Implementation of K-matrix formalism in the $D^0 \rightarrow K_S\pi^+\pi^-$ amplitude model

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Before The Physics…. Hi!

My name is Rudin from Dublin studying Physics (Undergraduate)
The big picture
Mixing and CPV

\[ D^0 \rightarrow \overline{D}^0 \rightarrow D^0 \rightarrow \overline{D}^0 \rightarrow D^0 \]  

Mixing

\( K^0 \) — strange quark
\( B^0 \) — bottom quark

\( B_s^0 \) mixing

CERN 2014
The big picture
Mixing and CPV

\[ D^0 \rightarrow \bar{D}^0 \rightarrow D^0 \rightarrow \bar{D}^0 \rightarrow D^0 \] Mixing

\[ K^0 \rightarrow \text{strange quark} \]
\[ B^0 \rightarrow \text{bottom quark} \]

\[ B_s^0 \text{ mixing} \]
The big picture
Mixing and CPV with Charm

- **2007** – First Evidence from *BaBar* and *Belle* for mixing in charmed neutral mesons

- **2012** – *LHCb* find > $5\sigma$ evidence for mixing in a single measurement
  - Mixing parameters very small
    \[ x = \frac{M_1 - M_2}{\Gamma}, \quad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma} \propto (10^{-2}) \]

- **No significant evidence for CPV in charm to date**
  - SM ‘predicts’ small CPV
    ⇔ Observation at current sensitivity would imply *New Physics*
$D^0 \rightarrow K_S \pi^+ \pi^-$

Allows measuring $x$ and $y$ parameters directly
$D^0 \rightarrow K_s \pi^+ \pi^-$

 Allows measuring $x$ and $y$ parameters directly

(modern notation)
\[ D^0 \rightarrow K_S \pi^+ \pi^- \]

 Allows measuring \( x \) and \( y \) parameters directly

- Protons collide in the VELO
- Very short life-time of charm quark
  \( D^0 \) travels \( \sim 2 \) mm, then decays
$D^0 \rightarrow K_s \pi^+ \pi^-$

*Allows measuring x and y parameters directly*

- Protons collide in the VELO

- Very short life-time of **charm quark**
  - $D^0$ travels $\sim 2$ mm, then decays

$D^0$ detected by TT tracker.
- bend through magnet.
- detected by T1, T2, T3.
\[ D^0 \rightarrow K_s \pi^+ \pi^- \]

*Allows measuring \( x \) and \( y \) parameters directly*

- Protons collide in the VELO
  
  \[
  P \quad \rightarrow \quad P
  \]

- Very short life-time of **charm quark**
  
  \( D^0 \) travels \( \sim 2 \) mm, then decays

- \( \pi^- \) detected by TT tracker.
  
  bend through magnet.
  
  detected by T1, T2, T3.

- \( K_s \)
  
  33\% – decay in VELO as well
  
  66\% – decay outside VELO
  
  \( \rightarrow \) harder to reconstruct

(many more elements)
\[ D^0 \rightarrow K_s \pi^+ \pi^- \]

Allows measuring \( x \) and \( y \) parameters directly

- **3-Body Decay**
  
  Unlike 2-body decays, energy of daughter particles not well-defined
  
  Range of possible values.

\[ m_{K_s \pi^+} \]

**Dalitz plot**

Graphical representation

LHCb Simulation
\( D^0 \rightarrow K_s \pi^+ \pi^- \)

Allows measuring x and y parameters directly

➢ 3-Body Decay

Unlike 2-body decays, energy of daughter particles not well-defined

Range of possible values.

\( D^0 \rightarrow K_s \pi^+ \pi^- \)

Dalitz plot

graphical representation

 Plot invariant mass of one pair against other pair
$D^0 \rightarrow K_s \pi^+ \pi^-$

Probability $|M|^2 \neq \text{const}$

Not homogenously populated with events
Some daughter energies more probable
What we observe:
Dalitz plot is characterised by Resonances + non-resonant background

Resonances (bands of high $|M|^2$)
Corresponds to

$D^0 \rightarrow r K_s \rightarrow K_s \pi^+ \pi^-$
Intermediate particle, e.g. $\rho^0$

$D^0 \rightarrow r \pi^+ \rightarrow K_s \pi^+ \pi^-$

$D^0 \rightarrow r \pi^- \rightarrow K_s \pi^+ \pi^-$
$|M|^2$ is clearly affected by resonances

**THE ISOBAR MODEL**

- Resonant decay treated as a superposition of 2-body decays
e.g. $D \to rK_s$ and $r \to \pi^+\pi^-$
- Each **resonance** has a probability amplitude $M_r$

$$M = \sum_r a_r M_r$$

$$M_r = \langle \pi^+\pi^- | r \rangle \Delta_r(m_{\pi^+\pi^-}) \langle K_s r | D^0 \rangle$$

- Approximate by a **relativistic Breit–Wigner** probability distribution function

$$D^0 \to K_s \pi^+\pi^-$$

Great for isolated Resonances!
PROBLEM

If Resonances overlap
Total probability > 1

⇒ Not good
⇒ Theorists are unhappy
⇒ Should probably do something about that…

BAD for overlapping resonances

Great for isolated Resonances!

ALTERNATIVE MODEL

Makes use of **K-Matrix formalism** to describe resonances
\[ D^0 \to K_s r \to K_s \pi^+ \pi^- \] with \( \text{spin}(r) = 0 \) \( (\pi\pi - S \text{ wave}) \)

Assumption:
\[ D^0 \to K_s \pi^+ \pi^- \] equivalent dynamics to \( K_s \) scattering off \( \pi^+ \pi^- \)

Need to consider all possible decays of \( r \) in the analysis
\[ r \to \pi\pi \]
\[ r \to KK \]
\[ r \to \pi\pi\pi\pi \]
\[ r \to \eta\eta \]
\[ r \to \eta\eta' \]
K- Matrix Formulation

\[ F = (I - iK\rho)^{-1}P \]

Pseudo-propagator
Comes from scattering data.

\[ K = K - \text{ Matrix} \]

\[ \rho = \text{phase-space factor} \]

Production Vector
Describes Couplings of resonances to \( D^0 \)

\[ M_r = \langle \pi^+\pi^- | r \rangle F \langle K_S r | D^0 \rangle \]

Upholds unitarity by construction!

Implementation: CODING

My Job:
Implement **K-matrix** description in **fitting** code for
\[ D^0 \rightarrow K_s \pi^+ \pi^- \]

implement \( F = (I - iK\rho)^{-1}P \) \( \rightarrow \) fit to data

**Programming language:** CUDA (\( \sim C++ \))

**Running on a** GPU (Graphical Processing Unit)
\(- extremely fast parallelisation\)
What are we fitting to data?  

\[ F = (I - iK\rho)^{-1}P \]

- **K** – K-matrix: *comes entirely from scattering data*
- **\(\rho\)** – phase space matrix: *depends only on masses*
- **\(P\)** – production vector: describes coupling to resonances

\[ P_j = \sum_{\alpha} \frac{\beta_0^\alpha}{m_\alpha^2 - m_{\pi\pi}^2} \frac{\alpha_j}{m_{\pi\pi}^2} + f_{\pi\pi j}^{pr} \frac{1 - s_0^{pr}}{m_{\pi\pi}^2 - s_0^{pr}} \]

**21 floating variables**

\(\beta_0^\alpha\) and \(f_{\pi\pi j}^{pr}\) are complex

Need to fit
STATUS

➢ Wrestling with errors
  • Observing things (printing to screen) changes variables.
    → memory issues probably to blame

➢ This Week: Code Working on a CPU!
  • Positive cross-checks (e.g. $D^0$ decay time $\tau$)
    → Implementation complete

➢ GPU memory issues currently being addressed
THANK YOU
Questions?
Spare Slides
What are we fitting to data?

**K-Matrix**

\[ K_{ij} = \left( \sum_{\alpha} \frac{g_{\alpha i} g_{\alpha j}^0}{m_{\alpha}^2 - m_{\pi\pi}^2} + f_{ij}^{sc} \frac{1 - s_{0}^{sc}}{m_{\pi\pi}^2 - s_{0}^{sc}} \right) \left[ \frac{1 - s_{0}^{A}}{m_{\pi\pi}^2 - s_{0}^{A}} \left( m_{\pi\pi}^2 - \frac{a_{A} m_{\pi}^2}{2} \right) \right] \]

**Production vector**

\[ P_{j} = \sum_{\alpha} \frac{\beta_{\alpha}^0 g_{\alpha j}^0}{m_{\alpha}^2 - m_{\pi\pi}^2} + f_{\pi\pi j}^{pr} \frac{1 - s_{0}^{pr}}{m_{\pi\pi}^2 - s_{0}^{pr}} \]

Sum over poles  Non resonant term  Correction term

21 floating variables  \( \beta_{\alpha}^0 \) and \( f_{\pi\pi j}^{pr} \) are complex \( \rightarrow \) both real and imaginary parts needed
Fig. 1. The widths and mass differences of the physical states of the flavoured neutral mesons. The width corresponds to the inverse lifetime while the mass difference determines the oscillation frequency.
\[ D \rightarrow P \]

- \( 1 = \pi \pi \)
- \( 2 = K \bar{K} \)
- \( 3 = \text{Multi-body} \)
- \( 4 = \eta \eta \)
- \( 5 = \eta \eta' \)

\[ (1 - iK \rho)^{-1} \]

\[ \pi^+ \rightarrow P \]

\[ \pi^- \]