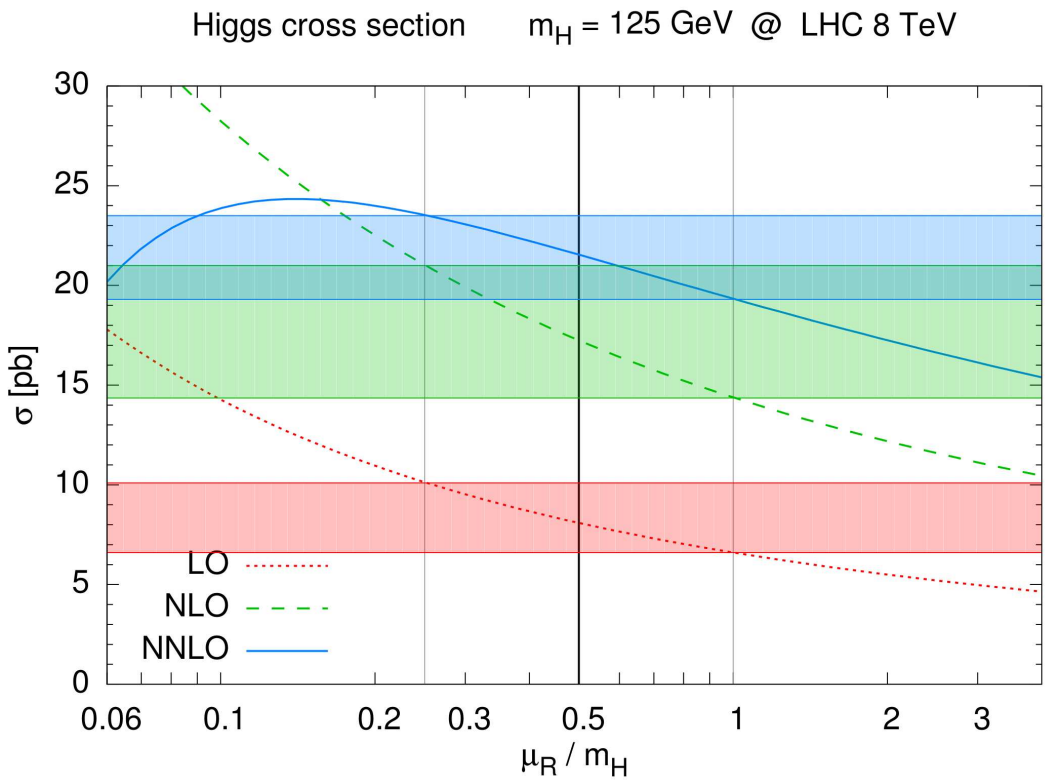
THE NNLO CROSS-SECTION



COMMON SETTINGS FOR PDFS, EW, HW, ETC ADOPTED;
NOT MEANT TO BE REALISTIC, FOCUS ON SIZE OF QCD CORRECTIONS

BEYOND NNLO

LO, NLO AND NNLO



- THE PERTURBATIVE SERIES FOR $gg \to H$ CONVERGES SLOWLY
- SCALE VARIATION UNDERESTIMATES SHIFT TO NEXT ORDER
- FACTORIZATION SCALE DEPENDENCE NEGLIGIBLE, ONLY RENORMALIZATION SCALE DEPENDENCE SIGNIFICANT

QUESTIONS

- DOES RESUMMATION IMPROVE THE FIXED-ORDER RESULT? HOW AMBIGUOUS IS IT?
- CAN WE CONSTRUCT A REASONABLE APPROXIMATION TO $\mathrm{N^3LO}$ BASED ON CURRENT KNOWLEDGE?

RESUMMATION

$$\sigma(N, m_H^2) \equiv \int_0^1 d\tau\, \tau^{N-2} \sigma(\tau, m_H^2); \quad z = \frac{M_h^2}{s} \qquad \left(\frac{\ln^{k-1}(1-x)}{1-x}\right)_+ \Leftrightarrow \ln^k N + O(\ln^{k-1} N) + O\left(\frac{1}{N}\right)$$

$$\sigma_{\rm res}(N, \alpha_s) = \sigma_0 g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right];$$

$$g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3);$$

N^{k}LL RESUMMATION: ALL g_i WITH $i < k+1$, ALL g_{0j} WITH $j < k$

TO ORDER α_s^n, $\ln^m N$ PREDICTED WITH $2(n-k) \leq m \leq 2n$

in SCET literature, this is called N^{k}LL', at N^{k}LL one less power of log: $2(n-k)+1 \leq m \leq 2n$

AMBIGUITIES

- SHOULD g_0 SIT IN THE EXPONENT, ENTIRELY OR IN PART?

  NOTE $g_{0,1}$ NATURALLY PART OF g_{i+2}

  $\Rightarrow$ DIFFERENCE SUBLEADING!

- HOW DOES ONE FIX SUBLEADING TERMS?

  $\ln N$, $\ln N + 1$, $\psi_0(N)$ (MELLIN OF $\left(\frac{1}{1-x}\right)_+$) BEHAVE IN THE SAME WAY AS $N \to \infty$, UP TO $O(1)$ & $O(1/N)$ TERMS

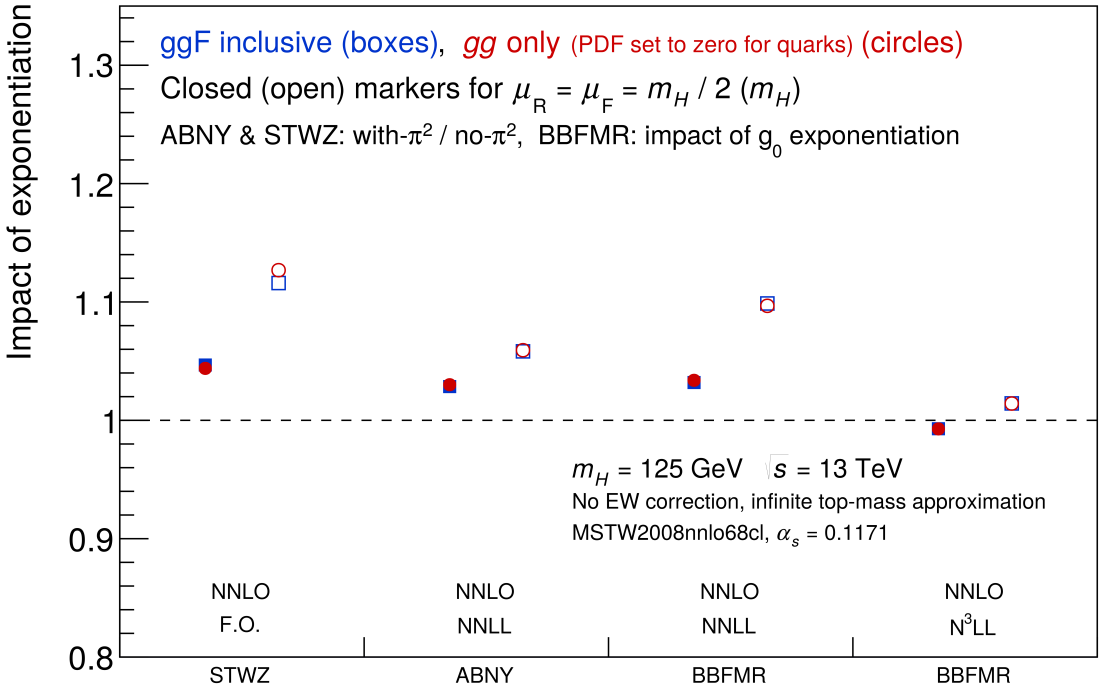
  $\Rightarrow$ DIFFERENCE SUBLEADING!

  NOTE IN SCET APPROACH, RESUMMATION NATURALLY IN z SPACE $\Rightarrow$ SUBLEADING DIFFERENCES

CONSTANT EXPONENTIATION

- IN SCET APPROACH, π^2 EXPONENTIATION W. IMAGINARY SCALE:
  STWZ: ADDED TO FIXED NNLO; ABNY: ADDED TO NNLO+NNLL
- IN DQCD RESUMMATION DIS-SCHEME CONSTANT EXPONENTIATION
  BBFMR: BOTH NNLO+NNLL AND NNLO+N^{3}LL AVAILABLE

EXPONENTIATED/UNEXPONENTIATED RATIO (NO RESUMMATION)



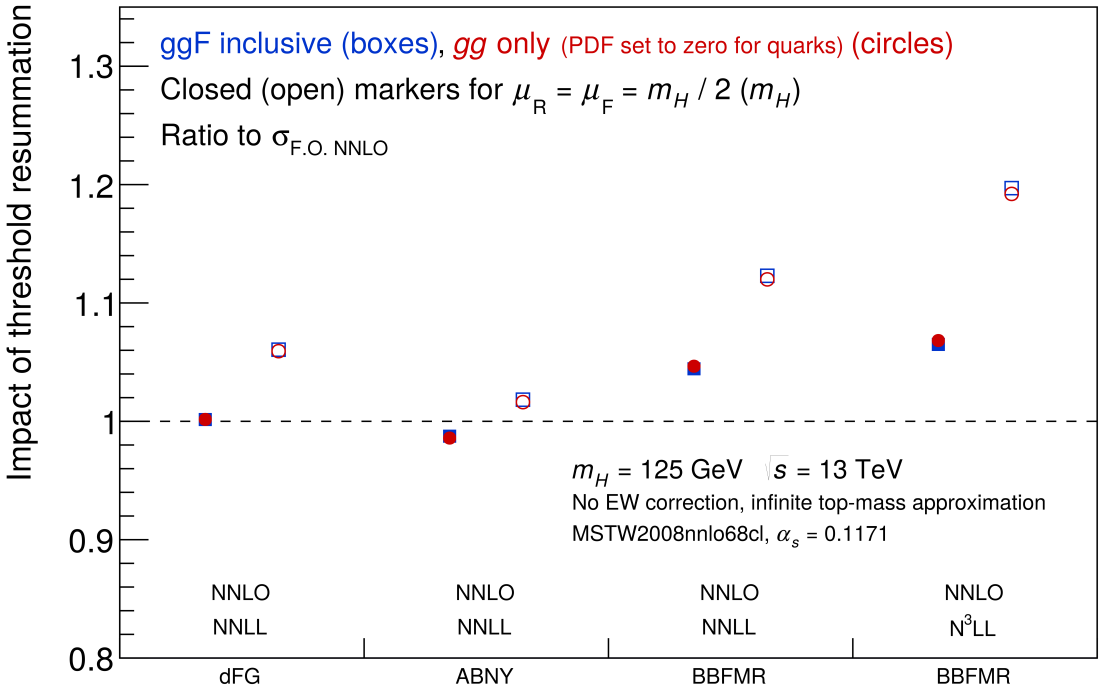
note k factor computed wr to NNLO at respective scale

- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- IMPACT SMALLER AT RESUMMED THAN AT FIXED ORDER
- IMPACT SMALLER AT N^{3}LL THAN AT NNLL

CHOICE OF LOGS

- dFG: CURRENT REFERENCE
- ABNY: SCET (z SPACE) APPROACH TO NNLL (REALLY N^{3}LL*, OR SCET-N^{3}LL)
- BBFMR: DIFFERS FROM dFG BECAUSE OF 'ANALYTIC' RESUMMATION
  (correct small-N singularities when expanded to finite order)
- BBFMR: N^{3}LL ALSO AVAILABLE

RESUMMED/UNRESUMMED RATIO (NO CONST. EXPONENTIATION)



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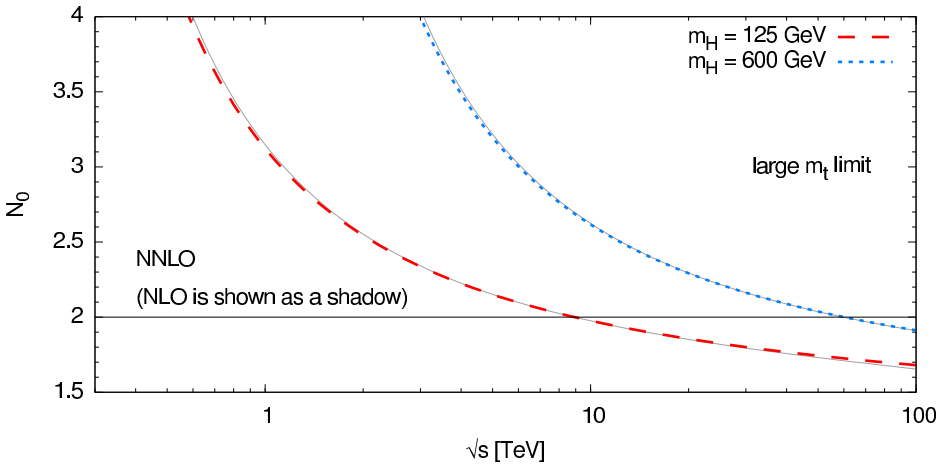
- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- z-SPACE: LITTLE EFFECT, ANALYTIC: LARGER EFFECT
- IMPACT LARGER AT N^{3}LL THAN AT NNLL

FROM RESUMMATION TO N^{3}LO

TECHNICAL INTERLUDE

- PARTONIC CROSS SECTIONS $\hat{\sigma}(z, \alpha_s)$ ARE DISTRIBUTIONS, $0 \leq z = \frac{M_h^2}{s} \leq 1$
- THEIR MELLIN TRANSFORMS $\hat{\sigma}(N, \alpha_s)$ ARE ORDINARY FUNCTIONS
- FOR GIVEN m_H^2 AND s ONLY ONE "SADDLE" N VALUE CONTRIBUTES
- BONUS: HAD. XSCT PRODUCT OF PARTONIC TIMES LUMI: $\sigma(N, m_H^2) = \hat{\sigma}\left(m_H^2, \alpha_s\right) \mathscr{L}(N)$

SADDLE VS COLLIDER ENERGY



Q: WHY ARE THERE LARGE RESUMMATION AMBIGUITIES?

A: BECAUSE $\beta_0 \alpha_s \ln 2 \approx 0.04 \ll 1$:RESUMMATION IS PERTURBATIVE!

IN ALL FLAVOURS OF RESUMMATION DISCUSSED ABOVE

BETWEEN 2/3 AND 3/4 OF THE NNLL OR N^{3}LL RESUMMATION COMES FROM N^{3}LO,

ALL REMAINING INFINITE ORDERS CONTRIBUTING $\sim 1 \div 2\%$

RESUMMATION IS AMBIGUOUS BECAUSE N^{3}LO IS APPROXIMATE!

N^{3}LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat{\sigma}^{(3)}(N) = c_{06} \ln^6 N + c_{05} \ln^5 N + \cdots + c_{02} \ln^2 N + c_{01} \ln N + c_{00} + \quad \text{SOFT (S)}$$
$$. \quad + \frac{1}{N} \left(c_{15} \ln^5 N + c_{14} \ln^4 N + \cdots + c_{10} \right) + \quad \text{NEXT-TO-SOFT (NS)}$$
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NSPACE c COEFFICIENTS DETERMINE THE z SPACE c' AND CONVERSELY:

- N-SPACE N^k SOFT $\Leftrightarrow$ z-SPACE N^k SOFT
- N-SPACE N^{k}LL $\Leftrightarrow$ z-SPACE N^{k}LL
- MELLIN OF z SPACE N^k SOFT EQUALS N-SPACE N^k SOFT PLUS N^{k+1} TERMS
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$$\hat{\sigma}^{(3)}(z) = c'_{06} \left(\tfrac{\ln^5(1-z)}{1-z} \right)_+ + \cdots + c'_{01} \left(\tfrac{1}{1-z} \right)_+ + c'_{00} \delta(1-z) + \quad \text{SOFT (S)}$$
$$. \quad + \left(c'_{15} \ln^5(1-z) + c'_{14} \ln^4(1-z) + \cdots + c'_{10} \right) + \quad \text{NEXT-TO-SOFT (NS)}$$
$$. \quad + (1-z) \left(c'_{25} \ln^5(1-z) + c'_{24} \ln^4(1-z) + \cdots + c'_{20} \right) + \ldots \quad \text{NNS}$$

- c_{06} TO c_{02} DETERMINED BY NNLL RESUMMATION
- c_{01} COMPUTED (Moch, Vogt, 2005; Laenen, Magnea, 2005)
- COMPLETE z-SPACE SOFT $\Rightarrow$ c'_{00} (Anastasiou et al. 2014)
- c_{15}, c_{24},... c_{51} KNOWN FROM LL RESUMMATION (Catani, deFlorian, Grazzini, 2001)
- c_{14} COMPUTED BASED ON CONJECTURE (deFlorian, Mazzitelli, Moch, Vogt, 2014)
- COMPLETE z-SPACE NS $\Rightarrow$ ALL c'_{1i} (Anastasiou et al. 2015) $\rightarrow$ CONJECTURE VALIDATED
- COMPLETE z-SPACE $\ln^4(1-z)$ & $\ln^3(1-z) \Rightarrow$ ALL c'_{i4}, c'_{i3} (Anastasiou et al. 2015) (FORMALLY LEADING LOGS)

N^{3}LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat{\sigma}^{(3)}(N) = c_{06} \ln^6 N + c_{05} \ln^5 N + \cdots + c_{02} \ln^2 N + c_{01} \ln N + c_{00} + \quad \text{SOFT (S)}$$
$$. \quad + \tfrac{1}{N} \left(c_{15} \ln^5 N + c_{14} \ln^4 N + \cdots + c_{10} \right) + \quad \text{NEXT-TO-SOFT (NS)}$$
$$. \quad + \tfrac{1}{N^2} \left(c_{25} \ln^5 N + c_{24} \ln^4 N + \cdots + c_{20} \right) + \ldots \quad \text{NNS}$$

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$$. \quad + \left(c'_{15} \ln^5(1-z) + c'_{14} \ln^4(1-z) + \cdots + c'_{10} \right) + \quad \text{NEXT-TO-SOFT (NS)}$$
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N^{3}LO: WHAT DO WE KNOW

THE HIGH-ENERGY EXPANSION

SMALL N POLES OR ABOUT $z = 0$

rightmost singularity at $N = 1$:

$$\hat{\sigma}^{(3)}(N) = \left(d_{13}\frac{1}{(N-1)^3} + d_{12}\frac{1}{(N-1)^2} + d_{11}\frac{1}{(N-1)}\right) + \quad \text{LEADING LOG } x \text{ (LL}x\text{)}$$

$$. \quad + \left(d_{06}\frac{1}{N^6} + \cdots + d_{01}\frac{1}{N}\right) + \quad \text{NLL}x$$

$$. \quad + \left(d_{-16}\frac{1}{(N+1)^6} + \cdots + d_{-11}\frac{1}{N+1}\right) + + \ldots \quad \text{NNLL}x$$

$$\hat{\sigma}^{(3)}(z) = \left(d_{13}\frac{\ln^2 z}{z} + d_{12}\frac{\ln z}{z} + d_{11}\frac{1}{z}\right) + \quad \text{LEADING LOG } x \text{ (LL}x\text{)}$$

$$. \quad + \left(d_{06}\ln^6 z + \cdots + d_{01}\ln z\right) + \quad \text{NLL}x$$

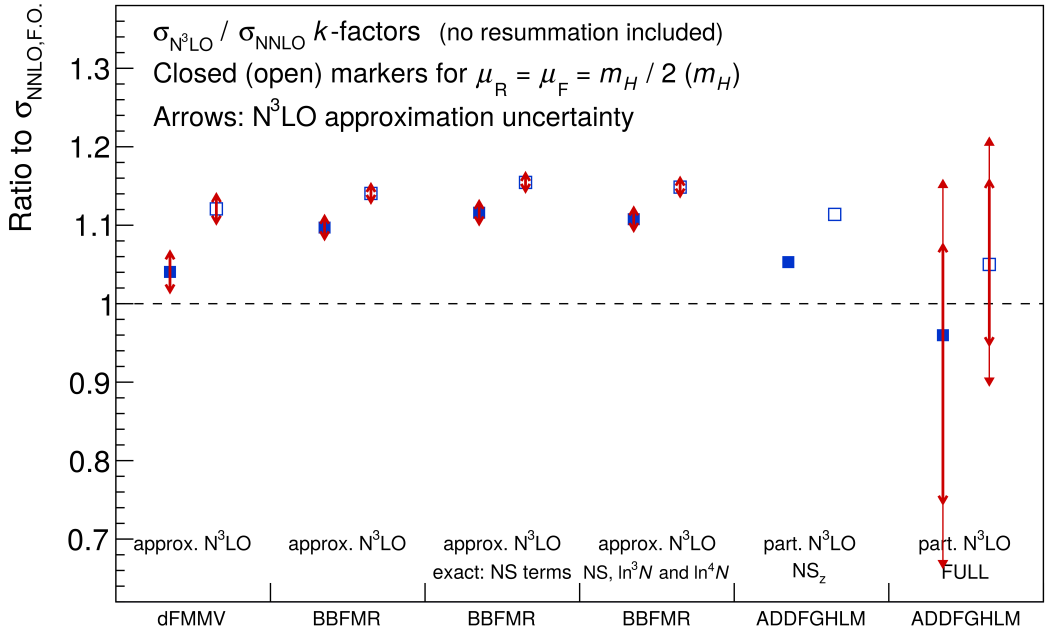
$$. \quad + z\left(d_{-16}\ln^6 z + \cdots + d_{-11}\ln z\right) + \ldots \quad \text{NNLL}x$$

- AS ABOVE, z-SPACE $\Leftrightarrow$ N-SPACE, ORDER BY ORDER (LOG & POWER), UP TO SUBLEADING TERMS
- ONLY d_{13} KNOWN EXACTLY FROM BFKL+SMALL x RESUMMATION
- CLASS OF DOMINANT (?) CONTRIBUTIONS TO d_{12}, d_{11} KNOWN FROM NL BFKL
- ALL SUBLEADING SINGS UNKNOWN

APPROXIMATE N^{3}LO

- dFMMV: N-SPACE APPROXIMATE NS; NOW NS KNOWN EXACTLY: SMALL EXPECTED CHANGE
- BBFMR: ALL KNOWN SINGULARITIES (SOFT & HIGH ENERGY):
  NS AND BEYOND APPROXIMATELY DETERMINED
  $\rightarrow$ IMPROVED WITH EXACT NS $\rightarrow$ IMPROVED WITH EXACT $\ln^4 N$, $\ln^3 N$ (SMALL CHANGES)
- ADDFGHLM: ALL KNOWN z SPACE TERMS (FULL) $\rightarrow$ PURE z SPACE NS ALSO SHOWN

N^{3}LO/NNLO k-FACTOR



note k factor computed wr to NNLO at respective scale

UNCERTAINTIES (ARROWS)

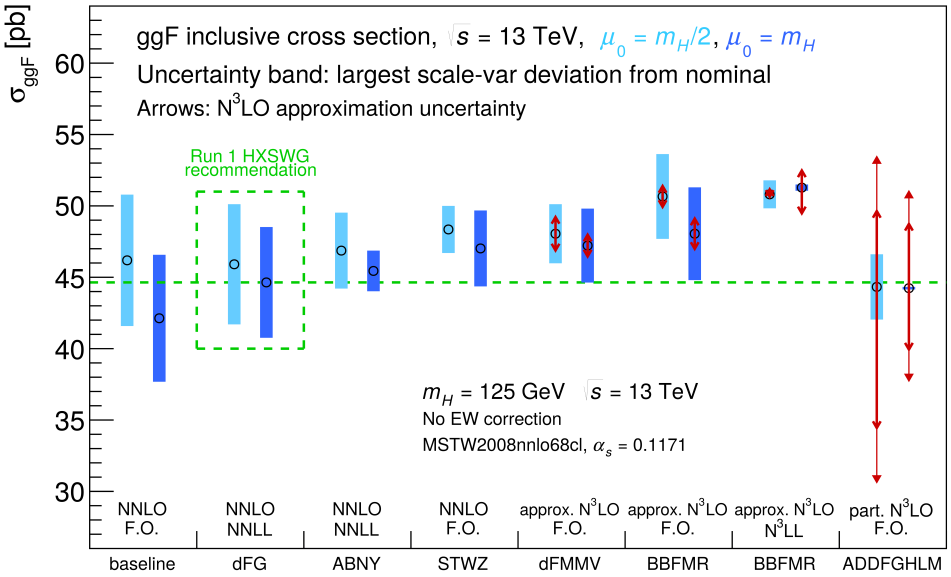
- dFMMV: S-NS BAND
- BBFMR: ESTIMATE OF SUBLEADING SINGULARITIES VALIDATED BY KNOWN ORDERS
- ADDFGHLM: ENVELOPE OF SCALE VAR. OF NS-FULL DIFFERENCE AT m_H, $\times$ 1.5 OR 1

SIZE OF UNCERTAINTY HOTLY DEBATED!!

WHAT IS THE CROSS-SECTION?

- STWZ: DIFFERS FROM NNLO BY CONST. EXPONENTIATION
- ABNY: DIFFERS FROM NNLO+NNLL (REF) BY CONST. EXPONENTIATION & DIFFERENT RESUMMATION
- DFMMV: APPROXIMATE N^3LO BASED ON APPROXIMATE N-SPACE NS; ARROWS: S-NS DIFFERENCE
- BBFMR: APPROXIMATE N^3LO BASED ON ANALYTIC APPROX (INCLUDING NS+BFKL); ARROWS: ESTIMATED MISSING SINGS+ FACT SCALE VARIATION FROM MISSING QUARK CHANNELS; $\rightarrow$ N^3LO+N^3LL BEST PREDICTION
- ADDFGHLM: ALL KNOWN z SPACE TERMS; ARROWS: ENVELOPE OF SCALE VARIATION OF NS-FULL DIFFERENCE

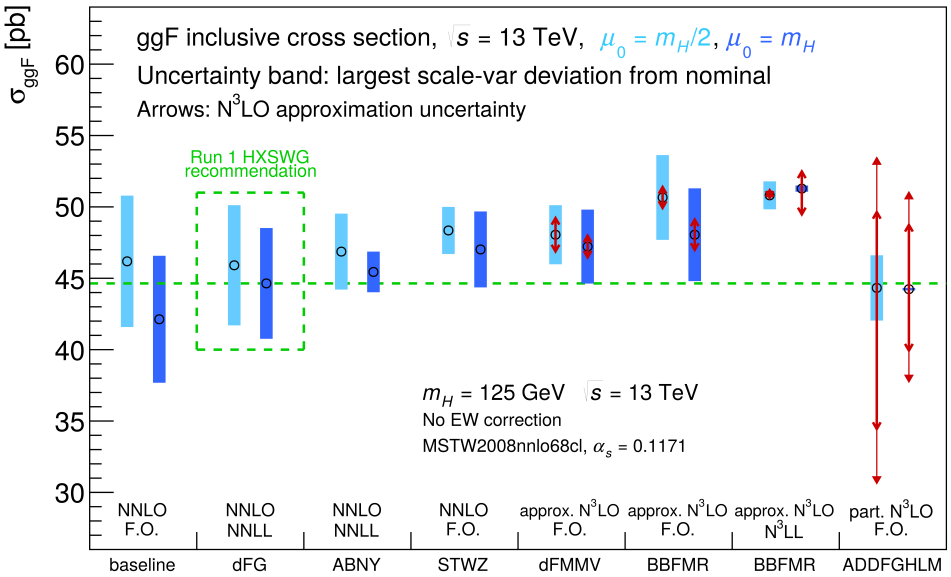
THE GLUON FUSION CROSS-SECTION



all numbers given with benchmark settings (K-factor to dFG very stable)
band is seven-point fact. and ren. scale variation

ISSUES/QUESTIONS/OPINIONS

THE GLUON FUSION CROSS-SECTION



- SHOULD WE INCLUDE RESUMMATION DESPITE ITS AMBIGUITIES? WHY? IN WHICH FORM?
- SHOULD WE TRUST APPROXIMATE RESULTS? WHICH UNCERTAINTY SHOULD WE ASSIGN TO THEM?

RESUMMATION

DOES IT HELP?

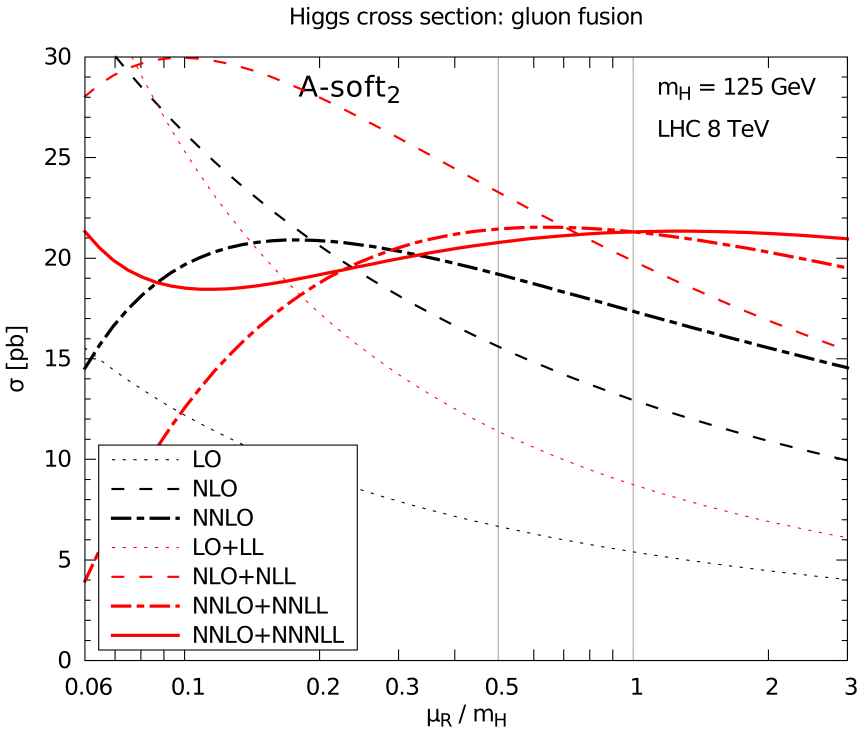
NO

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\ & 36\log^6 N && (\to 0.0013\%)\\
& + 170.679\ldots\log^5 N && (\to 0.0226\%)\\
& + 744.849\ldots\log^4 N && (\to 0.2570\%)\\
& + 1405.185\ldots\log^3 N && (\to 1.0707\%)\\
& + 2676.129\ldots\log^2 N && (\to 4.0200\%)\\
& + 1897.141\ldots\log N && (\to 5.1293\%)\\
& + 1783.692\ldots && (\to 8.0336\%)\\
& + 108\frac{\log^5 N}{N} && (\to 0.0105\%)\\
& + 615.696\ldots\frac{\log^4 N}{N} && (\to 0.1418\%)\\
& + 2036.407\ldots\frac{\log^3 N}{N} && (\to 0.9718\%)\\
& + 3305.246\ldots\frac{\log^2 N}{N} && (\to 2.9487\%)\\
& + 3459.105\ldots\frac{\log N}{N} && (\to 5.2933\%)\\
& + 703.037\ldots\frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

Anastasiou

- "LEADING" LOGS DO NOT PROVIDE THE DOMINANT CONTRIBUTION
- SOFT EXPANSION CONVERGES VERY SLOWLY

YES



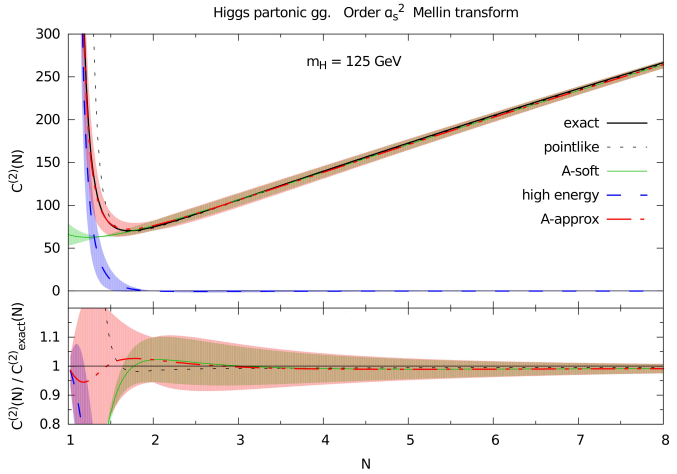
Bonvini

- RESUMMED EXPANSION CONVERGES FASTER THAN UNRESUMMED ONE
- SCALE DEPENDENCE ALWAYS SMALLER AT RESUMMED LEVEL

APPROXIMATE N^{3}LO

YES

COMBINE SMALL AND LARGE N SINGULARITIES,
ALLOWING FOR UNCERTAINTY DUE TO UNKNOWN ONES:

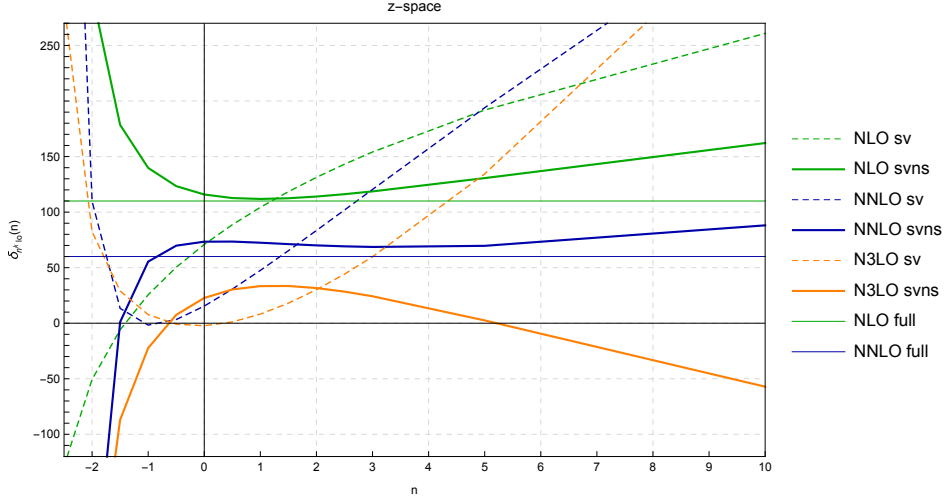


Bonvini

- ASYMPTOTIC CONSTRAINTS STRONGLY BOUND BEHAVIOUR IN THE MIDDLE
- AT KNOWN ORDERS, EXACT RESULT NICELY BRACKETED

NO

PERFORM SOFT EXPANSION OF $\frac{\hat{\sigma}(z)}{z^{n+1}}$
& STUDY THE z DEPENDENCE:



Anastasiou

- SOFT EXPANSION DEPENDS STRONGLY ON WHAT ONE EXPANDS
- NO SIGN OF CONVERGENCE WITHIN THE KNOWN ORDERS

IS THERE A PREFERRED WAY OF EXPANDING?

THE SOCRATIC PROBLEM

HOW DO WE ESTIMATE THE AMOUNT OF OUR IGNORANCE?

[21δ] ἐντεῦθεν οὖν τούτῳ τε ἀπηχθόμην καὶ πολλοῖς τῶν παρόντων: πρὸς ἐμαυτὸν δ' οὖν ἀπιὼν ἐλογιζόμην ὅτι τούτου μὲν τοῦ ἀνθρώπου ἐγὼ σοφώτερός εἰμι: κινδυνεύει μὲν γὰρ ἡμῶν οὐδέτερος οὐδὲν καλὸν κἀγαθὸν εἰδέναι, ἀλλ' οὗτος μὲν οἴεταί τι εἰδέναι οὐκ εἰδώς, ἐγὼ δέ, ὥσπερ οὖν οὐκ οἶδα, οὐδὲ οἴομαι: ἔοικα γοῦν τούτου γε σμικρῷ τινι αὐτῷ τούτῳ σοφώτερος εἶναι, ὅτι ἃ μὴ οἶδα οὐδὲ οἴομαι εἰδέναι.
ἐντεῦθεν ἐπ' ἄλλον ἦα τῶν ἐκείνου δοκούντων σοφωτέρων εἶναι καί

Plato. Platonis Opera, ed. John Burnet. Oxford University Press. 1903.

I am wiser than this man; for neither of us really knows anything fine and good, but this man thinks he knows something when he does not, whereas I, as I do not know anything, do not think I do either. I seem, then, in just this little thing to be wiser than this man at any rate, that what I do not know I do not think I know either.

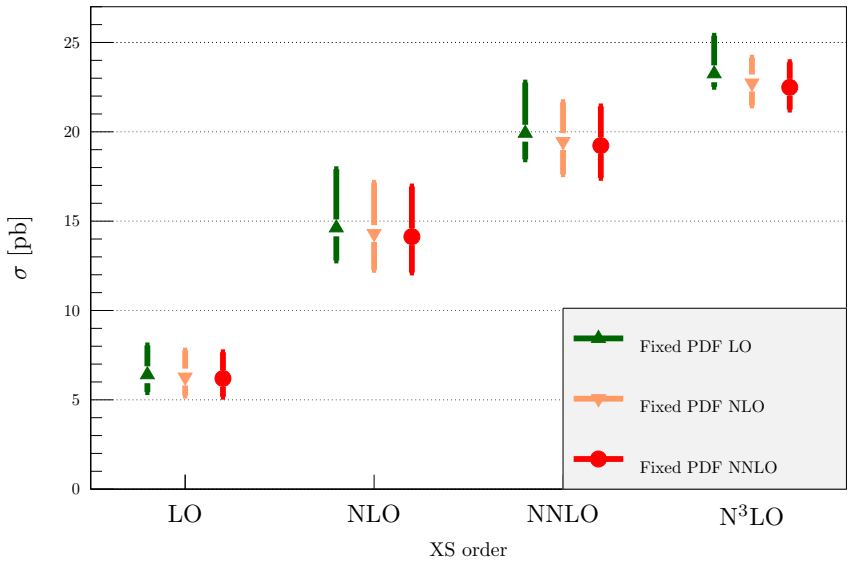
EXTRAS

DO WE NEED N^{3}LO PDFs?

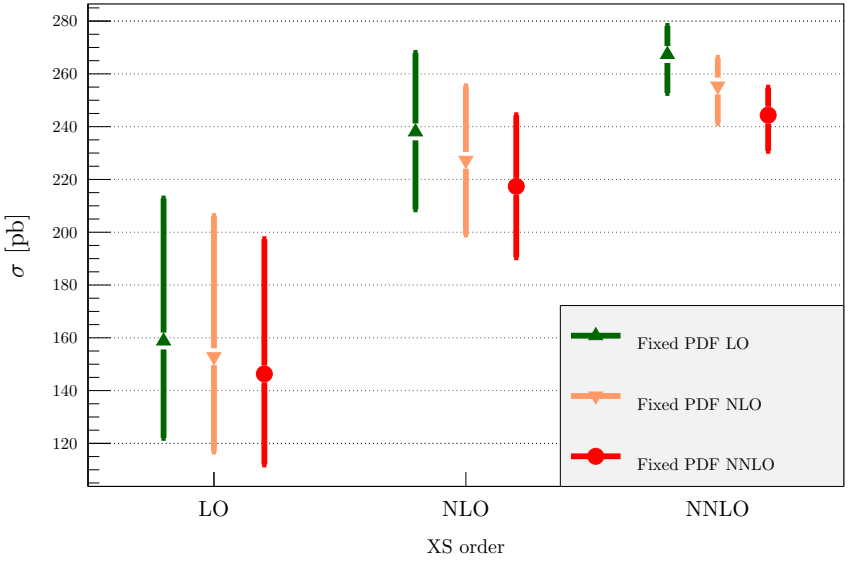
- IN PRINCIPLE, YES
- LOOK AT THE CROSS SECTION AS A FUNCTION OF THE PERTURBATIVE ORDER OF THE PDF AND THE MATRIX ELEMENT
- – HIGGS GLUON FUSION: PERTURBATIVE DEP. OF PDF NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT ⇒ TH. UNCERTAINTY ALMOST ENTIREY DUE TO MATRIX ELEMENT ⇒ NO!
  – TOP: PERTURBATIVE DEP. OF PDF SMALLER, BUT NOT NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT, ANTICORRELATED TO IT ⇒ COMBINED UNCERTAINTY SMALLER⇒ YES!

SCALE UNCERTAINTY & DEP. ON PERTURBATIVE ORDER

HIGGS



TOP



(s.f., Isgrò, Vita, 2014)