Stylized features of nuclear momentum distributions and the connection to observables

Camille Colle, Maarten Vanhalst, Wim Cosyn, Jan Ryckebusch

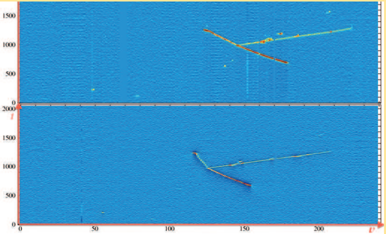
Department of Physics and Astronomy, Ghent University

Data Mining, August 2014

UNIVERSITEIT GENT

FACULTEIT WETENSCHAPPEN

Research goals



"hammer events" in $(\nu_\mu, \mu^- pp)$ (arXiv:1405.4261)

- develop an approximate flexible method for computing the nuclear momentum distributions for $A(N, Z)$
- use this method to study the mass and isospin dependence of SRC
- provide a unified framework to establish connections with measurable quantities that are sensitive to SRC
  1. inclusive $A(e, e')$ at $x_B \gtrsim 1.5$
  2. magnitude of the EMC effect
  3. two-nucleon knockout: $A(e, e'pN)$, $A(\nu_\mu, \mu^- pp)$
- learn about SRC physics in a unified framework

Correlated operators I

- shift complexity from wave functions to operators

$$| \Psi \rangle = \frac{1}{\sqrt{\mathcal{N}}} \widehat{\mathcal{G}} | \Phi \rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle \Phi | \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} | \Phi \rangle$$

  $| \Phi \rangle$ is an IPM single Slater determinant
- nuclear correlation operator $\widehat{\mathcal{G}}$

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 + \hat{l}(i,j) \right] \right) ,$$

- central (Jastrow), tensor, spin-isospin are the major source of correlated strength

$$\hat{l}(i,j) = -g_c(r_{ij}) + f_{\sigma\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j + f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j \ .$$

Correlated operators II

- expectation values between correlated states can be turned into expectation values between uncorrelated states

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

- conservation law of misery:

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger} \, \widehat{\Omega} \, \widehat{\mathcal{G}} = \left(\sum_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \right)^{\dagger} \widehat{\Omega} \left(\sum_{k<l=1}^{A} \left[1 - \hat{l}(k,l) \right] \right)$$

$\widehat{\Omega}^{\text{eff}}$ is an A-body operator

- truncation is required:
low-order correlation operator expansion (LCA)
- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

The LCA method explained

- the LCA effective operator corresponding with a one-body operator $\sum_{i=1}^{A} \widehat{\Omega}^{[1]}(i)$

$$\widehat{\Omega}^{\text{eff}} \approx \widehat{\Omega}^{\text{LCA}} = \sum_{i=1}^{A} \widehat{\Omega}^{[1]}(i) + \sum_{i<j=1}^{A} \left\{ \widehat{\Omega}^{[1],\text{l}}(i,j) + \left[\widehat{\Omega}^{[1],\text{l}}(i,j)\right]^{\dagger} + \widehat{\Omega}^{[1],\text{q}}(i,j) \right\}$$

- two-types of SRC corrections (two-body)
  1. linear in the correlation operator:

$$\widehat{\Omega}^{[1],\text{l}}(i,j) = \left[\Omega^{[1]}(i) + \Omega^{[1]}(j)\right] \hat{l}(i,j)$$

  2. quadratic in the correlation operator:

$$\widehat{\Omega}^{[1],\text{q}}(i,j) = \hat{l}^{\dagger}(i,j)\left[\widehat{\Omega}^{[1]}(i) + \widehat{\Omega}^{[1]}(j)\right]\hat{l}(i,j).$$

Norm $\mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi \rangle$

- LCA expansion of the norm $\mathcal{N}$

$$\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} {}_{\text{nas}}\langle \alpha\beta \mid \hat{l}^{\dagger}(1,2) + \hat{l}^{\dagger}(1,2)\hat{l}(1,2) + \hat{l}(1,2) \mid \alpha\beta \rangle_{\text{nas}}.$$

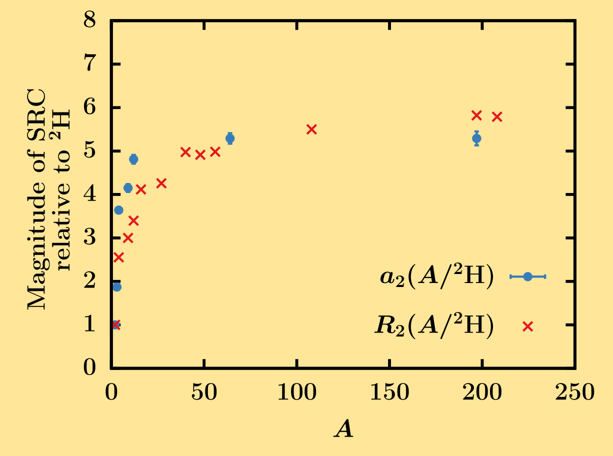
  1. $\mid \alpha\beta\rangle_{\text{nas}}$: normalized and anti-symmetrized 2N IPM-state
  2. $\sum_{\alpha<\beta}$ extends over all IPM states $\alpha \equiv n_\alpha l_\alpha j_\alpha m_{j_\alpha} t_\alpha$,

- $(\mathcal{N} - 1)$ is a measure for aggregated effect of SRC on IPM ground state
- aggregated quantitative effect of SRC in A relative to ^{2}H

$$R_2(A/^2\text{H}) = \frac{\mathcal{N}(A) - 1}{\mathcal{N}(^2\text{H}) - 1} ,$$

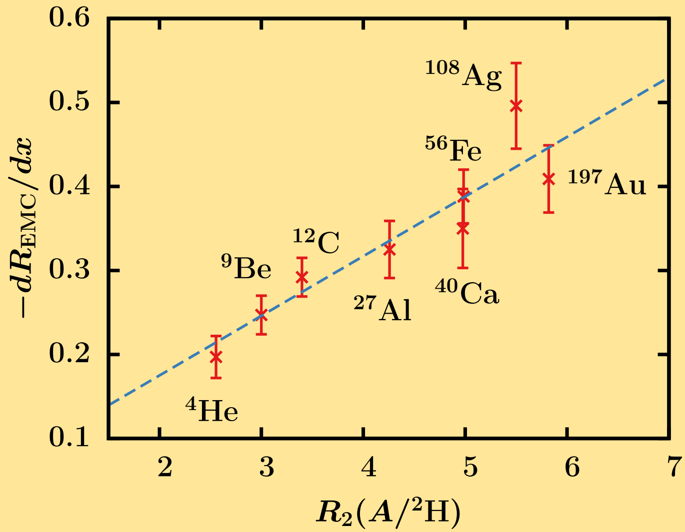
- input to the calculations:
  1. HO IPM states with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$
  2. A-independent universal correlation functions

$a_2(A/^2\text{H})$ from $A(e,e')$ at $x_B \gtrsim 1.5$ and $R_2(A/^2\text{H})$



1. $A \lesssim 40$: strong mass dependence in SRC effect
2. $A > 40$: soft mass dependence
3. SRC effect saturates for A large *(for large A aggregated SRC effect per nucleon is about 5× larger than in ^2H)*

Magnitude of EMC effect versus $R_2(A/^2\text{H})$



LCA can predict magnitude of EMC effect for any $A(N,Z) \geq 4$

Single-nucleon momentum distribution $n^{[1]}(p)$

- class of single-point correlation functions
- definition of $n^{[1]}(p)$

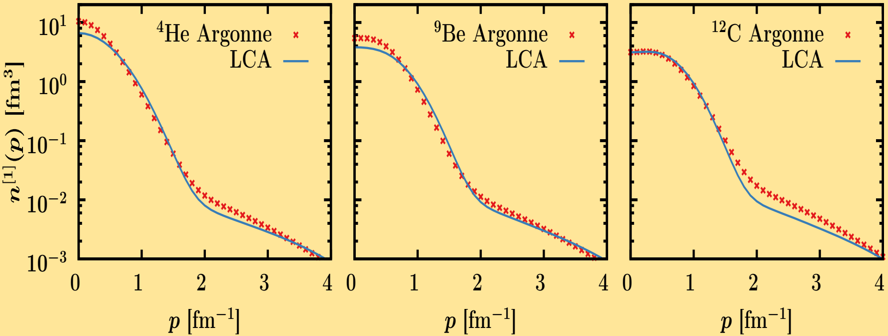
$$n^{[1]}(p) = \int \frac{d^2\Omega_p}{(2\pi)^3} \int d^3\vec{r}_1 \; d^3\vec{r}_1{}' \; d^{3(A-1)}\{\vec{r}_{2-A}\} e^{-i\vec{p}\cdot(\vec{r}_1{}'-\vec{r}_1)} \times \Psi^*(\vec{r}_1, \vec{r}_{2-A}) \Psi(\vec{r}_1{}', \vec{r}_{2-A}).$$

- corresponding single-nucleon operator $\hat{n}_p$

$$\hat{n}_p = \frac{1}{A} \sum_{i=1}^{A} \int \frac{d^2\Omega_p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{r}_i{}'-\vec{r}_i)} = \sum_{i=1}^{A} \hat{n}_p^{[1]}(i).$$

- effective correlated operator $\hat{n}_p^{\mathrm{LCA}}$ *(SRC-induced corrections to IPM $\hat{n}_p$ are of 2-body type)*
- normalization property $\int dp \, p^2 n^{[1]}(p) = 1$ can be preserved by evaluating $\mathcal{N}$ in LCA

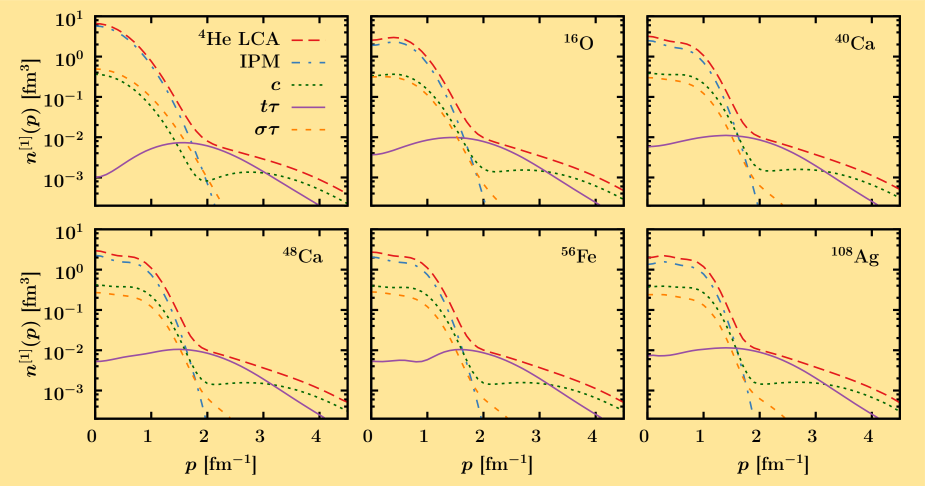
$n^{[1]}(p)$ for light nuclei



Argonne (QMC): PRC89 (2014) 024305 LCA: arXiv:1405.3814

1. $p \lesssim p_F = 1.25$ fm^{-1}: $n^{[1]}(p)$ is Gaussian (IPM PART)
2. $p \gtrsim p_F$: $n^{[1]}(p)$ has an "exponential" fat tail (CORRELATED PART)
3. fat tail in QMC and LCA are in reasonable agreement

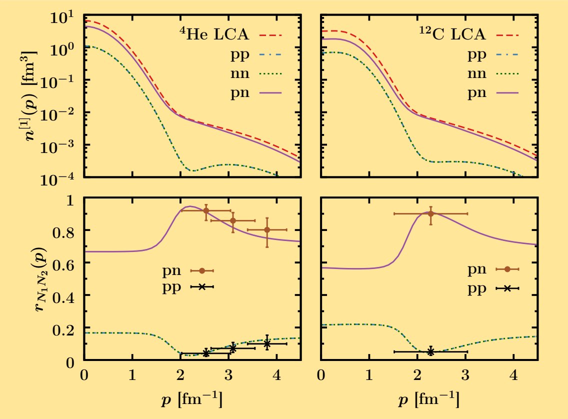
Source of correlated strength in $n^{[1]}(p)$



1. $1.5 \lesssim p \lesssim 3$ fm^{-1} is dominated by tensor correlations
2. central correlations substantial at $p \gtrsim 3.5$ fm^{-1}

Isospin dependence of correlations: pp, nn and pn

$$n^{[1]}(p) = n^{[1]}_{pp}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{pn}(p) \qquad r_{N_1 N_2}(p) \equiv n^{[1]}_{N_1 N_2}(p)/n^{[1]}(p)$$

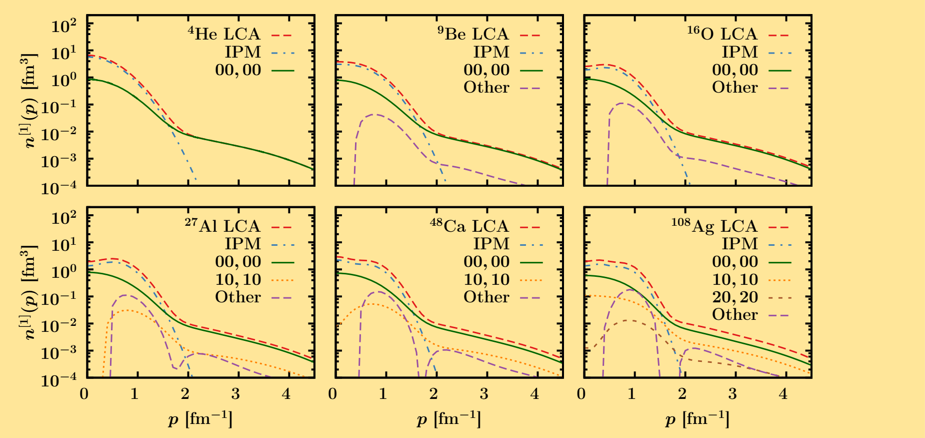


the “pn” dominance is momentum dependent!

- $r_{N_1 N_2}(p)$: relative contribution of $N_1 N_2$ pairs to $n^{[1]}(p)$ at p
- in a naive IPM:
  $r_{pp} = \frac{Z(Z-1)}{A(A-1)}$,
  $r_{nn} = \frac{N(N-1)}{A(A-1)}$,
  $r_{pn} = \frac{2NZ}{A(A-1)}$.
- data extracted from ^{4}He$(e, e'pp)/^4$He$(e, e'pn)$ (PRL 113, 022501) and $\frac{^{12}\text{C}(p,ppn)}{^{12}\text{C}(p,pp)}$ (Science 320, 1476) assuming that $r_{pp} \approx r_{nn}$

Quantum numbers of IPM pairs prone to correlations

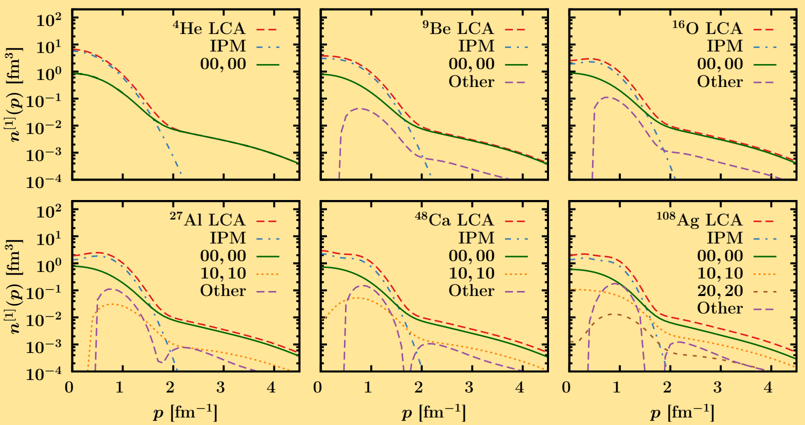
$n^{[1],\mathrm{corr}}$ stems from correlations acting on IPM pairs. What are relative quantum numbers (nl) of those IPM pairs?



$$\sum_{nl} \sum_{n'l'} n^{[1],\mathrm{corr}}_{nl,n'l'}(p) = n^{[1],\mathrm{corr}}(p)$$

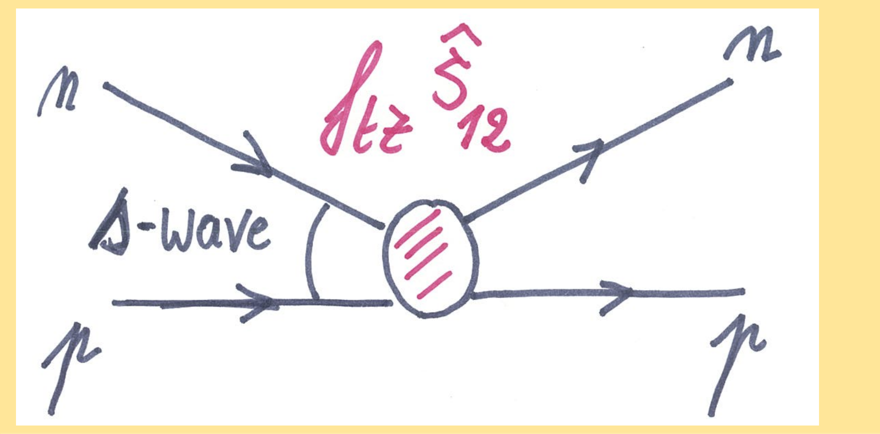
Quantum numbers of IPM pairs prone to correlations

$n^{[1],\text{corr}}$ stems from correlations acting on IPM pairs. What are relative quantum numbers (nl) of those IPM pairs?



major source of SRC: correlations acting on ($n = 0$ $l = 0$) IPM pairs

- physical picture from LCA: for $1.5 \lesssim p \lesssim 3$ fm^{-1} the correlations are due to tensor-induced short-distance scattering between IPM pn pairs in a relative *s*-state



Tan's contact: Ann. of Phys. 322 (2008) 2971-2990

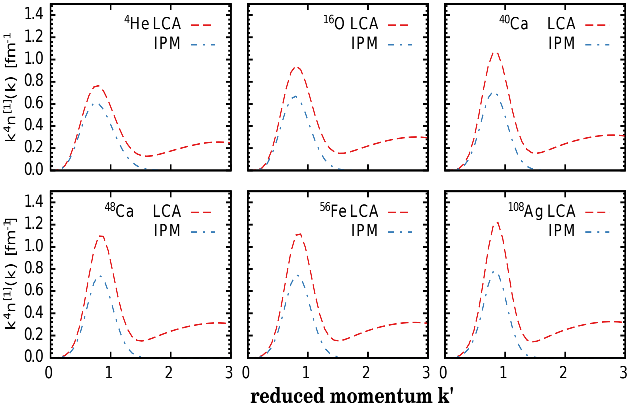
- physical picture from LCA: for $1.5 \lesssim p \lesssim 3$ fm^{-1} the correlations are due to tensor-induced short-distance scattering between IPM pn pairs in a relative *s*-state

- in tensor-dominated momentum range: nuclear Hamiltonian can be captured by the stylized Hamiltonian

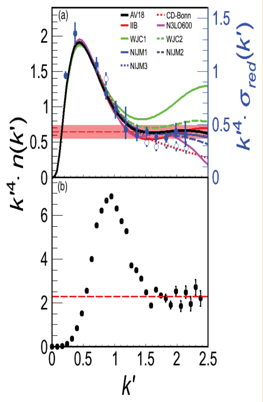
$$\begin{aligned} \widehat{\mathcal{H}} &\approx \sum_{\tau=p,n} \int d^3\vec{r}\psi^\dagger_\tau(\vec{r}) \left[-\frac{\hbar^2}{2m_N} \nabla^2_{\vec{r}} + U_\tau(\vec{r}) \right] \psi_\tau(\vec{r}) \\ &+ \int d^3\vec{r} d^3\vec{R} \psi^\dagger_p \left(\vec{R} + \frac{\vec{r}}{2} \right) \psi^\dagger_n \left(\vec{R} - \frac{\vec{r}}{2} \right) \psi_n \left(\vec{R} \right) \psi_p \left(\vec{R} \right) \lambda_{t\tau} \left(\vec{r} \right) \end{aligned}$$

- Physics of a two-component and strongly correlated Fermi gas subject to an *s*-wave contact interaction is described by Tan
- landmark of a contact interaction: $n^{[1]}(p) \sim Cp^{-4}$

p^4 scaling of the $n^{[1]}(p)$



arXiv:1407.8175



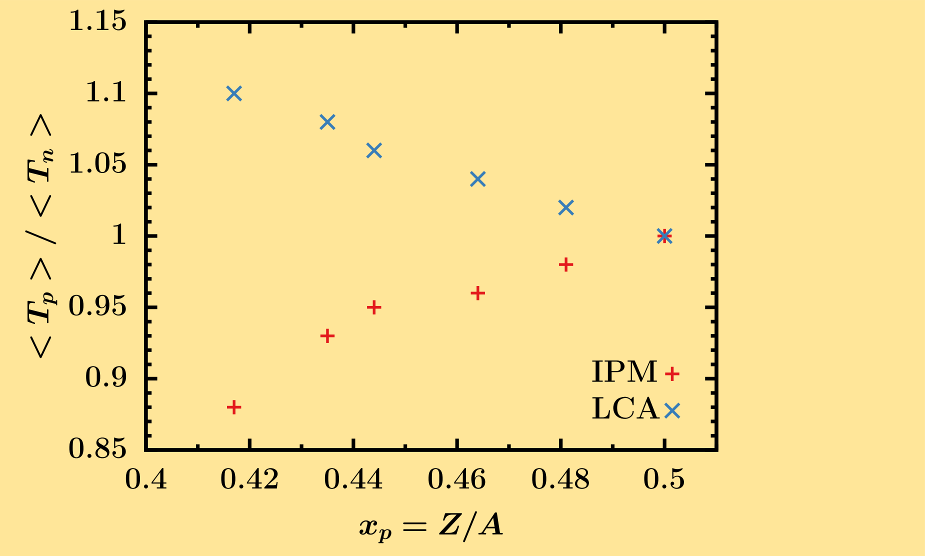
- momentum scale: $k' \equiv \frac{p}{p_F}$
- IPM is approximately Gaussian: stochastic collisions
- fat tail is the landmark of strong correlations (some approximate $\frac{1}{k^4}$ scaling is observed)

Average kinetic energy per nucleon

A	x_p	$\langle T_N \rangle$ (MeV)						$\langle T_p \rangle$ / $\langle T_n \rangle$	
		IPM (p)	IPM (n)	LCA (p)	LCA(n)	Perugia	UCOM	IPM	LCA
^{2}H	0.500	14.95	14.93	20.95	20.91			1.00	1.00
^{4}He	0.500	13.80	13.78	25.28	25.23		19.63	1.00	1.00
^{9}Be	0.444	15.81	16.58	28.91	27.33			0.95	1.06
^{12}C	0.500	16.08	16.06	28.96	28.92	32.4	22.38	1.00	1.00
^{16}O	0.500	15.61	15.59	29.48	29.43	30.9	23.81	1.00	1.00
^{27}Al	0.481	16.61	16.92	30.93	30.26		25.12	0.98	1.02
^{40}Ca	0.500	16.44	16.42	31.23	31.18	33.8	27.72	1.00	1.00
^{48}Ca	0.417	15.64	17.84	33.04	30.06		27.05	0.88	1.10
^{56}Fe	0.464	16.71	17.45	32.33	31.13	32.7		0.96	1.04
^{108}Ag	0.435	16.48	17.81	33.55	31.16			0.93	1.08

1. SRC increase $\langle T_N \rangle$!
2. minority component has largest $\langle T_N \rangle$

Predictions for $\langle T_p \rangle / \langle T_n \rangle$ ratio



Nuclear rms radii

A	IPM	LCA	UCOM	Expt
^{4}He	1.84	1.70	1.35	1.6755 ± 0.0028
^{9}Be	2.32	2.13		2.5190 ± 0.0120
^{12}C	2.46	2.23	2.36	2.4702 ± 0.0022
^{16}O	2.59	2.32	2.28	2.6991 ± 0.0052
^{27}Al	3.06	2.72	2.82	$3.0610 \pm 0{:}0031$
^{40}Ca	3.21	2.84	2.93	3.4776 ± 0.0019
^{48}Ca	3.47	3.05	3.20	3.4771 ± 0.0020
^{56}Fe	3.63	3.20		$3.7377 \pm 0{:}0016$
^{108}Ag	4.50	3.94		4.6538 ± 0.0025
^{197}Au	5.73	5.21		5.4371 ± 0.0038
^{208}Pb	5.83	5.28		5.5012 ± 0.0013

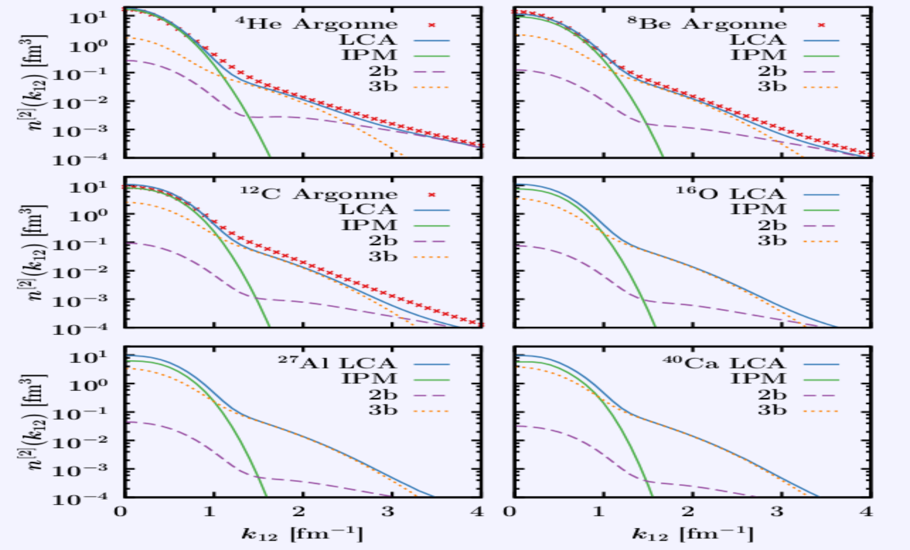
1. effect of SRC on rms radii is modest
2. we use global HO parameterization!

Two-nucleon momentum distribution(TNMD) $n^{[2]}\left(\vec{k}_{12},\vec{P}_{12}\right)$

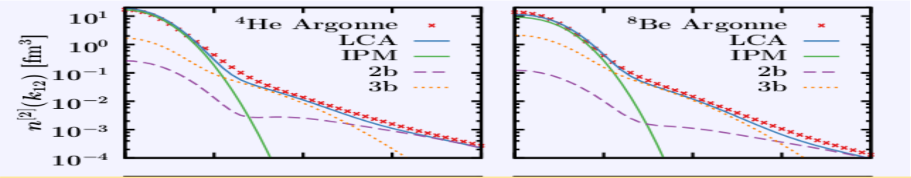
- belongs to the class of two-point correlation functions (two tagged nucleons)
- corresponding two-nucleon operator $\widehat{n}_{k_{12}P_{12}}$
- effective correlated operator $\widehat{n}^{LCA}_{k_{12}P_{12}}$ (SRC-induced corrections are two-body ("2b") and three-body ("3b") operators)
- relative TNMD: distribution of the relative momentum of the tagged pair

$$n^{[2]}\left(k_{12}\right)=\int d^3\vec{P}_{12}d^2\Omega_{k_{12}}n^{[2]}\left(\vec{k}_{12},\vec{P}_{12}\right)$$

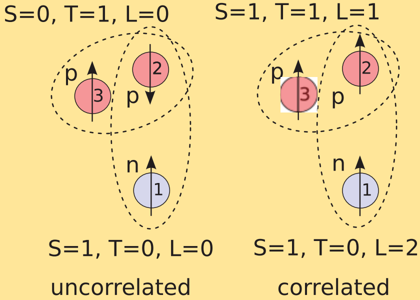
Relative TNMD: tail is dominated by “3b”



Relative TNMD: tail is dominated by “3b”

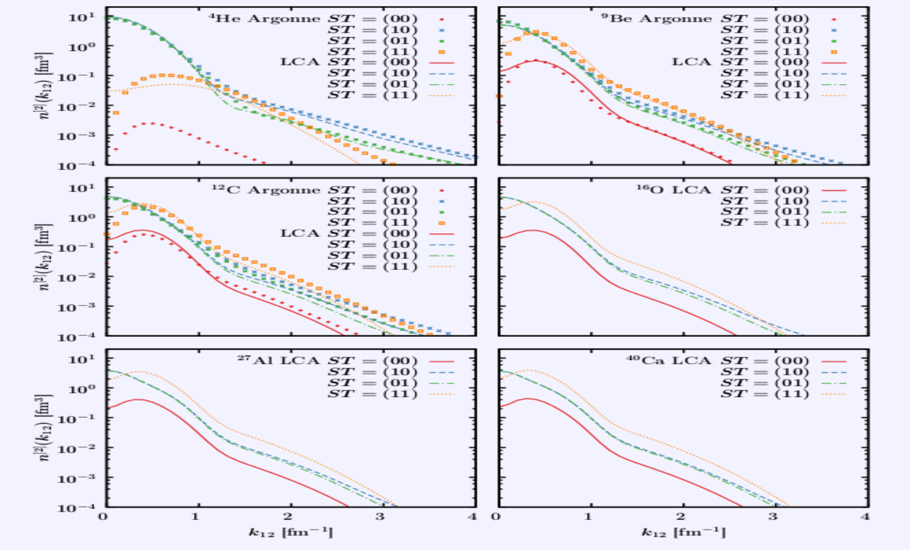


Correlations through the mediation of a third particle:

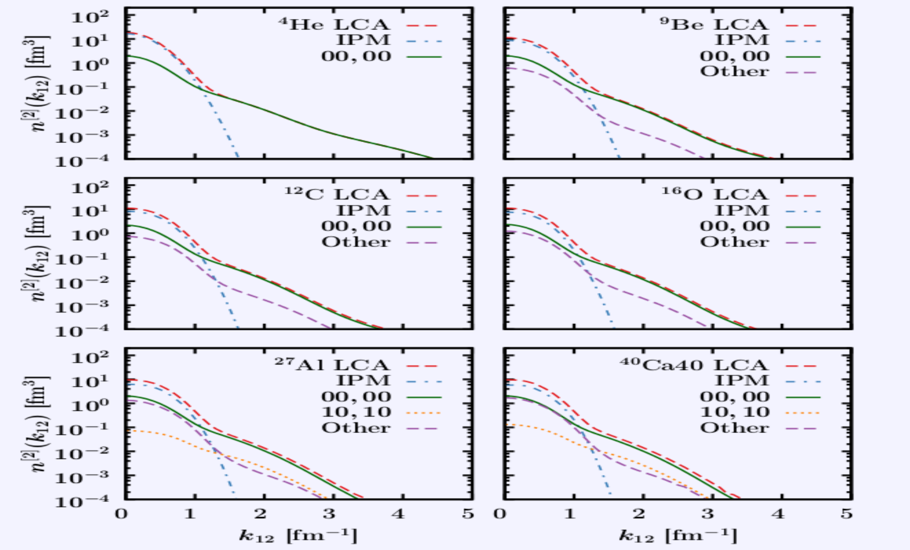


Feldmeier *et al.*, PRC 84 (2011), 054003

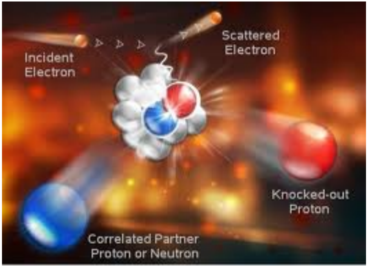
Relative TNMD: quantum numbers of tagged pairs $\neq$ quantum numbers of correlated pair

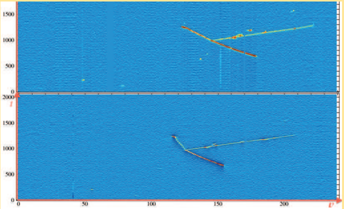


Correlated part of relative TNMD: dominated by *s*-wave scattering!



Exclusive two-nucleon knockout $A(e, e'NN)$





"hammer events" in $(\nu_\mu, \mu^- pp)$
(arXiv:1405.4261)

- The (virtual) photon-nucleon interaction is a one-body operator
- Two-nucleon knockout $A(e, e'NN)$ is the hallmark of SRC (one hits a nucleon and its correlated partner)
- Four particles in the final states!
- The ninefold $A(e, e'NN)$ differential cross sections is small and difficult to measure!

Exclusive $A(e, e'pp)$ reactions

The fact that SRC-prone proton-proton pairs are mostly in a state with relative orbital momentum $l_{12} = 0$ has important consequences for the EXCLUSIVE $A(e, e'pp)$ cross sections (PLB 383,1 ('96))!!

1. **The $A(e, e'pp)$ cross sections factorizes according to**

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e'pp) = E_1 p_1 E_2 p_2 f_{rec}^{-1} \times \sigma_{eN_1 N_2}(k_+, k_-, q) F_{h_1,h_2}(P)$$

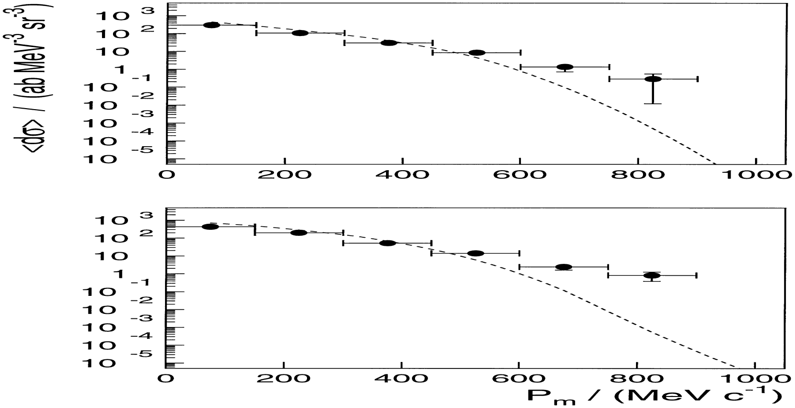
$F_{h_1,h_2}(P)$: probability to find a diproton with c.m. momentum P and relative orbital momentum $l_{12} = 0$!

2. **The A dependence of the $A(e, e'pp)$ cross sections is soft** (much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e, e'pp)}{^{12}\mathrm{C}(e, e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e, e'p)}{T_{^{12}\mathrm{C}}(e, e'p)}\right)^{1-2}$$

Factorization of the $A(e, e'pp)$ cross sections

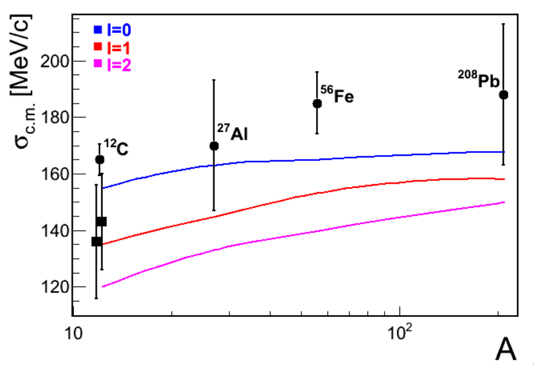
$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)



For $P \lesssim 0.5$ GeV c.m. motion of correlated pairs in ^{12}C is mean-field like $\left(\exp \frac{-P^2}{2\sigma^2_{c.m}}\right)$! Data prove factorization in terms of $F(P)$ (relative $l_{12} = 0$!).

C.m. motion of correlated pp pairs

DATA IS PRELIMINARY! (COURTESY OF O. HEN AND E. PIASETZKY)



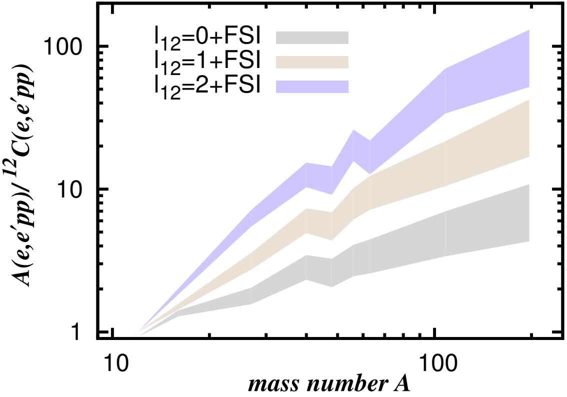
- analysis of exclusive $A(e, e'pp)$ for ^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb by Data Mining Collaboration at Jefferson Lab
- distribution of events against P is fairly Gaussian
- $\sigma_{c.m.}$: Gaussian widths from a fit to measured c.m. distributions
- theory lines: Gaussian fits to computed c.m. distributions for $I = 0, 1, 2$

Mass dependence of the $A(e,e'pp)$ cross sections

PREDICTION: A dependence of $A(e,e'pp)$ c.s. is soft

(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$

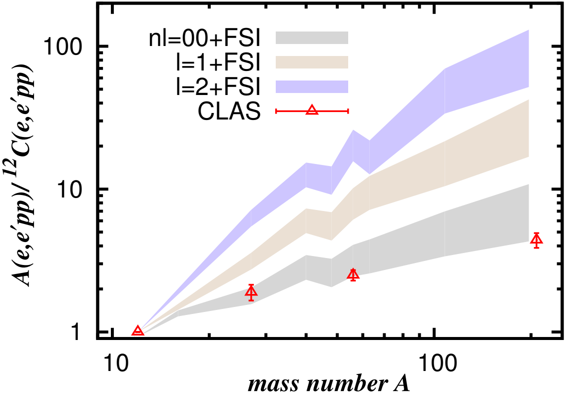


Mass dependence of the $A(e,e'pp)$ cross sections

PREDICTION: A dependence of $A(e,e'pp)$ c.s. is soft

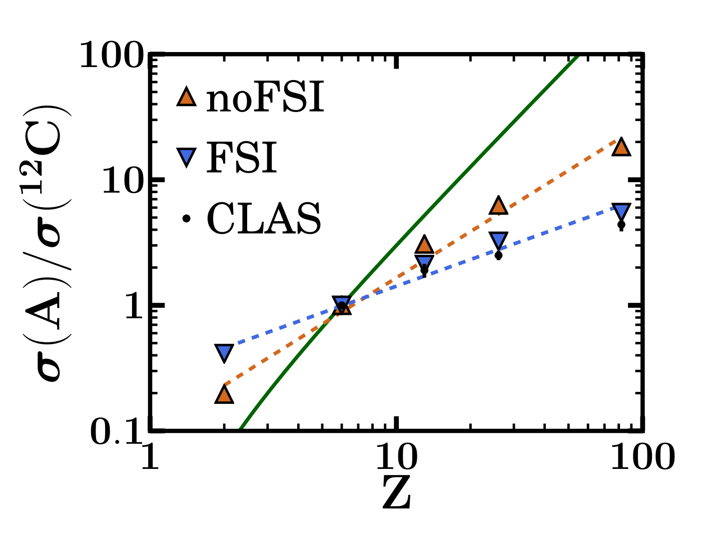
(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$



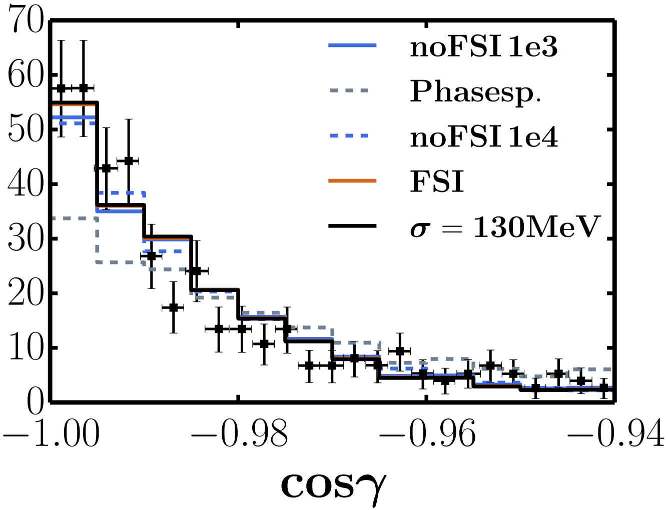
PRELIMINARY DATA (COURTESY OF O. HEN AND E. PIASETZKY) COMPATIBLE WITH ABSORPTION ON $l_{12}=0$ PAIRS!

Mass dependence of pp correlations



Effect of final-state interactions in the eikonal approximation!

Opening-angle distribution of ^{4}He$(e, e'pp)$



Selected publications

- M. Vanhalst, J. Ryckebusch, W. Cosyn
  Stylized features of single-nucleon momentum distributions
  arXiv:1405.3814
- C. Colle, W. Cosyn, J. Ryckebusch, M. Vanhalst
  Factorization of electroinduced two-nucleon knockout reactions
  arXiv:1311.1980 and Physical Review C **89** (2014), 024603.
- Maarten Vanhalst, Jan Ryckebusch, Wim Cosyn
  Quantifying short-range correlations in nuclei
  arXiv:1206.5151 and Physical Review C **86** (2012), 044619.
- Maarten Vanhalst, Wim Cosyn, Jan Ryckebusch
  Counting the amount of correlated pairs in a nucleus
  arXiv:1105.1038 and Physical Review C **84** (2011), 031302(R).
- Jan Ryckebusch
  Photoinduced two-proton knockout and ground-state correlations in nuclei
  Physics Letters **B383** (1996), 1.

http://inwpent5.ugent.be