RF: Generation, Transmission and Acceleration

Kevin Ronald, MICE and the
University of Strathclyde

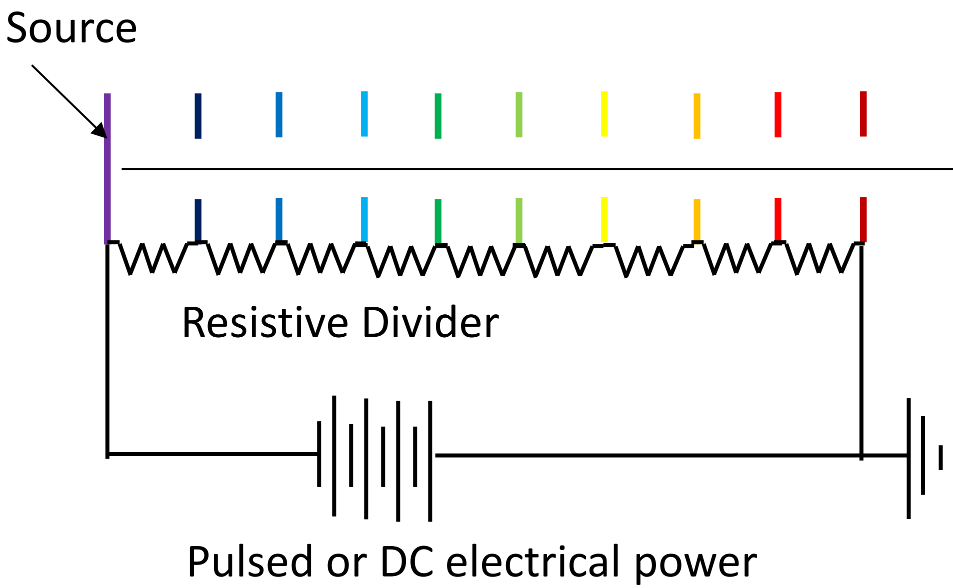
Acceleration Concepts

- To accelerate charged particles we merely need to impose an electric field
- We typically wish to accelerate the particles in some given direction
  - to boost energy we apply the E-field parallel (or anti parallel) to the velocity vector,

    $m_0c^2\, d\gamma/dt = q\, \underline{E} \cdot \underline{v}$
  - to ‘kick’ the beam we may pass it through a transverse deflecting E-field (though this is normally done by magnets)
    - OR choose the bunch to transit through a transverse cavity at a zero of the field- rotates the bunch from longitudinal to transverse- ‘crabbing’ the beam (e.g. work of Burt et al, Lancaster)
- Each role can be achieved using a number of technologies-
  - e.g. in CRT’s many of these functions are achieved electrostatically

Acceleration Concepts

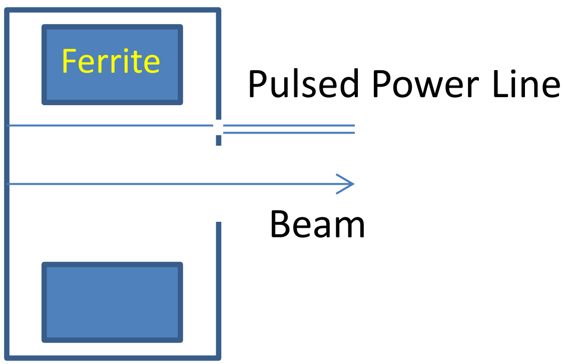
- Electrostatic, direct action, either pulsed or DC
  - To 10’s MeV- problems with high voltage breakdown, circuit components well above ground potential
  - Requires special insulation oils, gasses (e.g. SF_6)
  - Can run (in principle) continuously
  - Gaps can be ‘daisy chained’
    - But delivering energy to higher voltage cavities problematic
    - Nearby beam components (e.g. magnets) at risk of breakdown

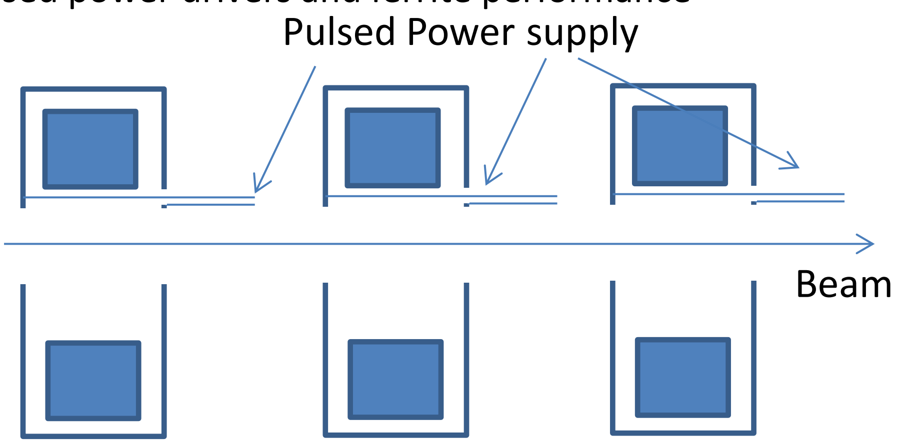


- Resistive Divider shown for simplicity
- More typically would use e.g.
  - Cockcroft-Walton (for DC)
  - Stacked pulsed power drivers (for pulsed system)
- Note one end is at a voltage many times the gap voltage- this is unavoidable in electrostatic system
  - Problematic to deliver energy

Acceleration Concepts

- Induction Linear Accelerator
  - Induction ‘cavities’ avoid high voltage electrodes
  - Pulsed voltages drive currents round chambers
  - ferrite cores (μ_r>1000) boost inductance- supports voltage across gap
  - Eliminates large DC external potentials
  - Gaps can be daisy chained- modulators can be ‘stagger fired’ in a long system/slow beam
  - No components in system removed more than one gap voltage above ground
  - Ferrite material limits achievable gradient
  - Pulse duration constrained by pulsed power drivers and ferrite performance





- RF acceleration
  - Similar benefits as Induction accelerator
  - Eliminates the need to use ferrites
  - High frequency operation in resonant system instead of ferrite loading

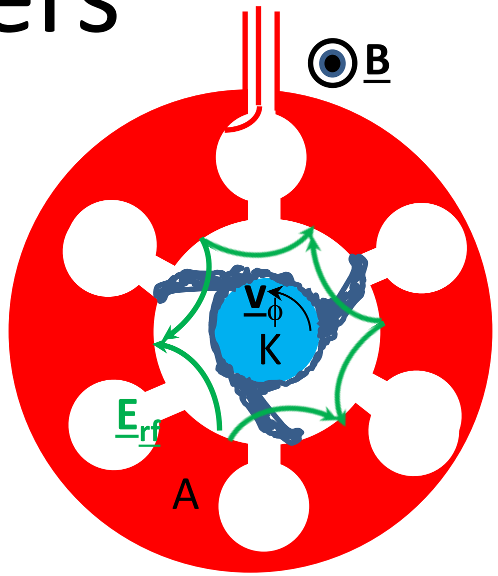
RF and Microwave Accelerator Systems

- Require a source of RF/microwave energy
  - Typically this need only have a narrow frequency range, since accelerator cavities are high Q and therefore have a narrow fractional linewidth ($\Delta\omega/\omega$)
- A transmission line to deliver the energy to the 'load'
  - This will be co-axial at low frequencies, waveguide at high frequencies
- A cavity to 'organise' the RF energy to produce a strong accelerating gradient
  - Structure of cavity can vary significantly, discrete (as in MICE), or distributed travelling wave & standing wave resonators
  - If several cavities are to be used then synchronisation is important
  - Demands energy sources to be amplifiers NOT free running oscillators
- A coupler to interface the cavity to the transmission line
  - This must use the RF energy to excite the cavity modes
  - Must provide matching of the transmission line

Drivers

- Typical drivers at low RF (up to VHF) frequencies are either individual triode/tetrode valve amplifiers or parallel banks of solid state transistor amplifiers
  - Operating in either pulsed or CW modes dependant on the application
  - Valves tend to be used in high power situations, SSPA's 'usually' at lower power
    - Devices often 'daisy chained' to achieve final required performance
    - Means high gain not required from high power device
  - MICE has a 2kW SSPA to boost a mW level signal to kW level, a tetrode amp. to reach 100's kW and a triode amp. to reach 2MW (ISIS Linac is very similar)
- At high RF (UHF) to low microwave (~3GHz) freq. typically one uses klystrons or diacrode/IOT valves (each a sophisticated type of tetrode): Diamond, ALICE use IOT's for example
  - If phase control is unimportant one may use Magnetron oscillators- e.g. for single cavity systems- typical in Medical Linacs, scientific applications often require too many stages to be synchronised
- At frequencies around/above 3GHz, typically one uses Klystrons as power amplifiers, novel developments include Gyroklystrons, two beam systems (CLIC), high power FEM's

Example RF drivers



- Magnetron oscillators frequently used to drive compact (single stage) RF generators.
  - Based on circulation of electrons in a coaxial chamber (central cathode, outer anode with cavities) with a uniform linking B-field
  - With correctly chosen B-fields the electrons cannot directly reach the anode- and can be caused (by ExB force) to rotate at a rate such that they move from cavity aperture to cavity aperture in sync with the oscillation of the magnetic field
  - Electrons which are in accelerating phase gain v_ϕ and are immediately returned to the cathode (by the increased Lorentz force), conversely electrons which are decelerated move towards the anode into higher regions of AC E field
  - These electrons are essentially recovering the kinetic energy lost to the wave by dropping potential energy, which is again immediately converted to wave field energy
  - Difficult (but not impossible) to phase control as they are free running oscillators

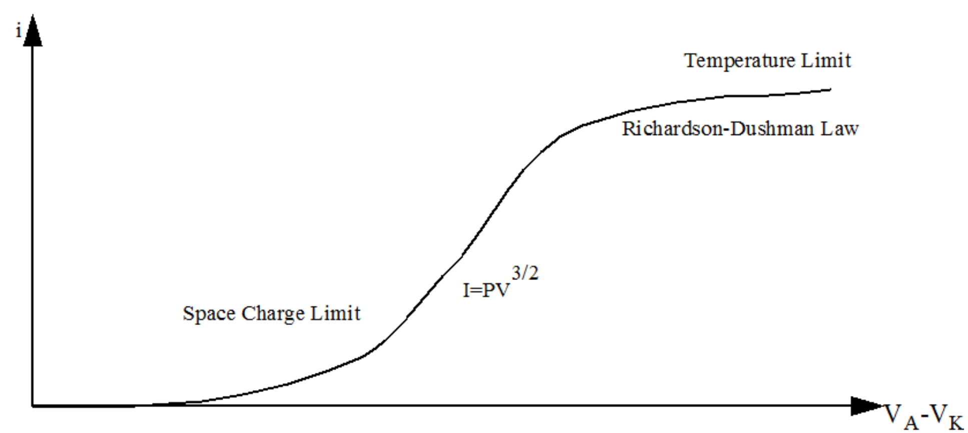
- Robust, small even at low frequencies (become too small for high power at high frequency)
- Inexpensive to make- few components
- Reliable
- Relatively high specific power density
- Efficiency very high, 10's % to approaching 90%

Valve amplifiers physical principles

- MICE requires more than one synchronised RF generation system
  - Implies amplifier system
- MICE uses valve (triode & tetrode) and transistor amplifiers
  - Transistor amplifiers based similar physical principles to valve amplifiers
  - Based on transit time effects
- A thermionic valve (the cathode is made warm) is governed by the Child-Langmuir law
  - Current is limited by the space charge depression of the field at the cathode
  - $i = \frac{4}{9}\frac{A\varepsilon_0}{d^2}\sqrt{\frac{-2e}{m}}(V_A - V_K)^{3/2}$ for // diode, usually written as, $i=PV^{3/2}$ where P is the Perveance
  - This holds until the voltage becomes so high we become constrained by the electron Fermi distribution function in the cathode- controlled by temperature

A: area, *d*: gap spacing

V_A-V_K: gap voltage

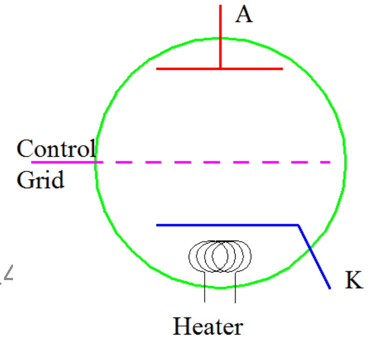


Valve amplifiers physical principles

- Modify a thermionic diode- add an intermediate ‘control’ grid (very close to the cathode) creating a triode
  - Typically the grid is below the cathode potential or no more than slightly above it
  - This ensures electrostatic lensing mitigates electron impact on the grid
    - Excessive grid current is very damaging to a valve
  - The potential applied to the mesh modifies the average E-field on the cathode
  - The grid also makes the E-field distribution on the cathode uneven
  - This can be (rather approximately) modelled by writing the Child law as

$$i = P\left(V_G + \frac{V_A}{\mu}\right)^{3/2}$$

  - We see we can now stop the current flow if V_G is sufficiently negative (μ refers to the relative importance of the grid and anode in setting cathode av. E field- a function of the gap spacings and the grid density).
  - Often a triode is shown as below in circuit diagrams- but recall, the grid is VERY close to the cathode in practice and most RF power valves are co-axial



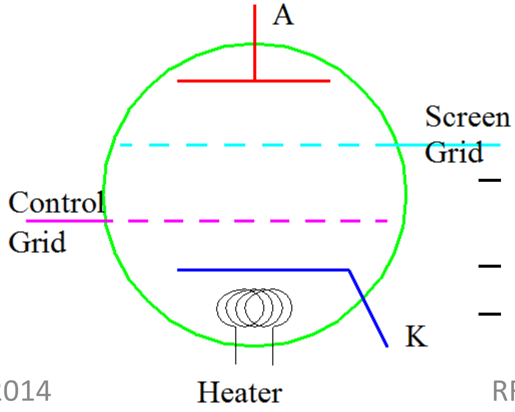
  - i.e. the cathode is the inner cylinder with the grid a cylindrical structure slightly outside the cathode
  - Note this proximity means contamination of the grid is an issue for valves

Valve amplifiers physical principles

- If we modulate V_G then we modulate 'i', Child law now looks like:

$$i = P\left((V_G + V_I(t)) + \frac{(V_A - V_O(t))}{\mu}\right)^{3/2} = P\left((V_G + V_{IA}\sin(\omega t)) + \frac{(V_A - V_{OA}\sin(\omega t))}{\mu}\right)^{3/2}$$

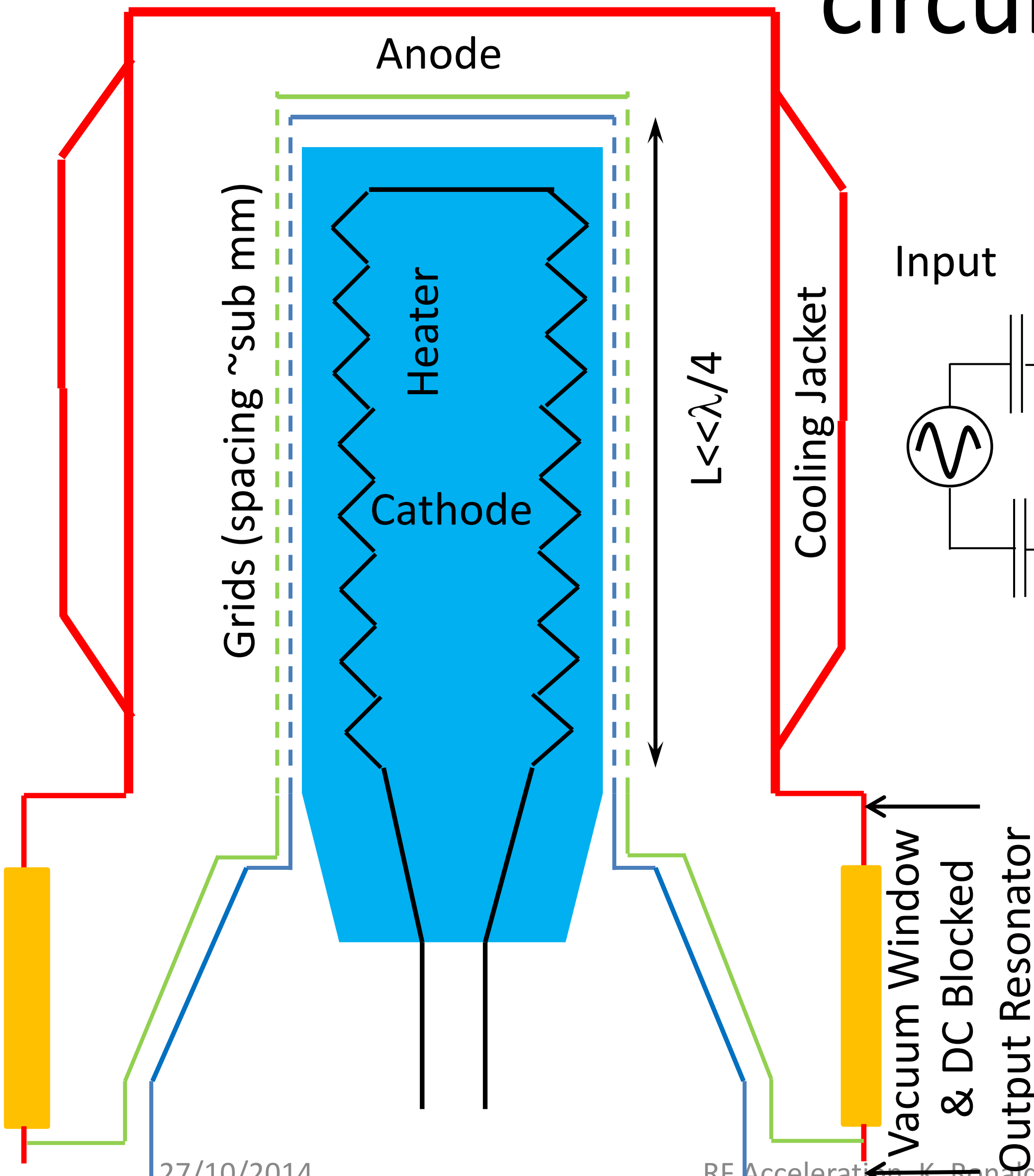
- if the anode is connected to a resonant circuit at the same frequency then an oscillation can grow in antiphase- so that the AC field opposes the DC field when the current is high, but enhances the DC field when the current is low
- As the anode voltage >> grid voltage, the anode swing can be much greater- amplification!
- BUT! If we have too much amplification the Anode swing will significantly affect the current- this limits the gain of a triode
- Circumvent this using a fourth electrode 'Screen' grid- now the control grid/cathode have much weaker capacitive coupling to the anode.
  - Simpler Child law and simpler analysis of the circuit behaviour of the device

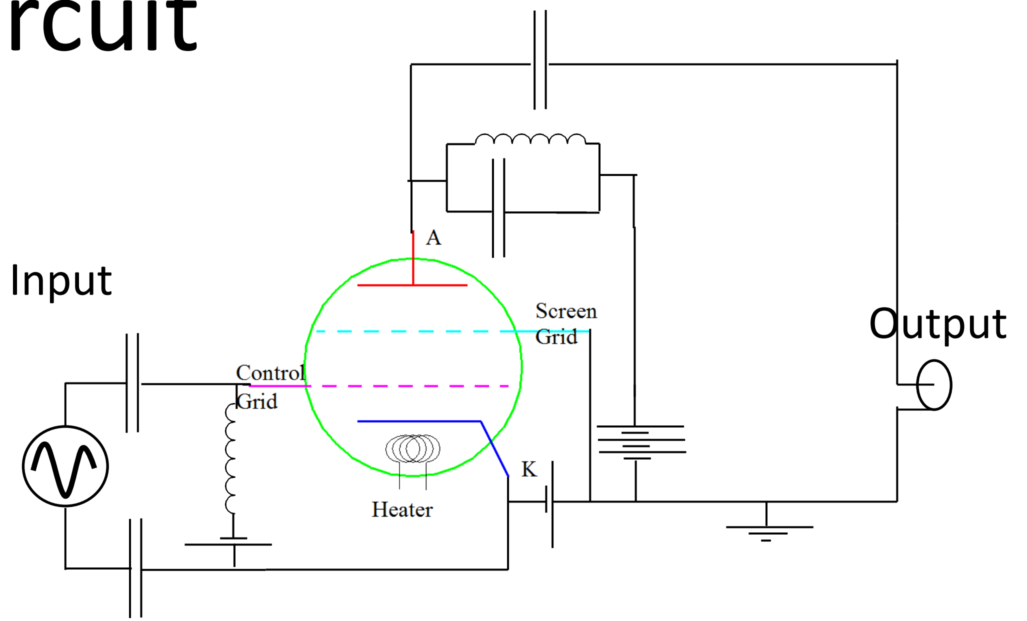


$$i = P\left((V_G + V_{IA}\sin(\omega t)) + \frac{V_{G2}}{\mu}\right)^{3/2}$$

- μ is now the relative significance of the screen and control grid- since V_{G2} is fixed by the PSU, the gain can be higher
- Interception must be mitigated on two grids
- Again RF power tetrodes are usually co-axial

Valve amplifier: physical structure and circuit



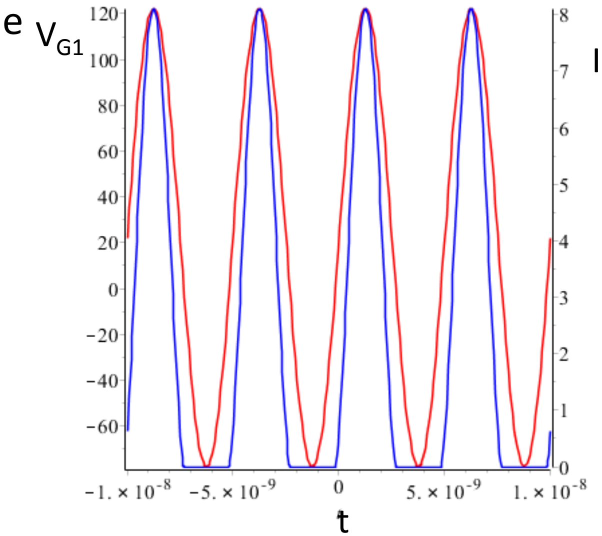


- Various circuits possible
- Here we illustrate 'grounded screen' config. for a tetrode
- Capacitive coupling or inductive blocks to isolate RF/HT supplies
- At high 'f' tank circuits replaced by resonant structures

Valve amplifier- waveforms, power, gain and efficiency

- One may attempt to (roughly) model a tetrode
  - We know anode bias, grid bias, μ, grid AC voltage swing (actually this is a bit harder to find than appears- we really know the input power), output impedance
  - We compute the current using the formula on previous slide
    - The Fourier current components come from evaluating the co-efficients of :

$$i = \frac{1}{2\pi}\left(\int_0^{2\pi} i(2\pi f t)\, d(2\pi f t)\right) + \sum_{n=1}^{\infty}\left[\left(\left\{\frac{1}{\pi}\int_0^{2\pi} i(2\pi f t)\cos(2\pi n f t)\, d(2\pi f t)\right\}\cos(2\pi n f t)\right) + \left(\left\{\frac{1}{\pi}\int_0^{2\pi} i(2\pi f t)\sin(2\pi n f t)\, d(2\pi f t)\right\}\sin(2\pi n f t)\right)\right]$$



  - The RMS grid modulation and anode modulation multiplied by the RMS first Fourier current harmonic give the input and output power (P=I*V/2), the ratio is the gain
  - Note the AC anode oscillation should be < 90% or so of the bias so electron velocity is never less than one third DC value- this implies the load impedance given the AC current term
  - The product of the bias voltage and the zeroth order Fourier current is the DC electrical power- efficiency is then $\eta=P_{out}/(P_{DC}+P_{in})$ (neglecting heater effects)

Valve amplifiers classes and constraints of operation

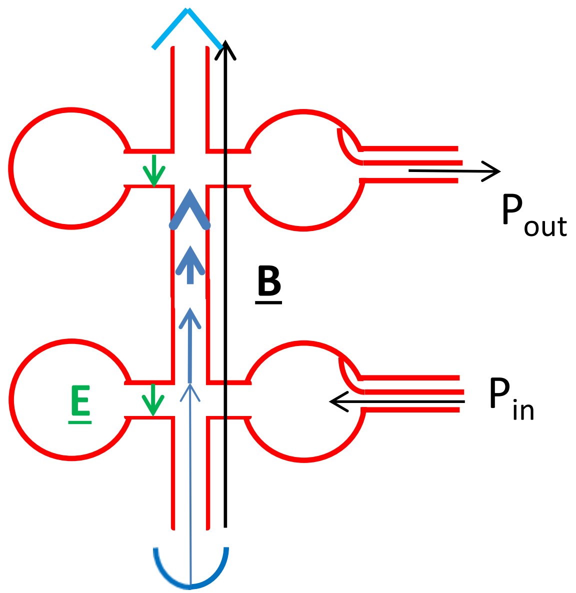
- Helpful features
  - note as in the previous example, a valve does not need to conduct for all of the cycle- there we saw a valve on for exactly half the cycle, we call this Class B
  - Class A is conduction for 360^o phase, AB is between 360 and 180, B is 180, BC is <180
  - In the case above, the fundamental AC current i_1=3.93A exceeds the DC current i_0=2.53A
  - Hence efficiency >50%. Also the amplifier will automatically dissipate NO heat (except for a small leakage current and the heater) if the RF drive is off!
- However
  - Typically as we reduce the conduction angle the gain, power and linearity get worse for similar parameters
- Other issues:
  - Cathode length, diameter fixed by max operating frequency (typically the cathode length $<<\lambda/4$, circumference also $< \lambda$- this limits current
  - Grids and cathode form low impedance open circuit terminated co-axial lines- requires matching to realistic RF feedlines- see later on termination of transmission lines
  - Max freq. also determined by transit time across gap $f<<1/T$, where T=gap/speed
  - Current is highest when velocity is lowest- periods of very high space charge in device

Valve amplifiers: practical points

- Gridded tubes and valves can have shorter lifetime than ungridded vacuum power devices
  - Very close proximity of the grid to cathode is a problem
  - Can distort due to thermal radiation from cathode, can be damaged by current interception
  - Grid can become contaminated with material from the cathode- loss of modulation control
  - Cathode burn out and poisoning can be an issue
  - Often cathode is running at high current density
  - Arcing can lead to tube failure
  - Typically increasing the voltages or the cathode temperature (at some point it may be necessary to do both together to increase power), duty, tend to reduce the lifetime
  - Water cooling is vital to the anode (plate) of any high power valve, and cooling will typically be required at a range of other locations in the circuit.
  - Nonetheless gridded tubes offer rather impressive specific power outputs and efficiency

Example RF drivers

- Klystrons often used for multistage accelerators (IOT's are hybrid klystron/linear tetrode devices)
  - Extended linear electron beam drifting along a confining B field
  - Beam formed in DC electron gun, high $v_{//}$ elsewhere
  - Transit time fast in all gaps- avoids limitations of tetrodes
  - Electrons transiting buncher cavity are velocity modulated, leading to space charge bunches after some distance
  - In the extraction cavity the E-field is excited by displacement of charge in the metal to the downstream end as the bunch enters the upstream 'beam pipe'
    - Ensures retarding field in the gap, reducing the kinetic energy
    - The cavity inverts during a period of tenuous electron current
    - Net work is done by the modulated beam on the wave
  - Addition of intermediate cavities enhances bunching, efficiency and gain



- Expensive to make
- Reliable
- Can achieve high frequency ~10GHz, ~100's MW
- Efficiency can be high, ~30% to 70%+, Gain can be very high (50+dB)

Cylindrical Waveguides/Cavities

- Maxwell's eqn's and boundary conditions constrain the transverse variation of the axial field components in hollow waveguide

$$\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial^2}{\partial r}) + \frac{1}{r}\frac{\partial^2}{\partial \phi^2}\right]\begin{pmatrix} E_{0,z} \\ H_{0,z} \end{pmatrix} = -k_c{}^2 \begin{pmatrix} E_{0,z} \\ H_{0,z} \end{pmatrix}$$

- These have solutions in the form of Bessel functions in '*r*' and trig functions in 'ϕ', TEM modes forbidden, only modes with E_z/H_z allowed
  - Infinite no. of independent solutions for modes with E_z and H_z
  - For waveguide (unconstrained in 'z')

$$H_Z = [A\sin(m\phi) + B\cos(m\phi)]J_n(k_c r)e^{-jk_z z}$$

$$E_Z = [A\sin(m\phi) + B\cos(m\phi)]J_n(k_c r)e^{-jk_z z}$$

  - For a cavity (closed in 'z' @ z=0,L)

$$H_Z = [A\sin(m\phi) + B\cos(m\phi)]J_n(k_c r)\sin(p\pi z/L)$$

$$E_Z = [A\sin(m\phi) + B\cos(m\phi)]J_n(k_c r)\cos(p\pi z/L)$$

  - Metallic boundary conditions quantise the axial and radial variation (Trig and Bessel f^{ns} respectively), symmetry quantises the azimuthal variation (Trig), modes are called $TE/TM_{m,n,p}$ where $m,p \in W$, $n \in N$, since $E_{//}$ and H_{perp} go cont. to zero at metal

Waveguides

- Maxwell's Curl eqn's give the transverse fields
  - In terms of E_z or H_z where k_z is the propagation vector and k_c the transverse wavevector ($k_c^2=k^2-k_z^2$), $k_c= \mu_{m,n}/a$ where *a* is the radius
  - Waveguide dispersion: $f = \frac{c}{2\pi}\sqrt{(\frac{\mu_{m,n}}{a})^2+{k_z}^2}$
  - $\mu_{m,n}$ given by radial B.C. as n'th root of $J_m(x)$ (TM modes) or of $J'_m(x)$ (TE modes)
  - Lowest Mode in Waveguide is $TE_{1,1}$ (lowest Bessel root)
  - $TE_{1,1}$ not useful for acceleration
    - E-field is perpendicular to axis
    - Can be used to 'kick' particles

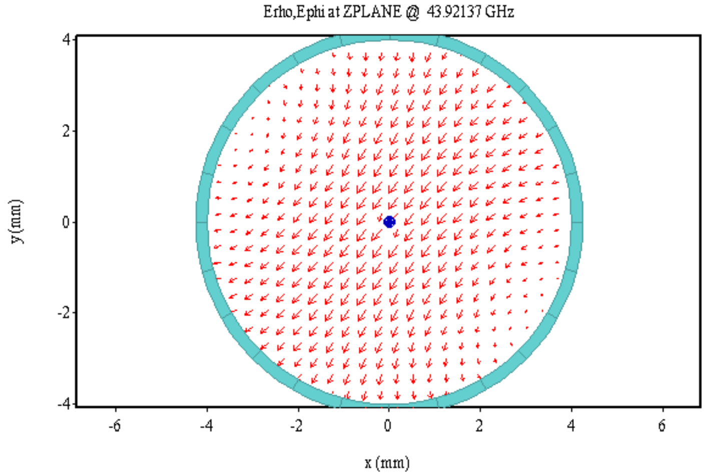
$$E_r = -\frac{j}{k_c^2}\left(k_z\frac{\partial E_z}{\partial r}+\frac{\omega\mu}{r}\frac{\partial H_z}{\partial \phi}\right)$$

$$E_\phi = -\frac{j}{k_c^2}\left(-\frac{k_z}{r}\frac{\partial E_z}{\partial \phi}+\omega\mu\frac{\partial H_z}{\partial r}\right)$$

$$H_r = \frac{j}{k_c^2}\left(\frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \phi}-k_z\frac{\partial H_z}{\partial r}\right)$$

$$H_\phi = -\frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial r}+\frac{k_z}{r}\frac{\partial H_z}{\partial \phi}\right)$$

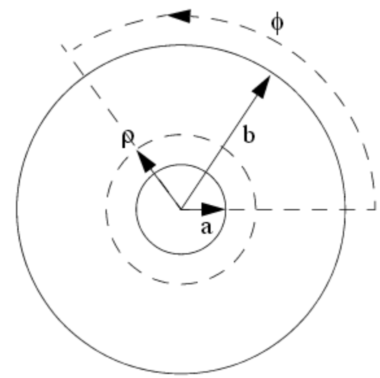
- Rectangular waveguide ('a x b' section)
  - Similar results, $TE_{m,n}$ and $TM_{m,n}$ modes
  - Transverse fields are Trig functions
  - dispersion: $f = \frac{c}{2\pi}\sqrt{(\frac{m\pi}{a})^2+(\frac{n\pi}{b})^2+{k_z}^2}$



Co-Axial Waveguides

- Maxwell's wave eqn constrains the transverse variation of the field components

$$E = \frac{V_0 \hat{\rho} e^{j\gamma z}}{\rho \ln \frac{b}{a}} \text{ and } H = \frac{I_0 \hat{\phi} e^{j\gamma z}}{2\pi\rho}$$

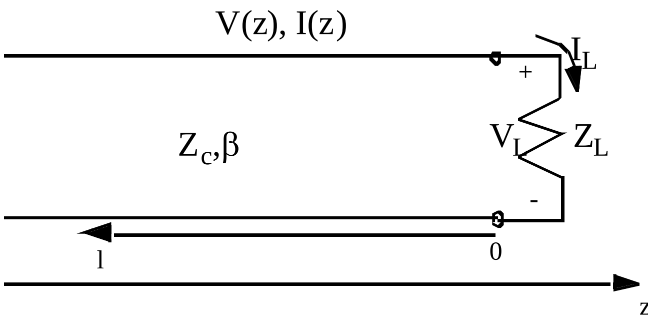


$$Z_C = \frac{V}{I} = \frac{E \ln \frac{b}{a}}{2\pi H} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{\ln \frac{b}{a}}{2\pi} = \frac{Z_W}{2\pi} \ln \frac{b}{a}$$

- The two conductor topology allows them to also support a TEM wave (no E_z or H_z), in addition to TE and TM modes
- Straightforward to show that this has the dispersion of the media enclosed
- Therefore field solution is the same as the static field solution
  - This Is the only mode allowed for $(2\pi/\lambda)$>~2/(a+b)
  - Useful for transporting energy- e.g. MICE transmission lines
  - Note as a/b decrease fields increase for given V and I- hence in MICE 6" lines for 1MW, 4" for 500kW
  - Avoids breakdown of gas (dry air at stp, max field is 30kV/cm)
  - Can also mitigate by filling with electronegative gasses
  - Voltage, current and power related by a characteristic impedance Z_c

Transmission Line Termination

- Waves on realistic transmission lines can be represented by superposition of counter propagating voltage and current waves on a two wire line (here β is the wavevector, Z_C is the line characteristic Impedance)



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \text{ and } I(z) = \frac{V_0^+}{Z_C} e^{-j\beta z} - \frac{V_0^-}{Z_C} e^{j\beta z}$$

$$\frac{V^-}{V^+} = \frac{\frac{Z_L}{Z_C} - 1}{\frac{Z_L}{Z_C} + 1} = \Gamma_L \qquad \bar{Z}_L = \frac{Z_L}{Z_C}$$

- The reflected amplitude V^- and phasor reflection co-efficient Γ_L are determined by continuity at the load
- Clearly unless the normalised impedance $\bar{Z}_L$ is unity there will be energy reflected back along the line, reducing the power delivered to the load to:

$$P = \frac{1}{2} \Re(V_L I^*_L) = \frac{|V^+|^2}{2Z_C}(1 - |\Gamma_L|^2)$$

Standing Wave Ratio

- A standing wave will be established on the line with nodes/antinodes spaced $\lambda/2$ apart (here $\rho=|\Gamma|$, θ is phase of Γ)

$$|V| = |V^+| \cdot \{1 + 2\rho\cos(\theta + 2\beta z) + \rho^2\}^{1/2}$$

- The ratio of minima to maxima amplitude is called the SWR (Standing Wave Ratio) and is often used to describe the matching conditions

$$SWR = {(1+\rho)}/{(1-\rho)}$$

- If we have a perfect reflection then the peak amplitude on the line doubles- can be important if we are near breakdown conditions
- A length 'l' of the transmission line transforms the phase (but not magnitude) of the reflection co-efficient, hence any imperfectly matched lossy load can be transformed to present pure reactance or resistance

$$\Gamma(l) = \frac{V^- e^{-j\beta l}}{V^+ e^{j\beta l}} = \Gamma_L e^{-2j\beta l} \quad \bar{\bar{Z}}_{in} = \frac{Z_{in}}{Z_C} = \frac{V}{IZ_C} = \frac{V^+ e^{j\beta l} + V^- e^{-j\beta l}}{V^+ e^{j\beta l} - V^- e^{-j\beta l}} = \frac{1+\Gamma(l)}{1-\Gamma(l)} = \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}}$$

$$\bar{\bar{Z}}_{in} = \frac{\bar{\bar{Z}}_L + j\tan\beta l}{1 + j\bar{\bar{Z}}_L \tan\beta l}$$

Impedance matching, measurement and visualisation

- A Smith chart is often used to visualise complex impedance/admittance and reflection data
  - Phasor reflection diagram in complex plane with contours of resistance and reactance superimposed
- Measurements of the circuit parameters are performed by an instrument known as a (vector or scalar) network analyser
- Impedance matching is desirable for many reasons
  - It enhances efficient energy transfer to the load
  - It prevents voltage uplifts on the transmission line (losses and breakdown issues)
  - If the source AND load are separately matched to the transmission line it prevents very narrow band cavity resonances on the transmission line
- Matching may be performed in several ways
  - L or T branch networks of capacitors/inductors
  - Stub tuners (inserts or branches placed into lines to balance the mismatches)
  - Quarter wave transformers (line segments $\lambda/4$ long with geometric mean impedance)

Cylindrical Cavities

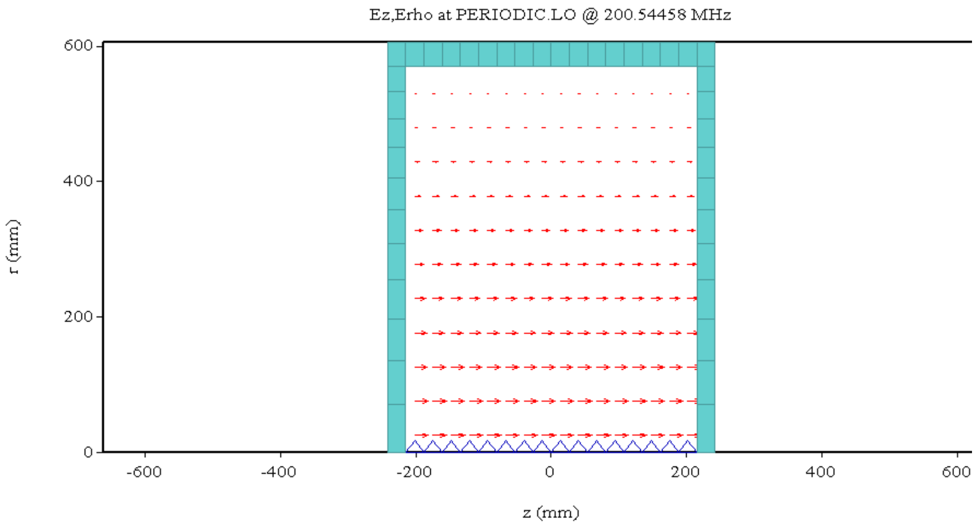
- Conducting plates on ends of waveguide- form a pillbox resonant cavity
  - Cavity eigenmodes are: $f = \frac{c}{2\pi}\sqrt{(\frac{\mu_{m,n}}{a})^2+(\frac{p\pi}{L})^2}$
  - Short cylindrical cavities, lowest mode is $TM_{0,1}$ (since p=0 forbidden by axial B.C. for TE modes)
  - This is convenient!
  - Strong axial and zero transverse E field- good for acceleration in place of induction system
  - These are a simple approximation to the MICE cavities
  - f tuned by radius NOT length

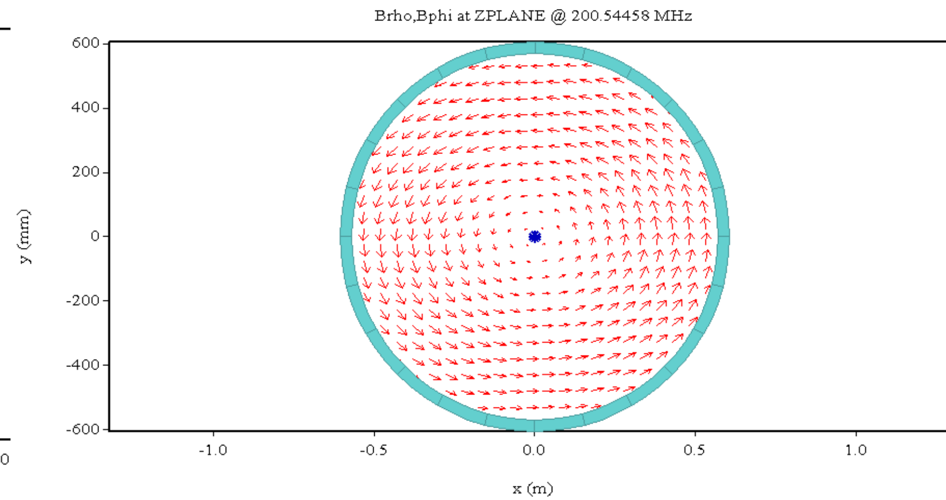
$$E_r = -\frac{j}{k_c^2}\left(k_z\frac{\partial E_z}{\partial r} + \frac{\omega\mu}{r}\frac{\partial H_z}{\partial \phi}\right)$$

$$E_\phi = -\frac{j}{k_c^2}\left(-\frac{k_z}{r}\frac{\partial E_z}{\partial \phi} + \omega\mu\frac{\partial H_z}{\partial r}\right)$$

$$H_r = \frac{j}{k_c^2}\left(\frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \phi} - k_z\frac{\partial H_z}{\partial r}\right)$$

$$H_\phi = -\frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial r} + \frac{k_z}{r}\frac{\partial H_z}{\partial \phi}\right)$$

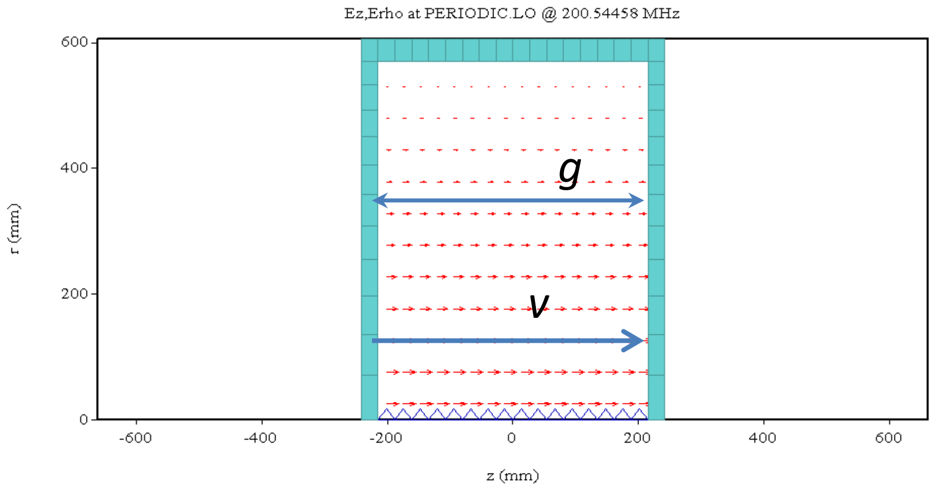




Transit Time

- Of course the particles take a finite time to cross the cavity
- Hence the electric field may be estimated by averaging over the transit time

$$E_{eff} = \frac{E_0}{\tau}\int_{-\tau/2}^{\tau/2}\cos(\omega t)dt = E_0\frac{\sin\left({}^{2\pi f g}/_{2v}\right)}{{}^{2\pi f g}/_{2v}}$$



- Here we have ignored the change in particle velocity
- In non-relativistic cases one might use an estimate of the average velocity

Energy and Dissipation

- The energy in the cavity is stored in the oscillating electric and magnetic fields
  - The total energy may be found by integrating the fields over the volume

$$U = \frac{1}{4}\int_V \left(\varepsilon|\underline{E}(r)|^2 + \frac{1}{\mu}|\underline{B}(r)|^2 \right) dV$$

  - The wall losses limit the gradient for a given drive power
    - When the cavity current is high enough that the power dissipated equals the feed power less reflected power
    - Minimising reflected power enhances the peak gradient
  - Wall losses are calculated integrating the tangential H field over the surface of the cavity

$$D = \frac{1}{2\sigma\delta}\oint_S \underline{H}_{\backslash\backslash} \cdot \underline{dS}$$ where $$\delta = \frac{\sqrt{2}}{\sqrt{\omega\mu\sigma}}$$ ıe skin depth

  - The Q is defined as 2.π times the ratio of the energy stored to the loss per cycle

$$Q_\Omega = \omega\frac{U}{D}$$

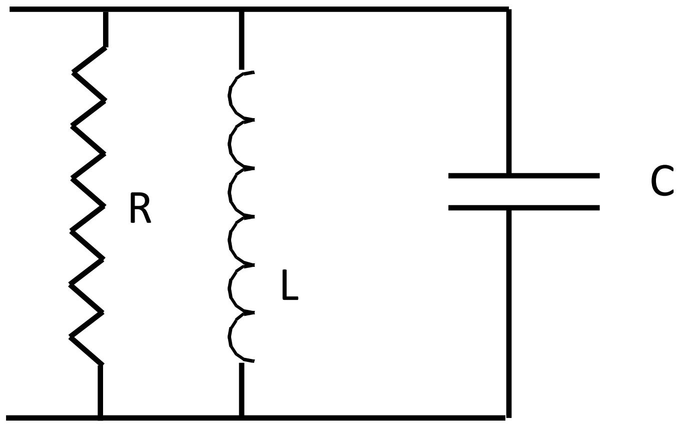
  - The integrals produces different results for TE and TM modes and for TM modes the result differs for the p=0 (axially invariant) mode and the other modes, for $TM_{0,1,0}$ (cylindrical resonator)

$$Q_\Omega = \frac{c\mu_{m,n}}{\pi f \delta_s (2 + {}^{2a}/_{L})}$$

  - If Q is high then we may also write $Q = \omega_0/\Delta\omega$, the ratio of the frequency to the linewidth

LRC Representation

- Close to resonance a cavity can be represented by a shunt LRC circuit



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

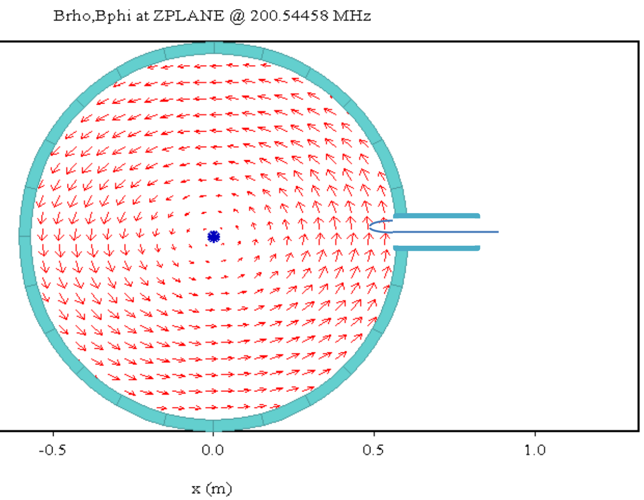
$$R = V^2/2P$$

$$Q_\Omega = R\sqrt{\frac{C}{L}}$$

- Here the capacitor nominally represents the opposite faces of the cavity, the inductor the current path round the cavity walls
- The resistance R (called the Shunt impedance) relates the dissipated power to the gap voltage amplitude
- In some sense R refers to the wall resistivity (can also account for beam loading- not relevant to MICE)- hence it is related to Ohmic Q
- Near resonance the impedance tends to R (even though it is typically quite a large value) since the LC combination has a very high impedance

Coupling of cavities to lines

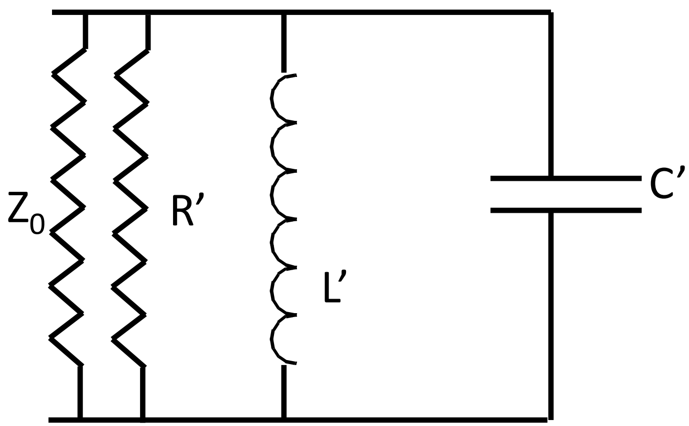
- To be useful we need to inject energy into a cavity
- This can be done by several techniques
  - If freq is fairly low, the transmission line is probably co-axial
  - Use probe coupling- insert inner of coaxial line into cavity // to E field
  - Or (as in MICE) place a loop coupler in the ‘cylindrical’ waist- excite H fields
  - At higher frequency the coupler is typically a waveguide aperture



- The loop coupler acts as a transformer on the end of the transmission line
- Transformer ratio is controlled by the polarisation of the coupler to the fields
- Adjusts the rather high impedance of the resonant cavity to the line impedance (say ~50Ω)
- The coupler acts both ways- it couples energy out as well as in

Coupling of cavities to lines

- We can again use an LCR model to understand the coupling process
- The cavity (impedance transformed through the mutual inductance of the coupler) is now connected to the line, characteristic impedance Z_0 (pure resistance)



- We now have an additional loss term
- The cavity is losing power to Z_0 and R

$$1/Q_l = D/U = (D_R + D_Z)/U = D_Z/U + 1/Q_\Omega$$

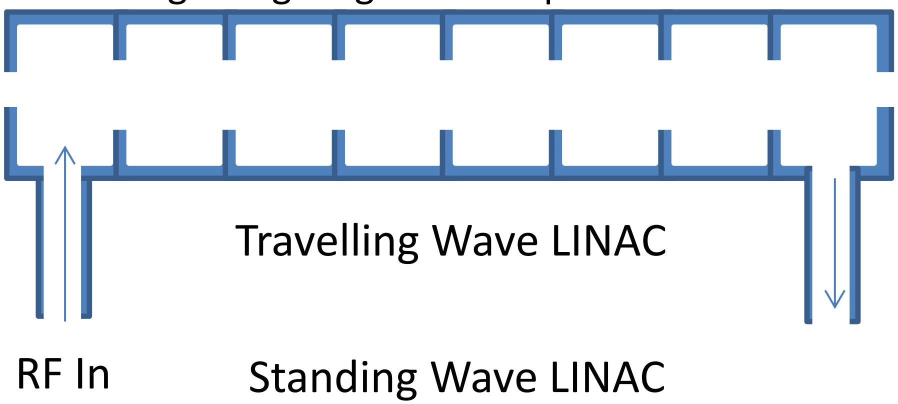
- Q_l is called the 'loaded Q' and D_z/U is called Q_e the external Q

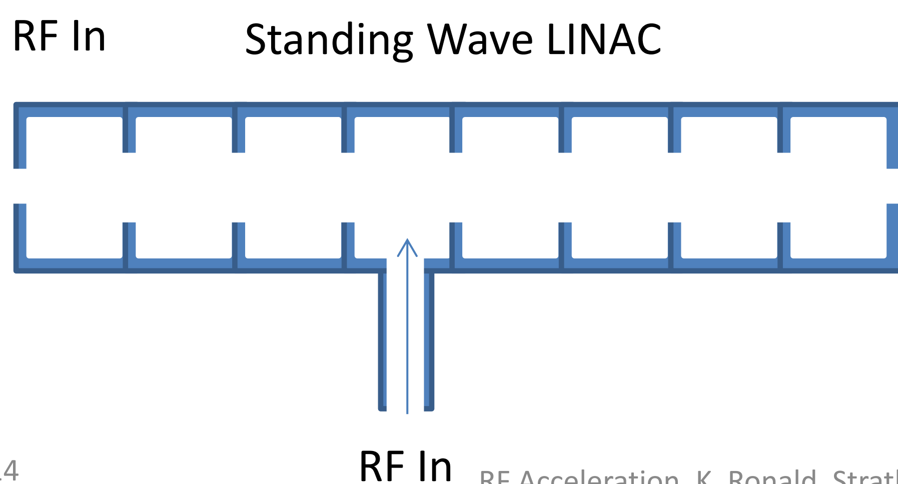
$$Q_l = Q_\Omega/(1+\beta), \text{ where } \beta = \frac{Q_\Omega}{Q_e} = \frac{Z_0}{R}$$

- If β is unity then the effective Q if the circuit is halved
- If β is unity we have achieved the required matching condition
  - Zero energy reflected from the cavity in steady state
  - Note this does NOT mean zero energy reflected during charge/discharge cycles

Types of RF Cavity

- There are a number of different cavity configuration- two shown here derive from disc loaded waveguides
  - Periodic loading reduces phase velocity, varying axial periodicity allows matching to beam velocity
  - Profiling along length can be performed as low energy particles are accelerated

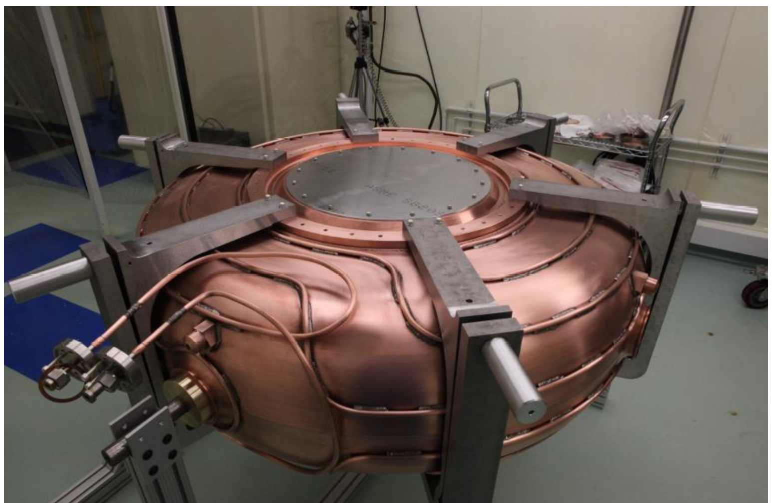


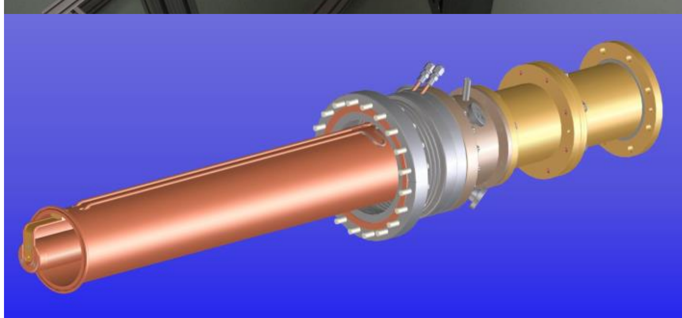


- Travelling wave accelerators, RF propagates with the beam
- RF extracted from end of cavity chain
- Can be recirculated in some circumstances
- Tuning of frequency controls energy
- Standing wave system, wave reflects from ends of cavity
- Not such direct control of acceleration
- Typically alternate cavities moved to sidearms to reduce length

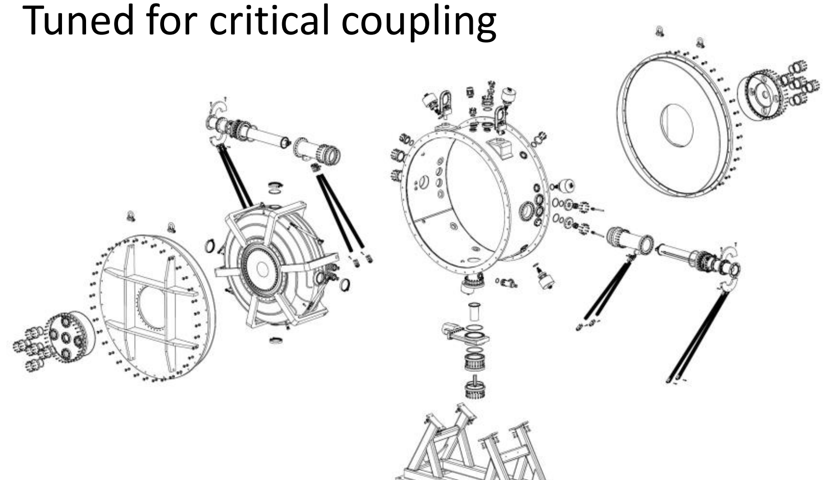
The MICE RF cavity

- The MICE cavity is an individual resonator with some similarities to the pillbox, but with profiled sides
- Such cavities may be solved numerically to predict their RF performance





- Each cavity has f_0~201.25MHz, Q~50,000
- Two couplers carrying ~500kW each for 8MV/m
- Gap is ~0.43m
- Tuning by rotation of polarisation of coupler loop
- Tuned for critical coupling



- Images ‘borrowed’ from Yagmur Torun and Allan DeMello