Constraints on the quark and lepton mixing matrices within a fourth generation scenario from a few simple constraints

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Introductory remarks

- Recent interest in a 4th fermion generation triggered by LHC startup:
  CKM and PMNS matrix constraints important for search strategies!

- Recent discussion of constraints on 4x4 CKM- & PMNS-elements:
  Kribs et al., Fourth generation and Higgs Physics, PRD76:075016, 2007

  CKM: Unitarity constraints + W-decays + D-mixing

  PMNS: $\mu \to e\gamma$, $\mu \to e$ conversion

- Several papers in the past, some even very recent:
  $Z \to bb$, B- & K-system (not considered here)

Content (no rigorous quantitative analysis yet)

*** W-decays (correlation between CKM and PMNS)**

*** τ- and μ-decays**

*** Other correlations in CKM- and PMNS-element extractions**

Mixing in quark sector

Directly measured matrix elements:

$$\left|V_{CKM}^{4\times4}\right| = \begin{pmatrix} 0.97418 & 0.2246 & 0.0039 & <0.04 \\ 0.22 & >0.78 & 0.041 & <0.6 \\ <0.08 & <0.40 & >0.78 & <0.65 \\ <0.1 & <0.60 & <0.65 & >0.78 \end{pmatrix}$$

at ~2σ

- Limiting factors:

 * $|V_{tb}|$ from single top + $R = \frac{\Gamma(t \to W + b)}{\Gamma(t \to W + q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$ not well constrained yet

 * $|V_{cs}|$ from sl D-decays: still large theoretical error (FF)

 (Leptonic D_s-decay: NP-signal? If yes: Not from 4th generation)

Mixing in quark sector

Directly measured matrix elements:

$$\left|V^{4\times4}_{CKM}\right| = \begin{pmatrix} 0.97418 & 0.2246 & 0.0039 & <0.04 \\ 0.22 & >0.75 & 0.041 & <0.6 \\ <0.08 & <0.40 & >0.78 & <0.65 \\ <0.1 & <0.60 & <0.65 & >0.78 \end{pmatrix}$$

- Constraints discussed by Kribs et al.:

  * $|V_{tb}|$

  * $\Gamma(W\to had)$

$$\left|V^{4\times4}_{CKM}\right| = \begin{pmatrix} 0.97418 & 0.2246 & 0.0039 & <0.04 \\ 0.22 & 0.965 & 0.041 & <0.2 \\ <0.08 & <0.15 & >0.78 & <0.65 \\ <0.09 & <0.17 & <0.65 & >0.78 \end{pmatrix}$$

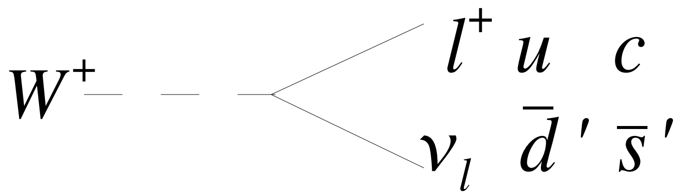
  * D-mixing (Golowich et al., PRD76:095009, 2007) $\rightarrow \left|V_{ud_4} V^*_{cd_4}\right| < 0.002$

  Limit dependent on d_4 mass (here 200 GeV used if I read off correctly)

  => Given the D0 sensitivity to long d_4 lifetimes
  more or less ok but deserves a closer look

Constraint from W-decays

* BF(W→had) used by Kribs et al. (PDG06) calculated from measured BF(W→lep)!



$$\frac{\Gamma(W\to l\nu)}{\Gamma(W\to All)} \approx \frac{1}{3+3\sum_{i=u,c}\sum_{j=d,s,b}|V_{ij}|^2(1+\alpha_s(M_W)/\pi)}$$

* Unitarity check in the first place

* #families>3: Strong constraint on $|V_{cs}|$!

* Consistent with 3x3-Unitarity: $\sum_{j=d,s,b}|V_{uj}|^2+\sum_{j=d,s,b}|V_{cj}|^2=2.002\pm0.027$

However:

* Formula (e.g. PDG06) assumes lepton universality!
* 4 generations: lepton universality possibly violated!

$$BF(W\to e\nu)=0.1075\pm0.0013$$
$$BF(W\to \mu\nu)=0.1057\pm0.0015$$
$$BF(W\to \tau\nu)=0.1125\pm0.0020$$

$$\frac{\Gamma(W\to l\nu)}{\Gamma(W\to All)} \approx \frac{\sum_{k=1,2,3}|U_{lk}|^2}{\sum_{l=e,\mu,\tau}\sum_{k=1,2,3}|U_{lk}|^2+3\sum_{i=u,c}\sum_{j=d,s,b}|V_{ij}|^2(1+\alpha_s(M_W)/\pi)}$$

Mixing in lepton sector: τ- and μ-decays

Constraints on 4th generation from τ mass & (leptonic) BF's:

* Dova, (Swain & Taylor), NP Proc.Suppl.76:133,1999; (hep-ph/9712383; PRD55:1,1997)

1) Since then: Significant improvements in m_τ & BF measurements

2) Assumption: Only significant mixing between 3rd and 4th family

W/o this assumption:

$$\Gamma(\tau^- \to l^- \bar{\nu}_l \nu_\tau) \propto G_F^2 \sum_{i=1,2,3} |U_{\tau i}|^2 \sum_{k=1,2,3} |U_{lk}|^2 \qquad l = e/\mu$$

$$\Gamma(\tau^- \to h \nu_\tau) \propto G_F^2 f_h^2 |V_{uj}|^2 \sum_{i=1,2,3} |U_{\tau i}|^2 \qquad j = d(\pi)/s(K)$$

$$\Gamma(h^- \to \mu^- \nu_\mu) \propto G_F^2 f_h^2 |V_{uj}|^2 \sum_{i=1,2,3} |U_{\mu j}|^2 \qquad j = d(\pi)/s(K)$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \propto G_F^2 \sum_{i=1,2,3} |U_{\mu i}|^2 \sum_{k=1,2,3} |U_{ek}|^2$$

CKM- & PMNS-matrix: W→lep & τ-/μ-decays

Constraints on PMNS elements:

$$\left|U^{4\times4}\right| = \begin{pmatrix} * & * & * & <0.2 \\ * & * & * & <0.2 \\ * & * & * & <0.2 \\ <0.25 & <0.25 & <0.25 & >0.9 \end{pmatrix}$$

IF G_F is constrained to vary by 5% around its standard value

PMNS constraints are sufficiently strong to improve

CKM constraints from $\Gamma(W\to l\nu)$ but less constraining compared to constraint assuming lepton universality in such a case

Constraints on CKM elements:

$$\left|V^{4\times4}_{CKM}\right| = \begin{pmatrix} 0.97418 & 0.2246 & 0.0039 & <0.04 \\ 0.22 & 0.965 & 0.041 & <0.3 \\ <0.08 & <0.3 & >0.78 & <0.65 \\ <0.09 & <0.3 & <0.65 & >0.78 \end{pmatrix}$$

Other areas to look at

- Determination of $|V_{ud}|$: $|V_{ud}| = 0.97418 \pm 0.00026$ (superallowed β-decays)

With 4^{th} generation:

$$\Gamma(\beta\text{-}decay) \propto G_F^2 \; |V_{ud}|^2 \sum_{k=1,2,3} |U_{ek}|^2$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \propto G_F^2 \sum_{i=1,2,3} |U_{\mu i}|^2 \sum_{k=1,2,3} |U_{ek}|^2$$

$$\to |V_{ud}| / \sqrt{\left(\sum_{k=1,2,3} |U_{\mu i}|^2\right)}$$

1σ-overestimation of $|V_{ud}|$: $|U_{\mu 4}| = 0.023$!

- Determination of $|V_{us}|$:

1. Separate averages for K_{e3} versus $K_{\mu 3}$ decays mandatory

2. $K_{\mu 2}/\pi_{\mu 2}$ decays: dependency on lepton sector cancels

Summary

- Combined 4^{th} generation analysis of CKM- & PMNS-matrix with a rigorous treatment of the correlations missing so far

  Quantitative analysis presented very preliminary: many things to be studied

- Leptons: μ- and τ-decays

  => some constraints on 4^{th} generation PMNS elements if G_F could be constrained using additional constraints

- Quarks: $\Gamma(W \rightarrow l\nu)$ together with μ- & τ-decays

  => Constraint on $|V_{cs}|$ only as good as one using lepton universality if G_F can be constrained

  => 3^{rd} ($1^{st}/2^{nd}$) -4^{th} mixing can be still very (reasonably) large (--> searches)

- Other CKM element extractions need to be considered as well, e.g.:

  $|V_{ud}| \rightarrow |V_{ud}|/\Sigma|U_{\mu i}|$, K_{e3} versus $K_{\mu 3}$ decays

  ($K_{\mu 2}/\pi_{\mu 2}$: dependency on lepton sector cancels)