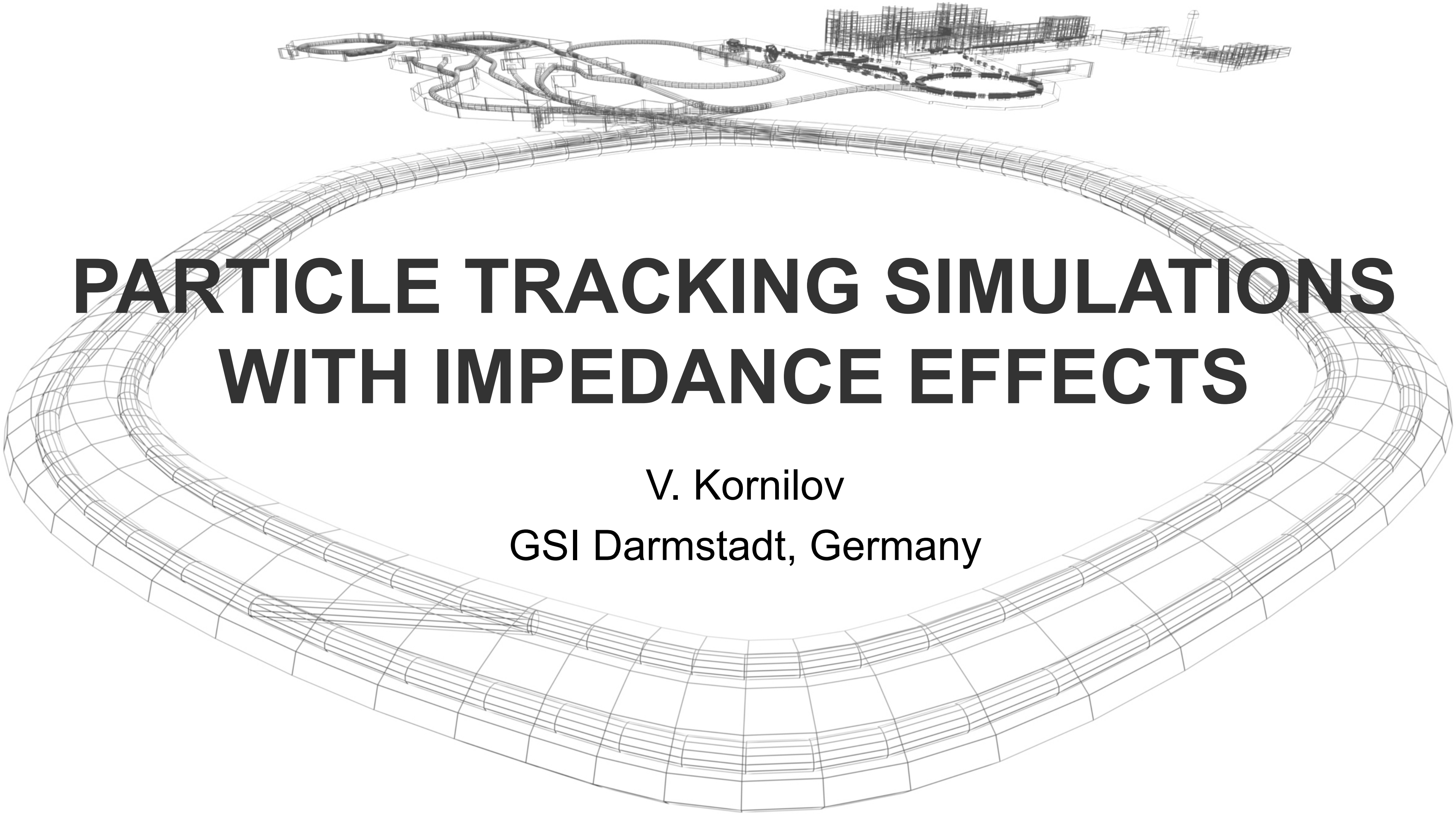


PARTICLE TRACKING SIMULATIONS WITH IMPEDANCE EFFECTS

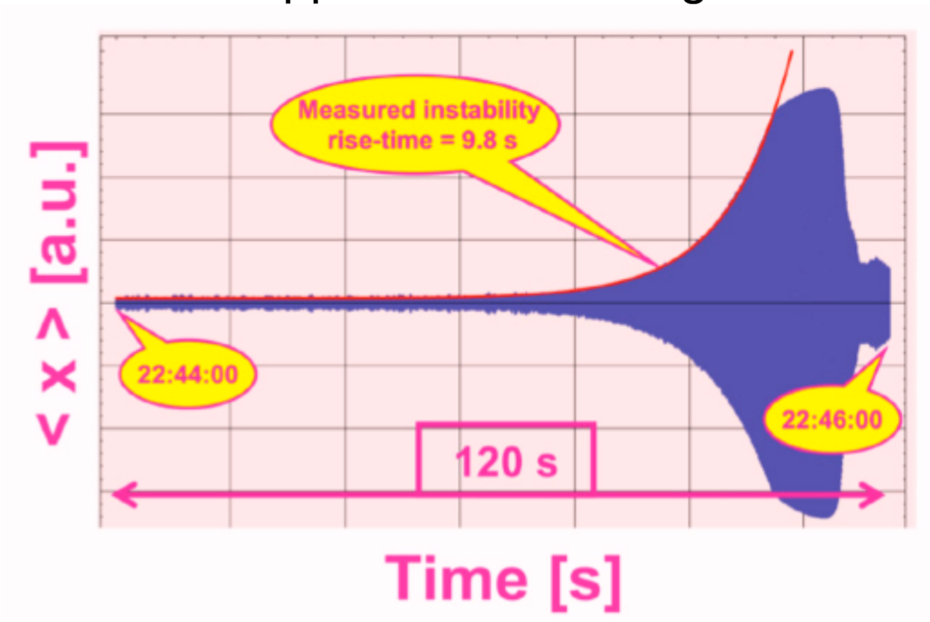
V. Kornilov
GSI Darmstadt, Germany

OUTLINE

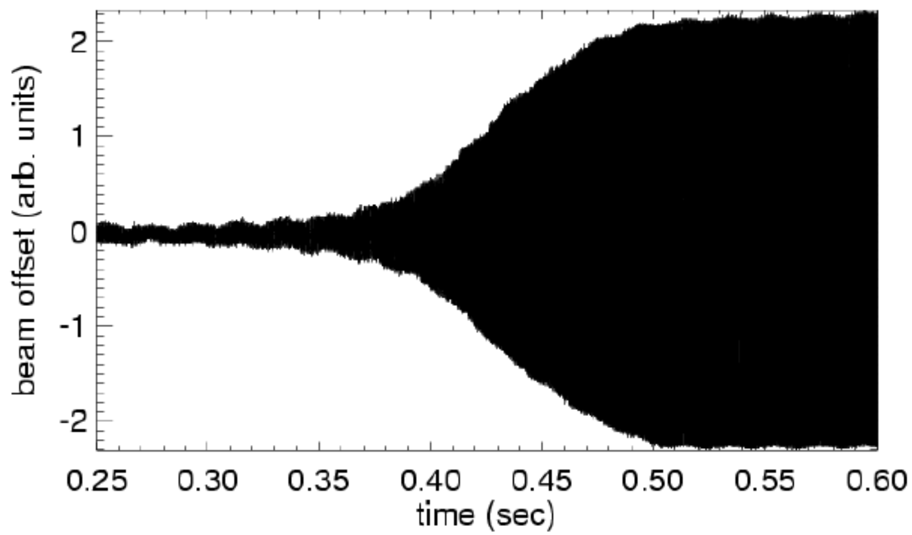
- Wake Fields, Impedances
- Effect of Impedances on Beams
- Implementation in Tracking Simulations
- Simulations with Longitudinal Impedances / Wakes
- Simulations with Transverse Impedances / Wakes

INTRODUCTION

Even if the focusing, acceleration, etc. is perfect in your machine, often happens the following:



Instability in LHC, CERN
Metral, et. al., IPAC2011

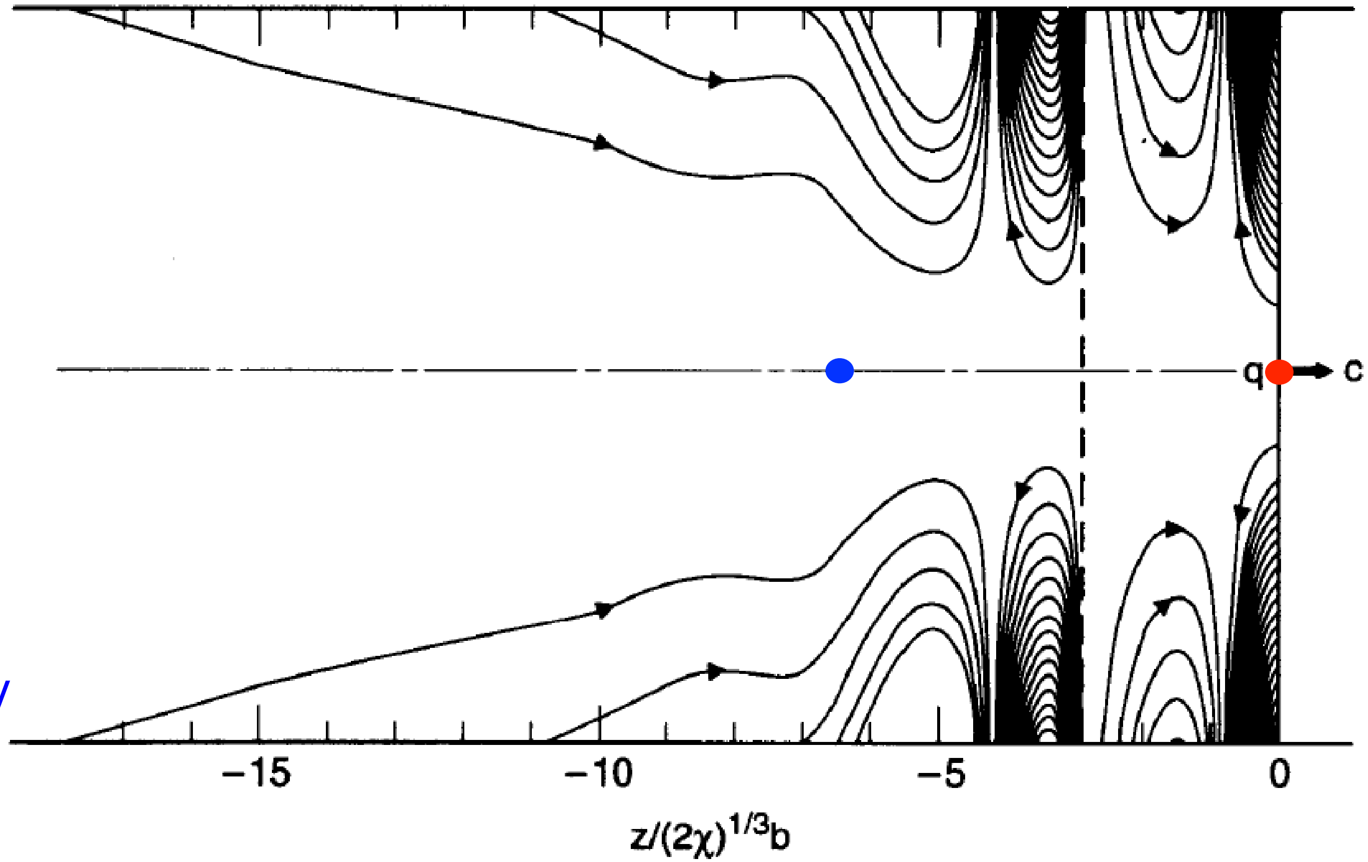


Instability in SIS18, GSI
V.Kornilov, 2009

The reason is: IMPEDANCES of the machine:
the vacuum pipe, kickers, collimators, cavities, bellows, diagnostics, … everything

WAKE FIELDS

- Leading charge
- Trailing charges
- Leading charge generates electromagnetic fields
- Leading charge is loosing energy
- Trailing charge is gaining/loosing energy



Electric field pattern for a resistive wall pipe
A.Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993

WAKE FUNCTIONS

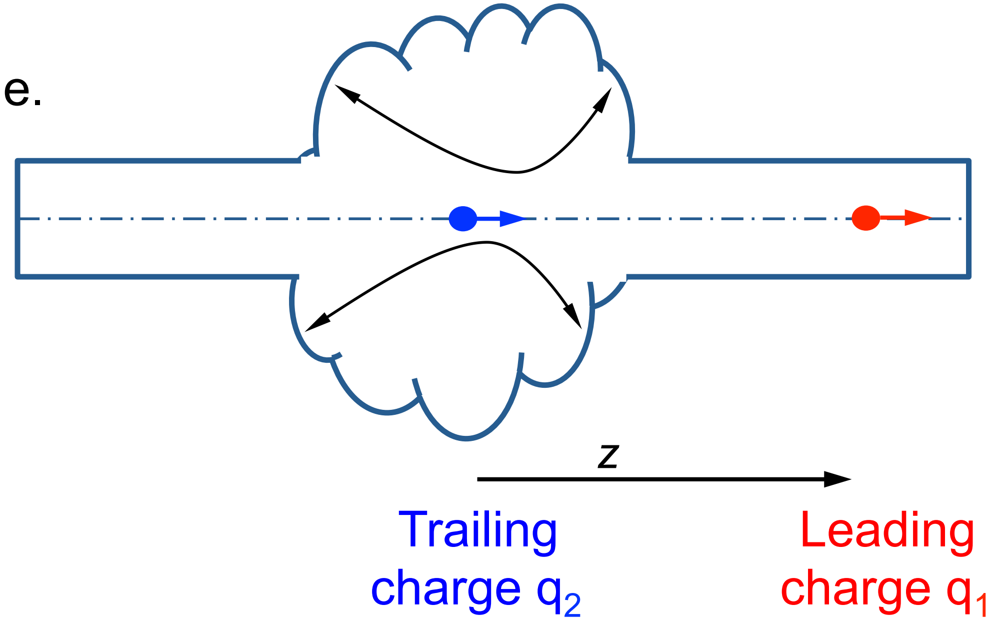
In order to describe the effect of the wake fields, we define the “wake function”.

The energy loss/gain of the trailing charge.

The longitudinal Wake Function:

$$\int F_{\|} ds = \Delta\mathcal{E}_2 = -q_1 q_2 W_{\|}(z)$$

$$F_{\|}(s,z) = q_2 E_z(s,z)$$



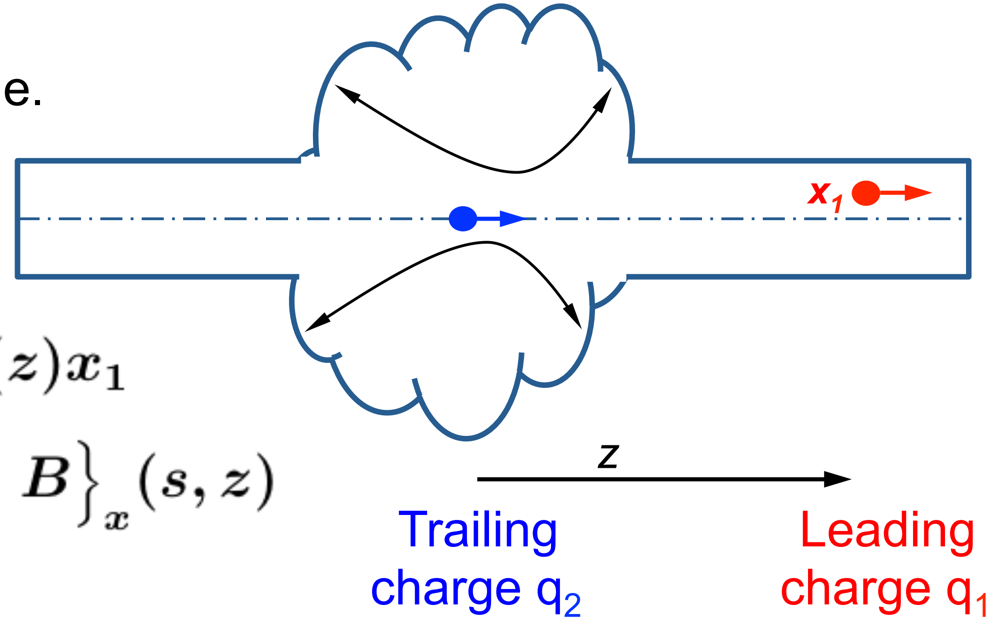
The “lumped” (localized) impedance:
one interaction per turn.
Field integral is over the structure elements.

In order to describe the effect of the wake fields, we define the “wake function”.

The energy loss/gain of the trailing charge.

The transverse Wake Function:

$$\int F_x ds = \mathcal{E}_0 \Delta x_2' = -q_1 q_2 W_x(z) x_1$$
$$F_x(s,z) = q_2 E_x(s,z) + q_2 \{v \times B\}_x (s,z)$$



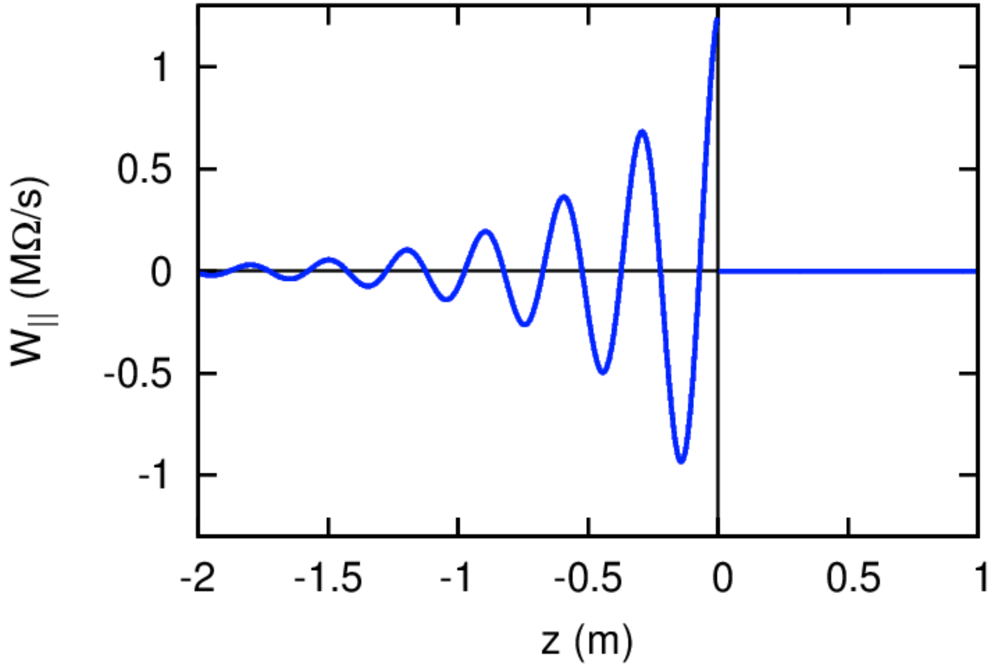
The dipole impedance: the offset of the leading particle produces the wake, which does not depend on the trailing particle offset.

WAKE FUNCTIONS

An example:
Resonator Model
(equivalent LRC circuit) with

- Resonant frequency f_r
- Quality factor Q
- Shunt impedance R_s
- Damping rate α

Real function,
(ultrarelativistic) non-zero for z<0,
behind the leading charge (causality)



f_r= 1GHz, Q=5, R_s=1kΩ

$$W_{||}(z) = \frac{R_s c k_r}{Q} e^{\alpha z} \left\{ \cos(\bar{k}_r z) + \frac{\alpha}{\bar{k}_r} \sin(k_r z) \right\}$$

$$k_r = \frac{\omega_r}{c}, \quad \alpha = \frac{k_r}{2Q}, \quad \bar{k}_r = \sqrt{|k_r^2 - \alpha^2|}$$

IMPEDANCES

Moving to the frequency space, we define the **coupling impedances:**
complex functions of the frequency

$$Z_{\|}(\omega) = \int e^{-i\omega z/v} W_{\|}(z) \frac{dz}{v}$$

$$Z_{\perp}(\omega) = i \int e^{-i\omega z/v} W_{\perp}(z) \frac{dz}{v}$$

$$\begin{aligned} \overline{Z_{\|}(\omega)} &= Z_{\|}(-\omega) \\ \overline{Z_{\perp}(\omega)} &= -Z_{\perp}(-\omega) \end{aligned}$$

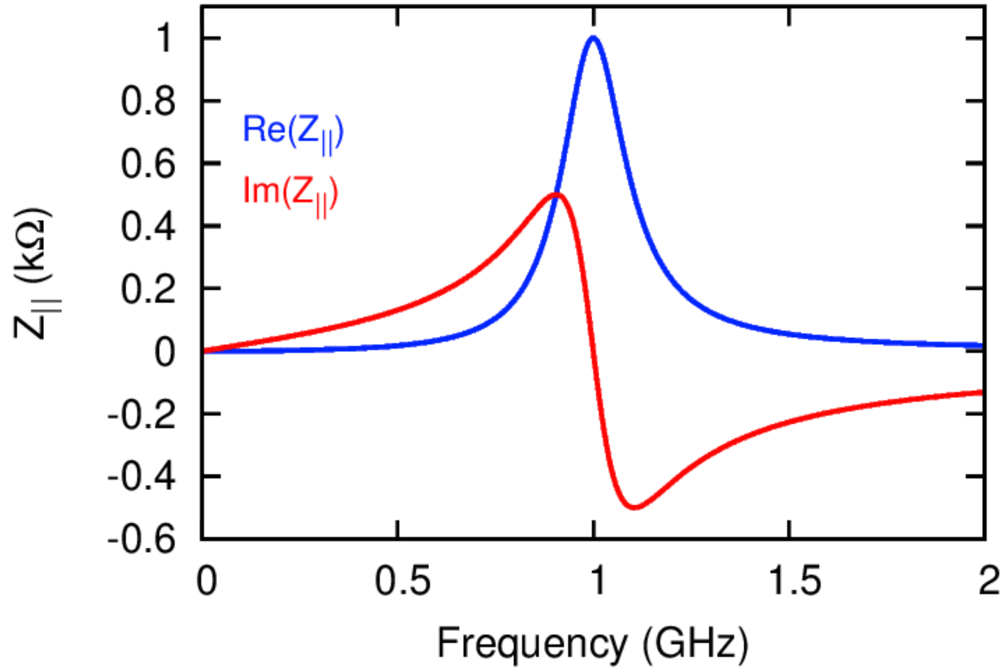
Many other properties following from the symmetry and causality

A.Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993

IMPEDANCES

An example:
Resonator Model
(equivalent LRC circuit) with

- Resonant frequency f_r
- Quality factor Q
- Shunt impedance R_s
- Damping rate α



f_r = 1GHz, Q=5, R_s=1kΩ

Complex function

$$Z_{||}(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

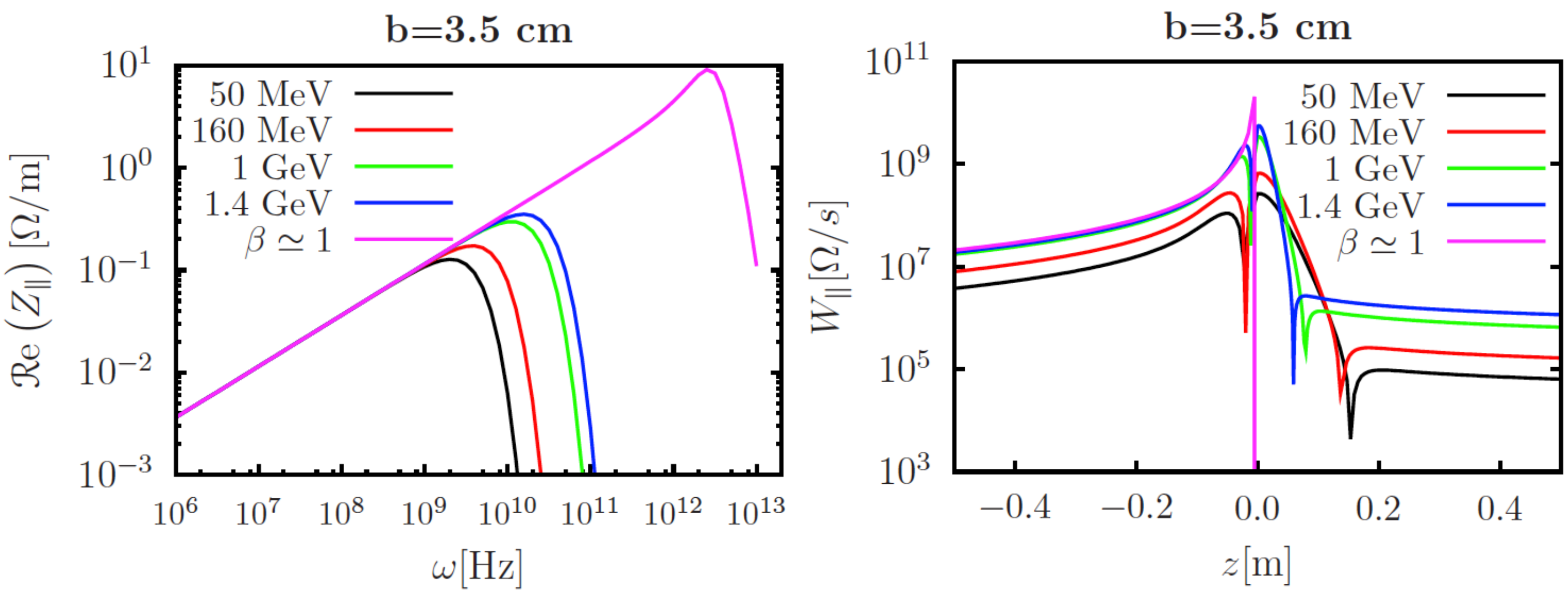
Often, it is can be expressed as:

$$\Omega = \Omega_{\rm Re} + i\Omega_{\rm Im} = \Omega_{\rm mode} + \kappa Z_{\rm Im}^{\rm eff} + i\kappa Z_{\rm Re}^{\rm eff}$$

beam oscillation | frequency shift | growth rate

Things are more complicated 1

The beam is non-ultrarelativistic ***β<1***
There is a wake ahead of the leading charge ***z>0***
The Wake Functions and the Impedances are much more involved.

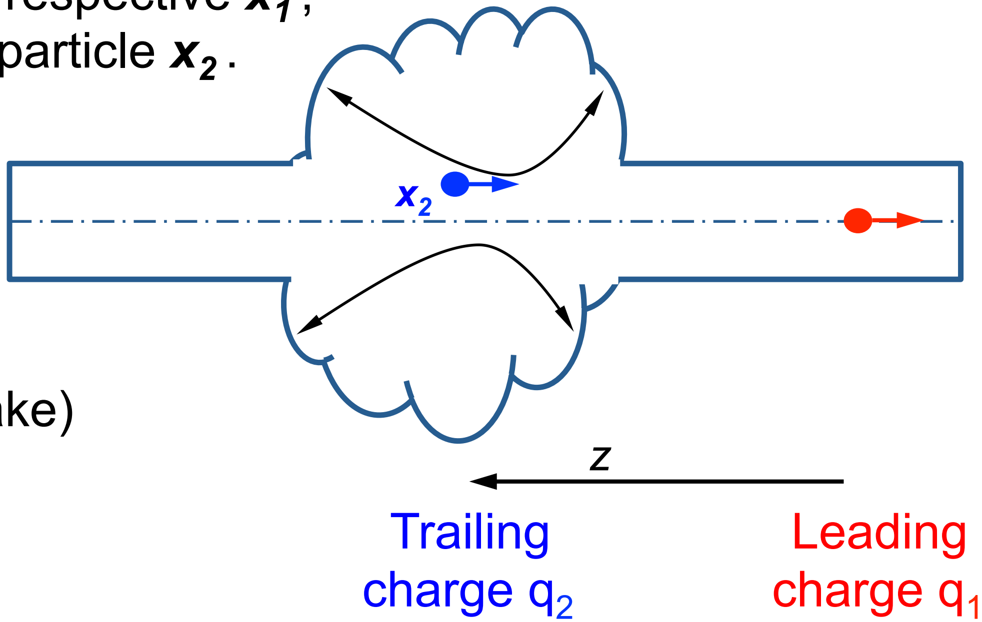


Resistive Wall Impedance and Wake Function for the PS Booster,
D.Quatraro, PhD Thesis, CERN, Bologna U. (2011)

Things are more complicated 2

Asymmetric structure produce wakes irrespective x_1, proportional to the offset of the trailing particle x_2.

$$\int F_x ds = -q_1 q_2 D_x(z) x_2$$



- $D_x(z)$ is the **detuning** wake function (sometimes called quadrupolar wake)
- $W_x(z)$ is the **driving** function (sometimes called dipolar wake)

$D_x(z)$ and $W_x(z)$ are very different in the effect on the beam and in the simulation.

Generalized impedances can be defined.

A.Burov, V.Danilov, Phys. Rev. Let. 82, 2286 (1999)

Things are more complicated 3

The leading charge is ***cos(mθ),***
there are higher order wakes and impedances.

$$\int F_{||} ds = -q_2 I_m W_{||m}(z) r^m \cos(m\theta)$$

Usually, the concern is about the lowest order (also discussed so far):

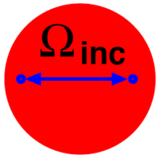
- Longitudinal monopole wake and impedance ***m=0***
- Transverse dipole wake and impedance ***m=1***

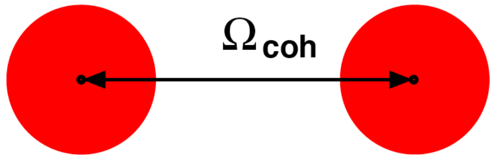
A.Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993
K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006

IMPEDANCE EFFECTS

incoherent (single-particle) / coherent (collective)

oscillations





interactions

$$\overline{F^{\perp}_{\text{inc}}} = 0 \qquad F^{\perp} = F^{\perp}_{\text{inc}} + F^{\perp}_{\text{coh}} \qquad \overline{F^{\perp}} = F^{\perp}_{\text{coh}}$$

impedance

incoherent (single-particle):

- The detuning wake / impedance
- Different for every particle
- Does not directly change $x_{beam}(t)$
- Can result in beam size growth or damping for collective oscillations

coherent (collective):

- The driving wake / impedance
- The same for the beam slice
- Directly changes $x_{beam}(t)$
- Can result in unstable collective oscillations

space charge

- Direct Space Charge: changes Ω_{inc}, does not Ω_{coh}
- Image Charges in the beam pipe: changes Ω_{icoh}, does not Ω_{inc}

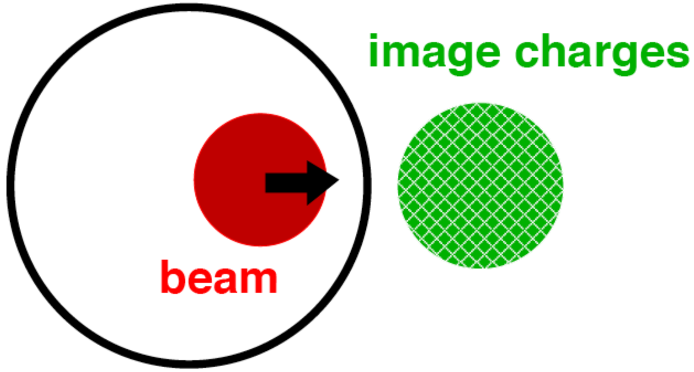
IMAGE CHARGES

A special impedance

- Shifts Ω_{icoh}, in the symmetric case does not change Ω_{inc};
- Corresponds to a purely imaginary impedance;
- Sometimes considered as a part of space charge (not correct);
- Can be implemented in the impedance (wake) module,
  or in the space charge module;
- The wake function is the delta-function;
- The trailing particles are the leading particles;
- Is it a driving wake or a detuning wake?

$$W^{\perp}(z) = \frac{Z_0 c L}{2\pi\gamma^2 m} \frac{\delta(z)}{h^2}$$

$$Z^{\perp} = -i \frac{Z_0 C}{2\pi\gamma^2\beta^2} \frac{2\xi_{x,y}}{h^2}$$



A.Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993
K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006

IMPEDANCES AND WAKE FUNCTIONS

Where do you get impedances?

- Literature: analytical formulas;
- EM Simulations: numerical data;
- Bench Measurements: numerical data;
- Beam measurements: usually parameters of an impedance model.

Usually it is easier to get the impedance, then you can calculate the wake function:

$$W_{\parallel}(\omega) = \frac{1}{2\pi}\int e^{i\omega z/v} Z_{\parallel}(\omega) d\omega$$

$$W_{\perp}(\omega) = -\frac{i}{2\pi}\int e^{i\omega z/v} Z_{\perp}(\omega) d\omega$$

Displaced beam between two infinite plates: [5] at $y=\pm h/2$. $\gamma \gg 1$, $[h/k^2, h-2y_0] \gg \delta_c$.	$Z_0^{\parallel} = \frac{1-\mathrm{sgn}(\omega)i}{\pi h}\sqrt{\frac{\lvert\omega\rvert\mu_r Z_0}{2c\sigma_c}} f_z$, $Z_1^{\perp} = \frac{\pi(\mathrm{sgn}(\omega)1-i)}{\sqrt{2\lvert\omega\rvert\sigma_c/(c\mu_r Z_0)}} f_\perp$ $f_z = 1+\frac{\pi y_0}{h}\tan\frac{\pi y_0}{h}$, $f_\perp = \frac{f_z}{h^3\cos^2(\pi y_0/h)}$, beam at $y=y_0$
Thin dielectric coating of thickness Δh.	$Z_0^{\parallel} = -\frac{i\omega Z_0(\epsilon_r\mu_r-1)\Delta h}{\pi c\epsilon_r h} f_z$, $Z_1^{\perp} = -\frac{i\pi Z_0(\epsilon_r\mu_r-1)\Delta h}{\epsilon_r} f_\perp$
Metallic coating on ceramic pipe: [6] compared with all metal pipe $Z_0^{\parallel}(\mathrm{met})$. $t_{m,c}$ = metal/ceramic thickness $\ll b$. $\gamma \gg 1$, $[(\epsilon_r-1)t_c^2, (1-\epsilon_r^{-1})bt_c] \ll \sigma_\ell^2$. Loss P/L is max. at $V=0.82$.	$Z_0^{\parallel} = Z_0^{\parallel}(\mathrm{met})\frac{A+\tanh(\nu t_m)}{1+A\tanh(\nu t_m)}$, $A=\left(1-\frac{1}{\epsilon_r}\right)\nu t_c$, $\nu = \frac{1-\mathrm{sgn}(\omega)i}{\delta_c}$ $\frac{P}{L} = \frac{Z_0 I_b^2 t_c(\epsilon_r-1)}{4\sqrt{\pi}b\sigma_\ell^2\epsilon_r}\left[V-\sqrt{\pi}V^2e^{V^2}\mathrm{erfc}(V)\right]$, $V=\frac{\epsilon_r\sigma_\ell}{(\epsilon_r-1)Z_0\sigma_c t_m t_c}$ Field penetration through pipe, $\frac{E_{z,\mathrm{out}}}{E_{z,\mathrm{in}}} = \frac{1}{\sqrt{1+4(1-1/\epsilon_r)t_m t_c/\delta_c^2}}$, becomes significant when $t_m \lesssim t_{\mathrm{crit}} = \delta_c^2/t_c$. P/L is at maximum at t_{crit}.

Elliptical hole: major and minor radii are a and d. $K(m)$ and $E(m)$ are complete elliptical functions of the first and second kind, with $m=1-m_1$ and $m_1=(d/a)^2$. For long ellipse perpendicular to beam, major axis $a \ll b$, beam pipe radius, because the curvature of the beam pipe has been neglected here [29].	$\alpha_e+\alpha_m = \begin{cases} \frac{\pi a^3 m_1^2[K(m)-E(m))]}{3E(m)[E(m)-m_1K(m)]} \\ \frac{\pi a^3[E(m)-m_1K(m)]}{3[K(m)-E(m)]} \end{cases} \overset{m\to 1}{\underset{\text{long ellipse}}{\Longrightarrow}} \begin{cases} \frac{\pi d^4[\ln(4a/d)-1]}{3a} & \parallel \text{ beam } d \ll b \\ \frac{\pi a^3}{3[\ln(4a/d)-1]} & \perp \text{ beam } a \ll b \end{cases}$ $\alpha_e+\alpha_m \underset{m\to 0}{\overset{\text{circular}}{\longrightarrow}} \frac{2a^3}{3}$ circular hole $a=d\ll b$ Above are for $t \ll a$. When $t \geq a$, $\times 0.56$ when hole is circular and $\times 0.59$ when hole is long-elliptic. For higher frequency correction, add to $\alpha_e+\alpha_m$ the extra term, $+\frac{2\pi a^3}{3}\left[\frac{11k^2a^2}{30}\right]$ circular, $\begin{cases} -\frac{\pi a d^2}{3}\left[\frac{k^2a^2}{5}\right] & \parallel \text{ beam long ellipse} \\ +\frac{2\pi a^3}{3}\left[\frac{2k^2a^2}{5[\ln(4a/d)-1]}\right] & \perp \text{ beam long ellipse} \end{cases}$
Rectangular slot: length L, width w.	$\alpha_e+\alpha_m = w^3(0.1814-0.0344w/L)$ $t \ll a$, $\times 0.59$ when $t \geq a$
Rounded-end slot: length L, width w.	$\alpha_e+\alpha_m = w^3(0.1334-0.0500w/L)$ $t \ll a$, $\times 0.59$ when $t \geq a$

Handbook of Accelerator Physics and Engineering, 2nd edition, (2013)

LONGITUDINAL IMPEDANCES

The longitudinal equation of motion

$$\phi'' + \frac{\omega_{s0}^2}{V_0}(V_{\rm rf}(\phi) - V_{\rm sc}(\phi)) = \kappa_{\|} \sum_n \int \lambda(z) W_{\|}(z + nC) dz$$

Integration in the particle tracking

$$\frac{dz}{dt} = -\frac{\beta c \eta}{p_0}\Delta p$$
$$\frac{dp}{dt} = \frac{q}{C}(V_{\rm rf} - V_{\rm sc} + V_{Z_{\|}})$$

We need to apply the related kicks.
Reality: $\boldsymbol{V_{rf}}$ is from a cavity, $\boldsymbol{V_Z}$ is from a lumped impedance, $\boldsymbol{V_{sc}}$ is distributed

Synchrotron tune $Q_s=\omega_{s0}/\omega_0$ is typically $10^{-4}-10^{-2}$,
so one kick per turn is enough.

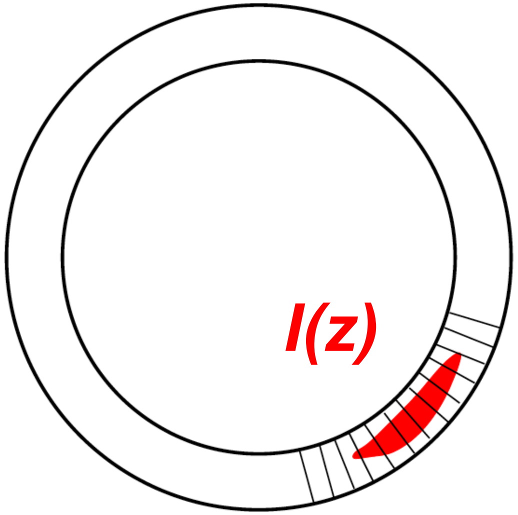
LONGITUDINAL IMPEDANCES

Two ways to implement the kick due to impedance:

- Wake Function (Space domain)
- **Impedance (Frequency domain)**

$$V_{Z_{\parallel}}^{(\omega)} = Z_{\parallel}(\omega)\ I^{(\omega)}$$

$$V_{Z_{\parallel}} = \mathrm{FFT}^{-1}\left\{\mathrm{FFT}(I_{\mathrm{beam}}) \times Z_{\parallel n}(\omega)\right\}$$



The bunch $I^{(\omega)}$=FFT*[I(z)]* is every turn different.

$V_{rf}(z)$ is does not change, $V_{Z}(z)$ is every turn different

Implementation of : ORBIT code, J. Holmes et. al., EPAC 2002.
ESME code, J. A. MacLachlan, Fermilab, FN-446 (1987)

Side remark: Space charge self-consistent and correct in the impedance implementation!
(**not the case** transversally)

$$\frac{Z_{\parallel \mathrm{sc}}}{n} = i\frac{Z_0 g_0}{\gamma^2 \beta}$$

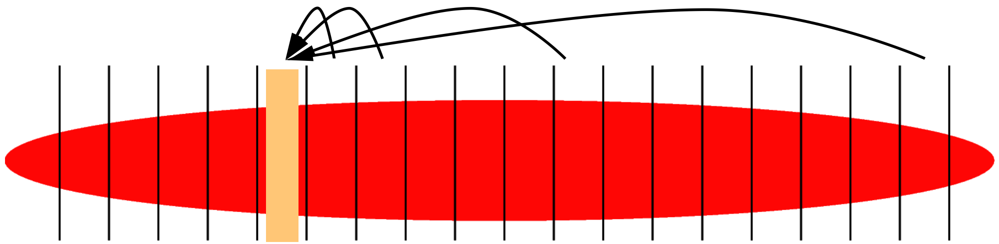
LONGITUDINAL IMPEDANCES

Two ways to implement the kick due to impedance:

- **Wake Function (Space domain)**
- Impedance (Frequency domain)

Apply the wake function per definition

$$\int F_{\|} ds \;=\; \Delta\mathcal{E}_2 = -q_1 q_2 W_{\|}(z)$$



- not the usual implementation;
- mostly, the impedances are available;
- especially the non-ultrarelativistic ***β<1*** case is better using impedances;
- space charge is implemented with impedances.

TRANSVERSE IMPEDANCES

$$\frac{d^2x}{dt^2} + \omega_\beta^2 x = \frac{F_\perp}{m\gamma}$$

- Each particle has its own incoherent tune,

$$Q_{\text{inc}} = \frac{\omega_\beta}{\omega_0} = Q_0 + \Delta Q_{\text{inc}}$$

ΔQ_{inc} depends on particle amplitude, ***z***-position.
ΔQ_{inc} is given by space-charge, detuning wakes

- The beam coherent spectrum can have a lot of components, mostly around

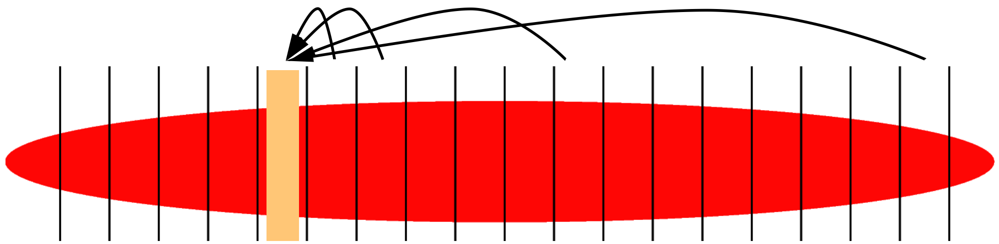
$$Q_{\text{coh}} = \left[n \pm (Q_0 + \Delta Q_{\text{coh}}) \right]$$

ΔQ_{coh} depends on the collective mode excited.
ΔQ_{coh} is given by image charges, driving wakes.
ΔQ_{coh} has an imaginary part (the growth rate).

IMPLEMENTATION

Two ways to implement the kick due to impedance:

- **Wake Function (Space domain)**
- Impedance (Frequency domain)



$$\Delta x' = \frac{2\pi\kappa}{R}\int_{-\tau_b}^{\tau_0}\frac{\lambda(t,\tau)}{\lambda_0}\overline{x}(t,\tau)W(\tau_0-\tau)d\tau =$$

$$= \frac{2\pi q_{\rm ion}\kappa}{\lambda_0 R^2}\sum_{i>j}\sum_n N_i\overline{x}_i W(z_j - z_i - nC)$$

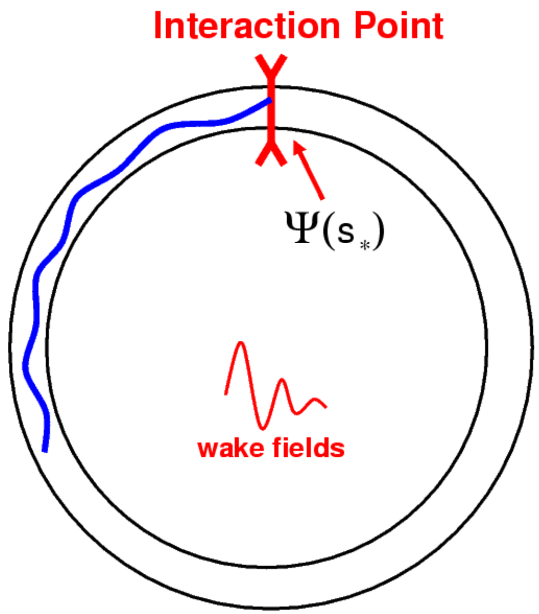
$$\kappa = \frac{I_0 q_{\rm ion}}{2\pi\gamma m\beta c\omega_0^2}$$

- Usual implementation
- This is the driving wake (x_1)
- One kick for the slice
- Similar for the detuning wake: individual particle kicks

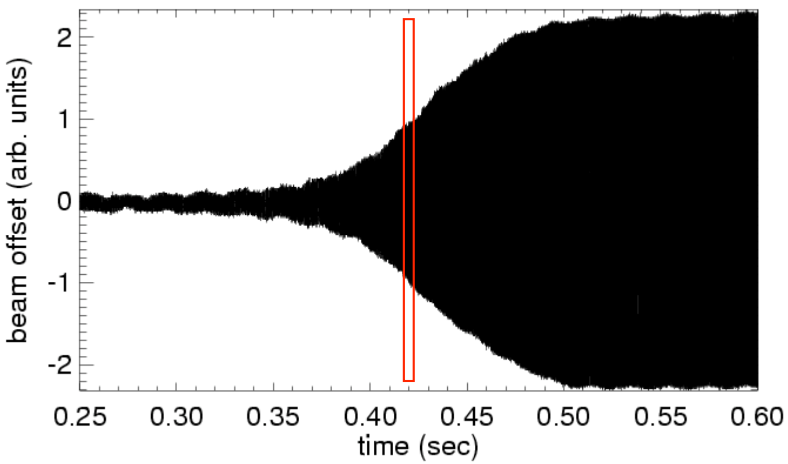
BEAM IN TIME / BEAM IN SPACE

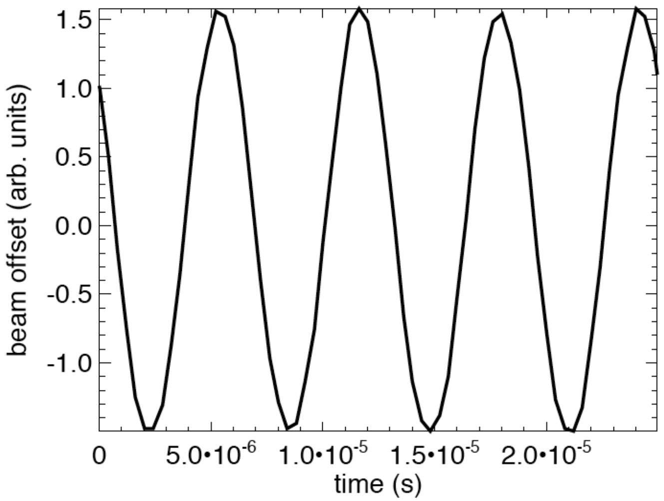
One aspect to keep in mind:

- At the interaction point we observe $\boldsymbol{x_{beam}(t)}$:
  Fixed-Location Signal
- Often we operate with $\boldsymbol{x_{beam}(z)}$ at a given time:
  Snapshot Signal



Example: Instability in SIS18. n=4 is expected.
On turn is T_0 = 4.7 μs
The mode rotation corresponds to Δt = 25 μs
n = 4 !





IMPLEMENTATION

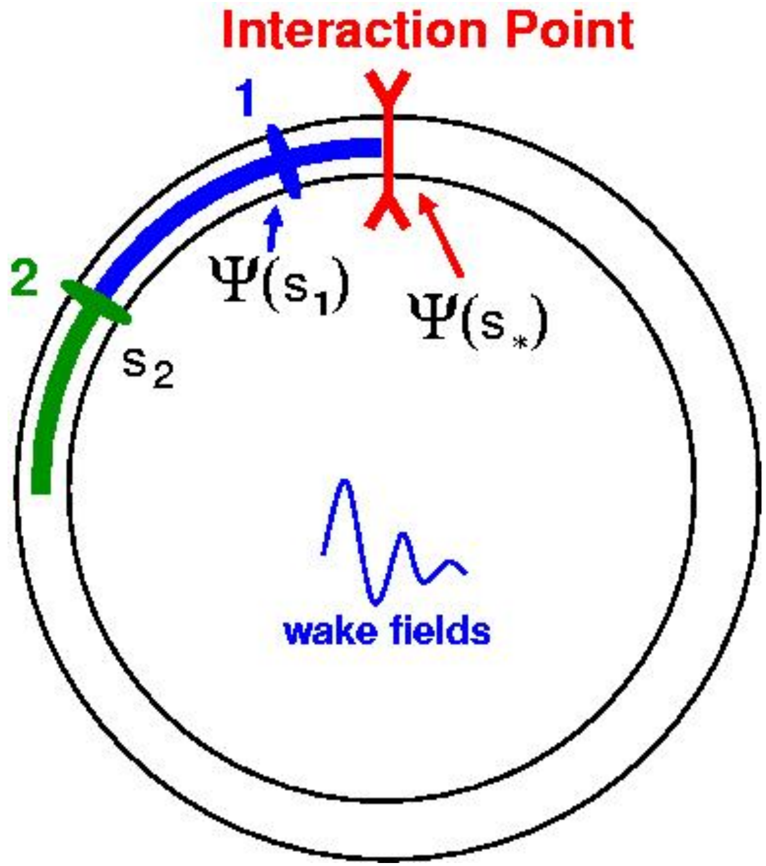
Resistive Wall (thick-wall) wake field

$$W_{\mathrm{RW}}(z) = -\frac{cL_{\mathrm{rw}}}{b^3}\left(\frac{\beta}{\pi}\right)^{3/2}\sqrt{\frac{Z_0}{|z|\sigma}}$$

Essential for long bunches
two approaches:
with / **without** phase advance:

$$\Delta p_{12} = \Psi(s_1)\, W_{\mathrm{RW}}(z)$$

$$\Delta p_{12} = \Psi(s_*)\, W_{\mathrm{RW}}(z)$$

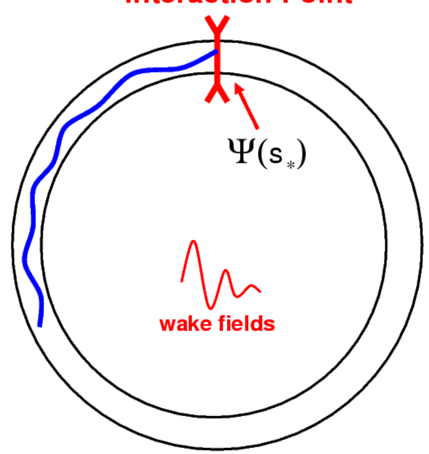


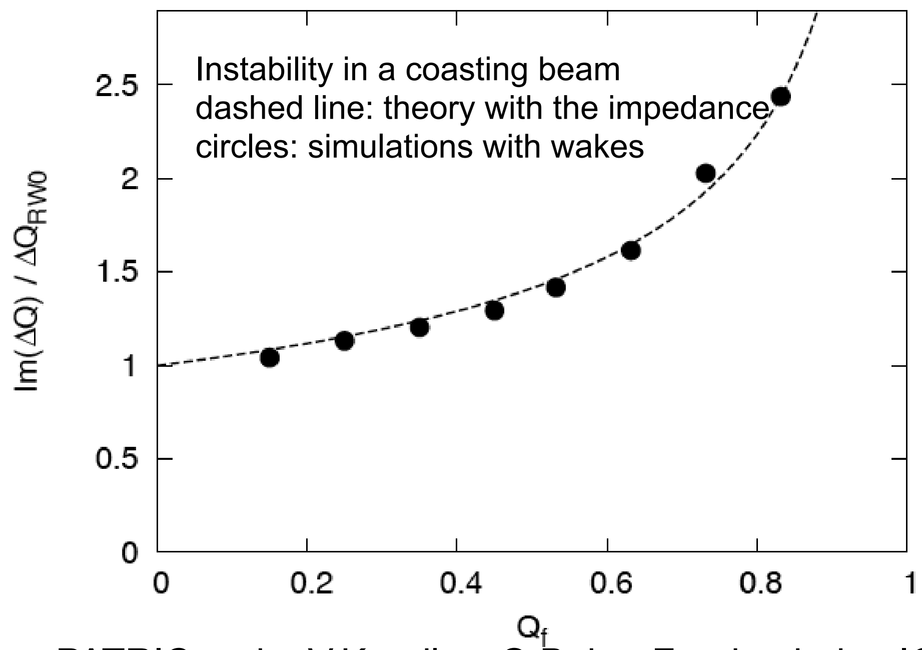
Also important: include previous turns.

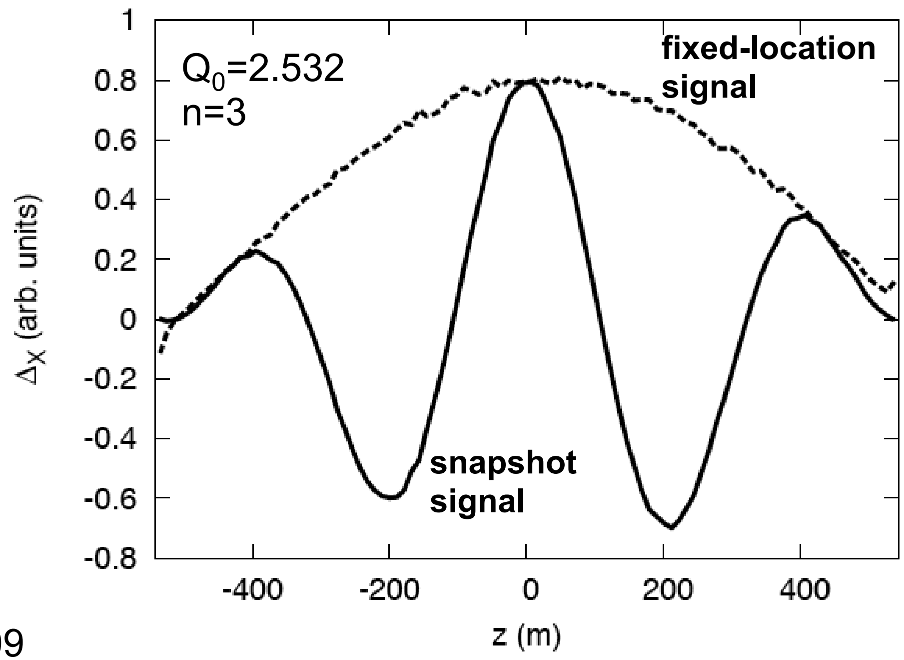
SIMULATIONS WITH IMPEDANCES

After the Implementation,
The most important is: **Verification**

If possible, with a theory







PATRIC code, V.Kornilov, O.Boine-Frankenheim, ICAP2009

IMPLEMENTATION

Two ways to implement the kick due to impedance:

- Wake Function (Space domain)
- **Impedance (Frequency domain)**

The main idea:

$$\Delta x' = \mathrm{Re}\left\{ i\Psi^{(\omega)} Z_\perp^{(\omega)} \right\}$$

where ***Ψ=Δx I(z)*** should be decomposed into cos-type (a_n) and sin-type (b_n).

The kick is the convolution:

$$2\sum \Psi^{(\omega)} Z_\perp^{(\omega)} = \sum_n Z_\perp([n+Q_0]\omega_0)(a_n - ib_n)e^{i(n+Q_0)\omega_0 t} + \\ + \sum_n Z_\perp([n-Q_0]\omega_0)(a_n + ib_n)e^{i(n-Q_0)\omega_0 t}$$

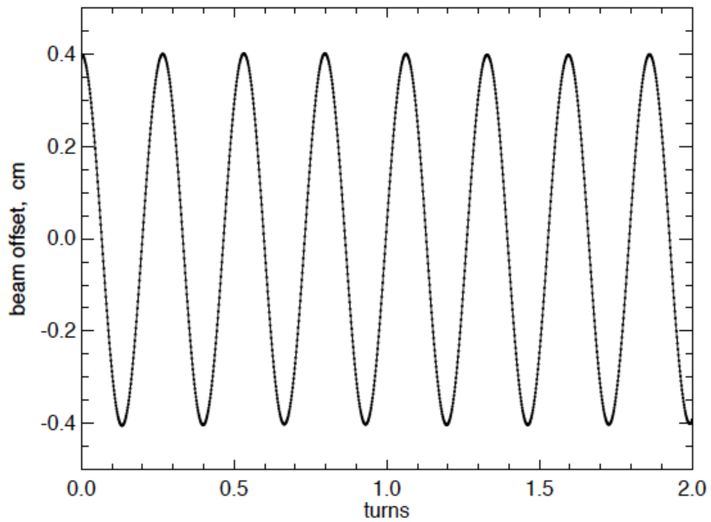
ORBIT code, V.Danilov, J.Galambos, J.Holmes, PAC2001
PATRIC code, O.Boine-Frankenheim, V.Kornilov, ICAP2006

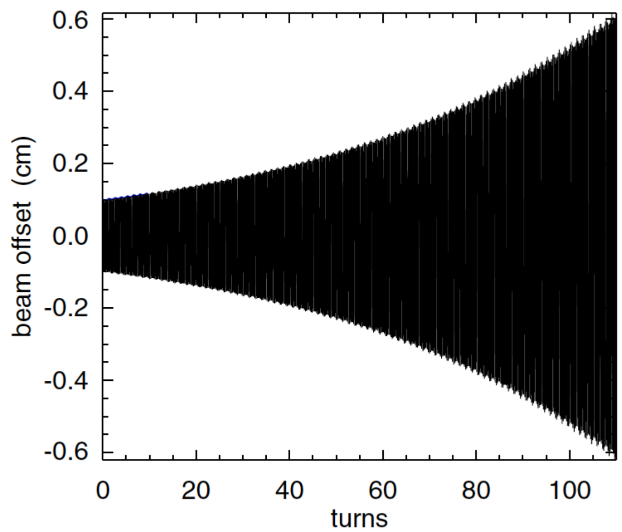
SIMULATIONS

In the case of a coasting beam,
the problem becomes 2D and easier:

$$\Delta x'(t) = \frac{Nq^2}{p_0 C}\left[-\overline{x}(t)\ \mathrm{Im}(Z^{\perp}) + \frac{\mathrm{d}\overline{x}}{\mathrm{d}t}(t)\frac{\mathrm{Re}(Z^{\perp})}{\mathrm{Re}(\omega)}\right]$$

The impedance is probed at one frequency.
Recall the same results with the Wake Function.





V.Kornilov, O.Boine-Frankenheim, prstab 11, 014201 (2008)
PATRIC code, O.Boine-Frankenheim, V.Kornilov, ICAP2006

SUMMARY

- **Understanding** of physics first

- Relevant frequencies / length scales
- The wake: driving, detuning?
- The forces / tune shifts: coherent or incoherent?
- Do you need non-ultrarelativistic wakes /impedances?
- ***m***-order of your wakes
- Wakes / Impedances: Space / Frequency ?
- Separate from the space charge implementation
- Causality
- Previous turns / multibunch
- Beam in the Snapshot or in the Fixed-Location?
- Try to make your impedance part simpler (e.g. 2D, …)

- **Verification,** ideally with an analytical theory