Dedicated Finite-Element Simulation for Accelerator Magnets

TECHNISCHE UNIVERSITÄT DARMSTADT

Herbert De Gersem

Example: GSI-SIS-100 magnet

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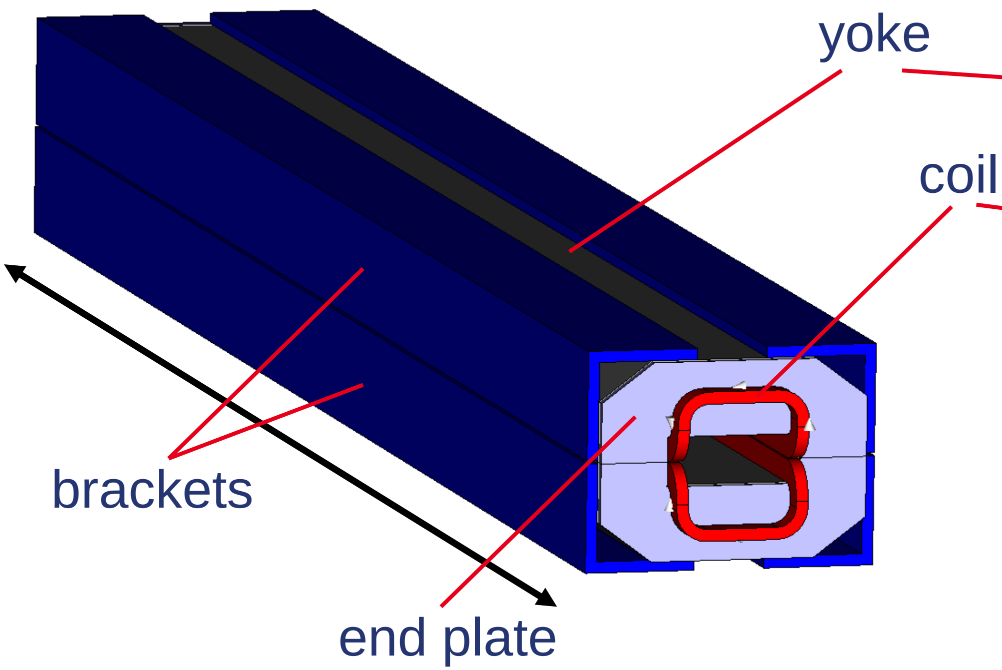

SIS100 dipole (prototype)

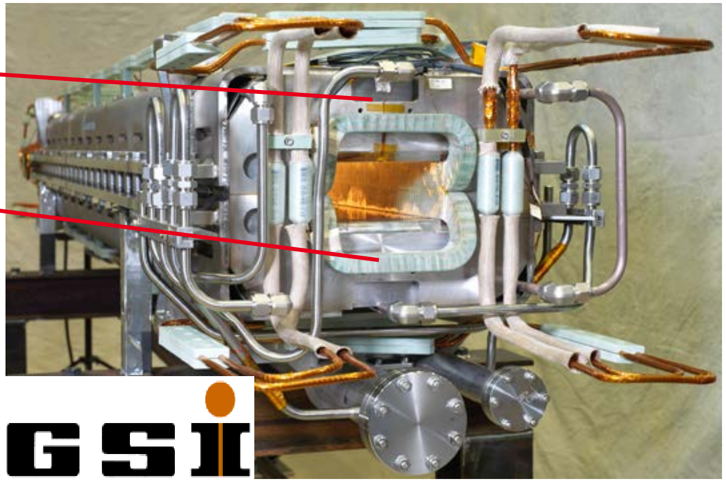

photo: J. Guse, GSI (www.gsi.de)

length: 3 m

Example: GSI-SIS-100 magnet

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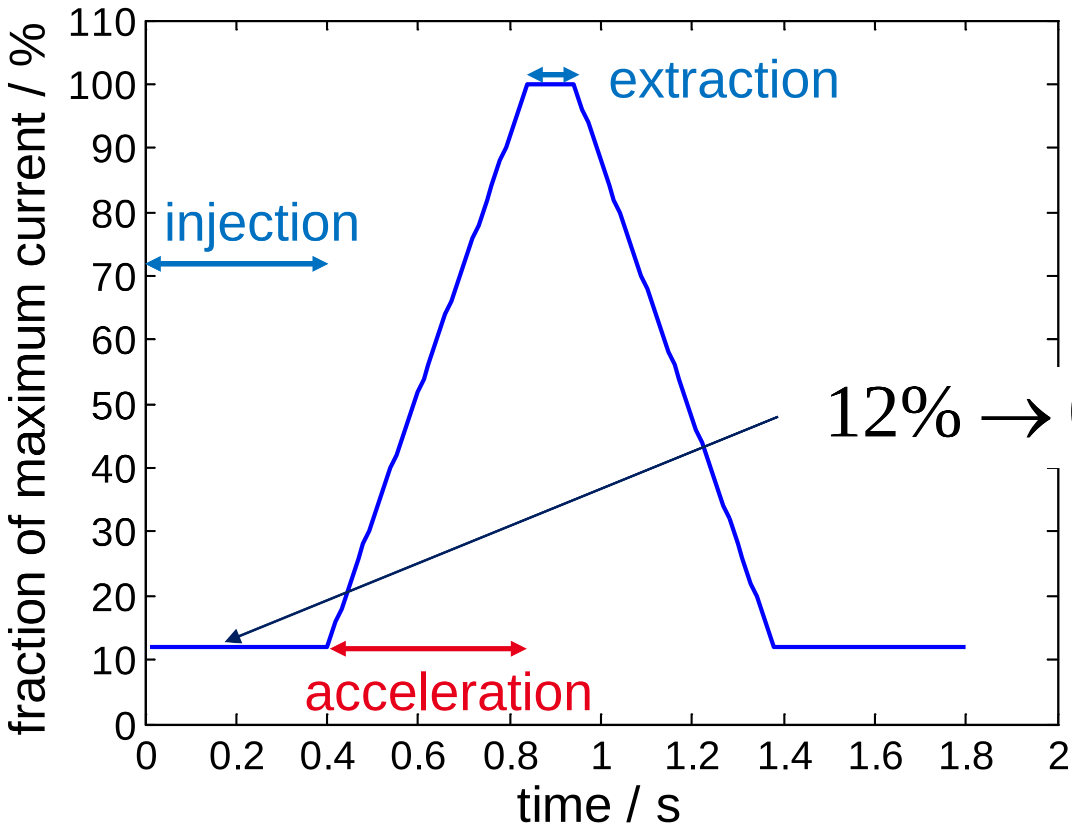
excitation profile



$$12\% \rightarrow 0.24\ \mathrm{T}$$

Magnetoquasistatic formulation

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differential equation:

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_{\mathrm{s}}$$

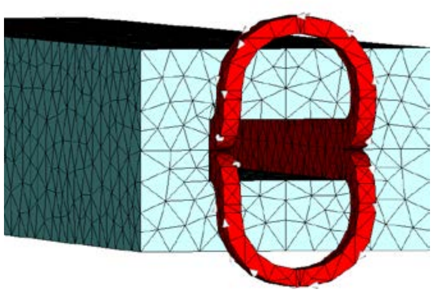
- ν: reluctivity
- $\vec{A}$: magnetic vector potential
- σ: conductivity
- $\vec{J}_{\mathrm{s}}$: applied current density

Discretisation in space

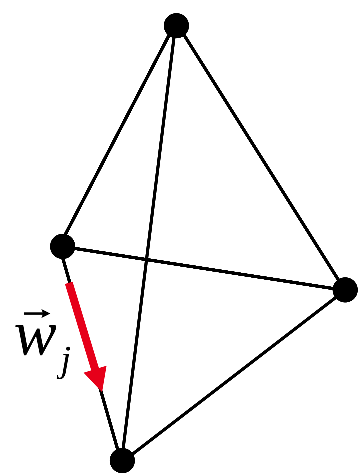
differential equation: $$\nabla \times \left(\nu \nabla \times \vec{A}\right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_{\mathrm{s}}$$

spatial discretisation $$\vec{A} \approx \vec{A}_{\mathrm{FE}} = \sum_j \widehat{a}_j \vec{w}_j$$

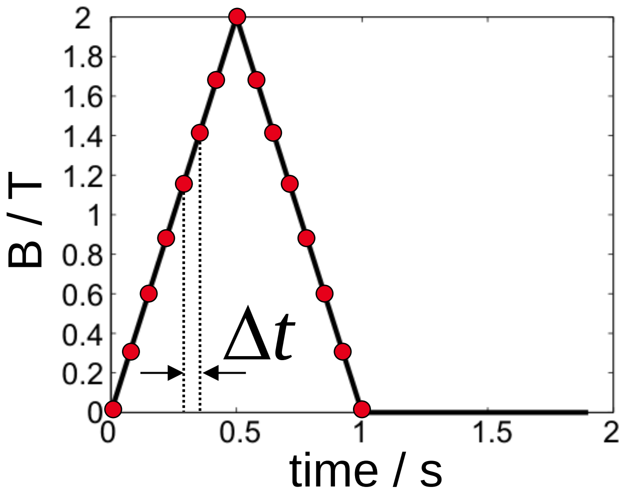
semi-discrete system: $$\mathbf{K}_\nu \widehat{\mathbf{a}} + \mathbf{M}_\sigma \frac{\mathrm{d}\widehat{\mathbf{a}}}{\mathrm{d}t} = \widehat{\mathbf{j}}_{\mathrm{s}}$$



shape functions:
edge finite elements
(curl-conforming)



Discretisation in time



differential equation

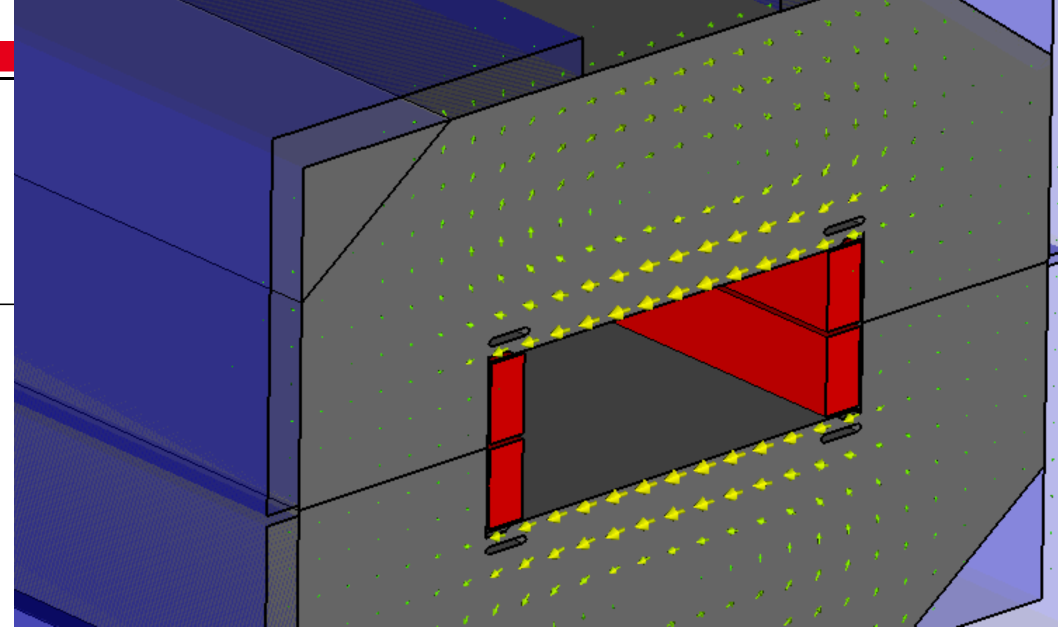
spatial discretisation

semi-discrete system: $$\mathbf{K}_\nu \hat{\mathbf{a}} + \mathbf{M}_\sigma \frac{\mathrm{d}\hat{\mathbf{a}}}{\mathrm{d}t} = \hat{\hat{\mathbf{j}}}_\mathrm{s}$$

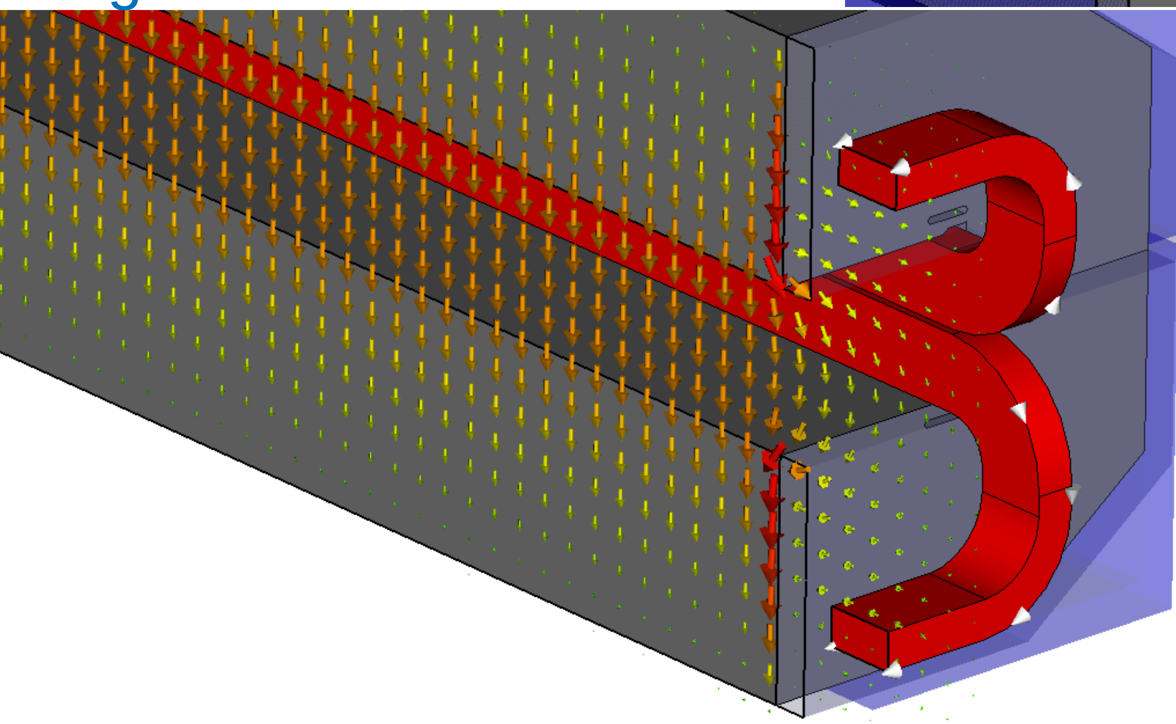
temporal discretisation

discrete system: $$\left(\mathbf{K}_\nu + \alpha \mathbf{M}_\sigma\right)\hat{\mathbf{a}}_{k+1} = \mathrm{RHS}$$

Results



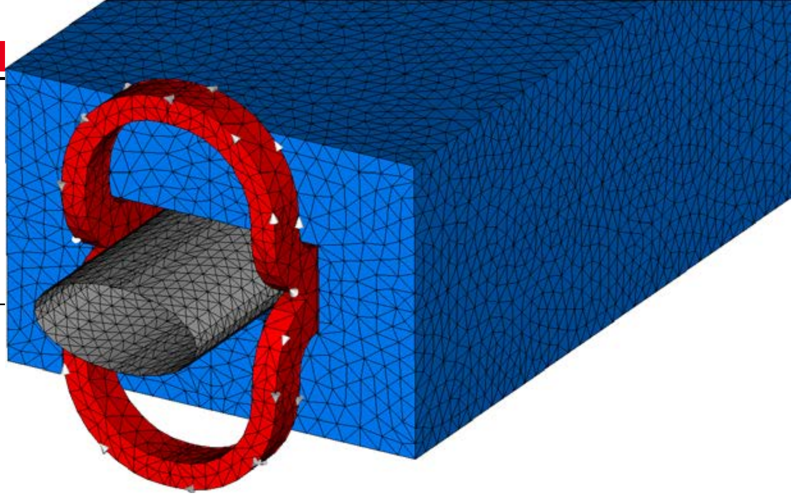
magnetic field



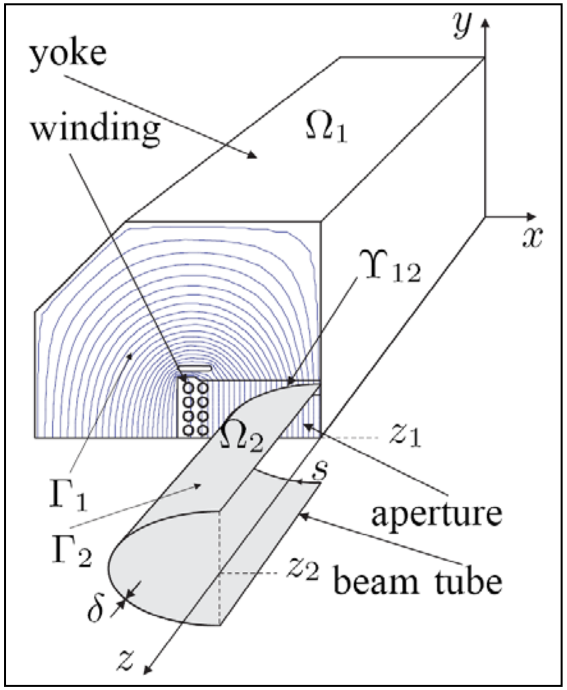
eddy currents
in the end plane

simulation by
CST EMStudio®

Overview

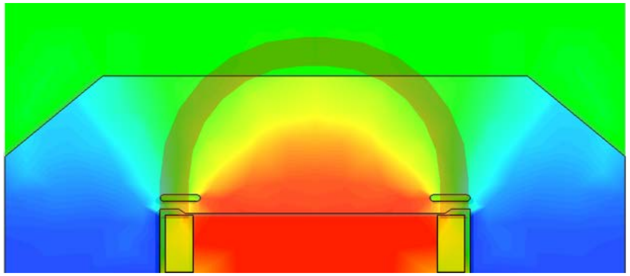


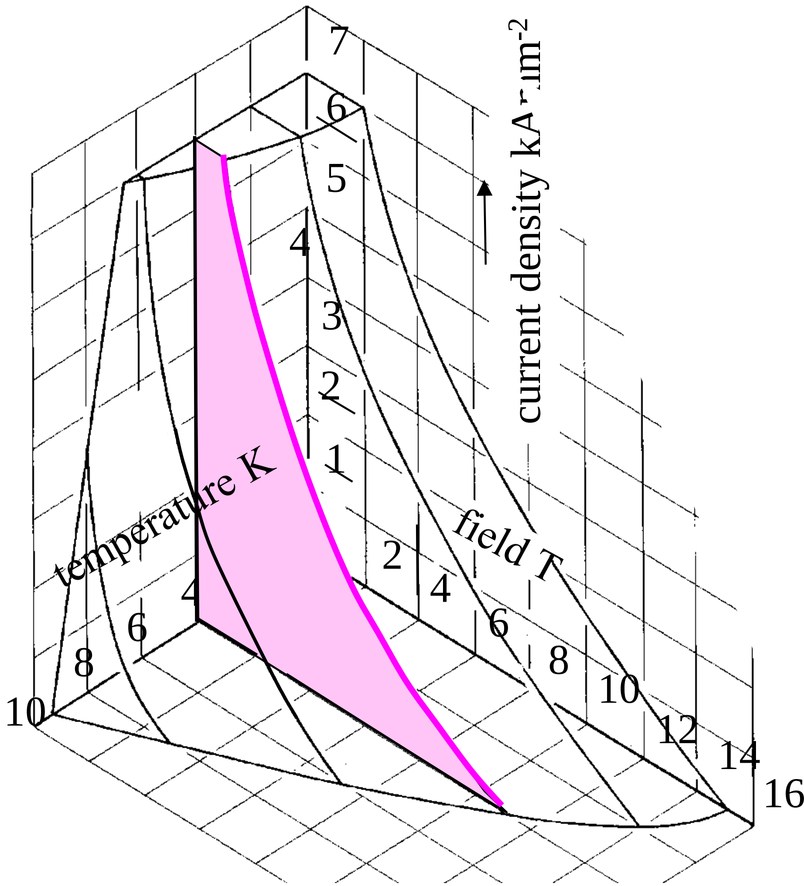
- magnet simulation (standard 3D FE solver)
- challenges
- 3D FE solver
- dedicated simulation tricks + appropriate numerics
- conclusions



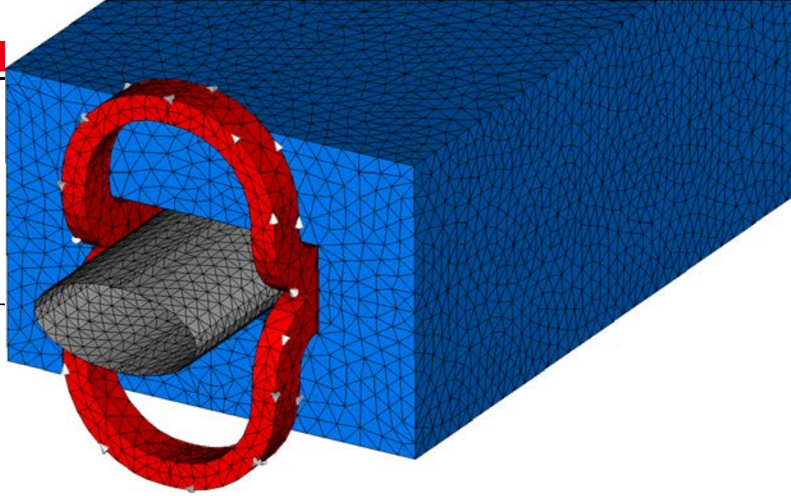
Challenges

- detailed geometry (e.g. end windings, coil position, beam tube)
- materials (e.g. composites, hysteretic, superconducting)
- multi-physics (e.g. EM, thermal, mechanical)
- multi-scale (in space and time)
- high accuracy requirements





Overview



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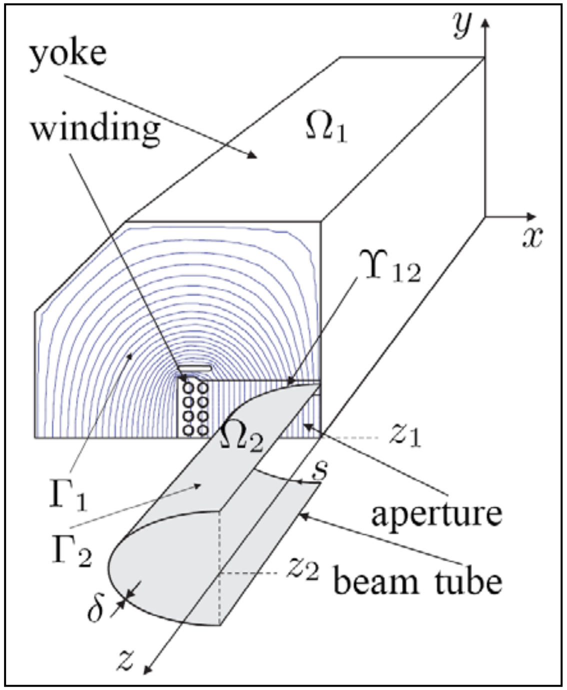
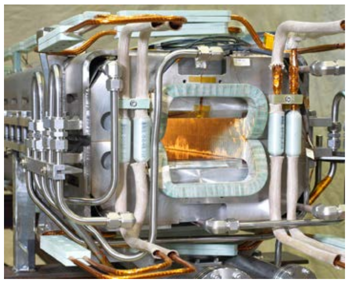
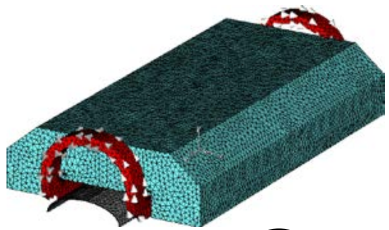


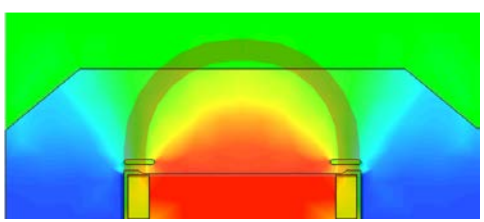
Dedicated Simulation Tool

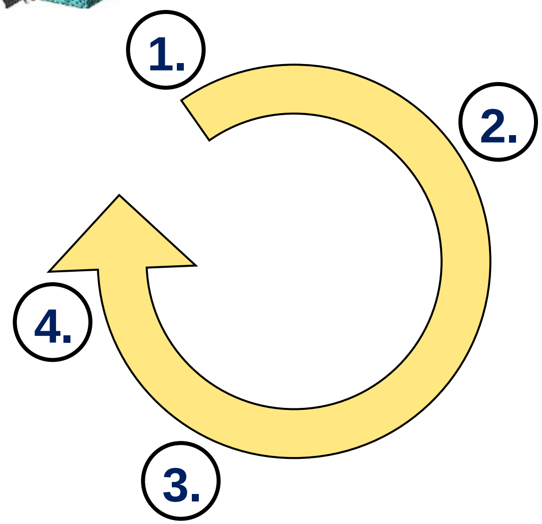




CST Studio Suite®
- CAD modelling
- meshing

- visualisation





FEMSTER, LLNL

TRILINOS, Sandia Labs

own software
- FE assembly (higher order FEs)
- transient solver
- nonlinear materials
- system solver

Matlab
- postprocessing
- visualisation

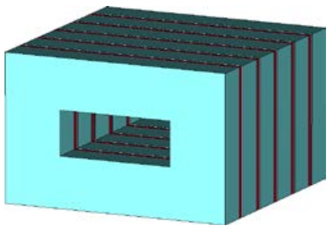
parallelisation

+ Stephan Koch, Jens Trommler

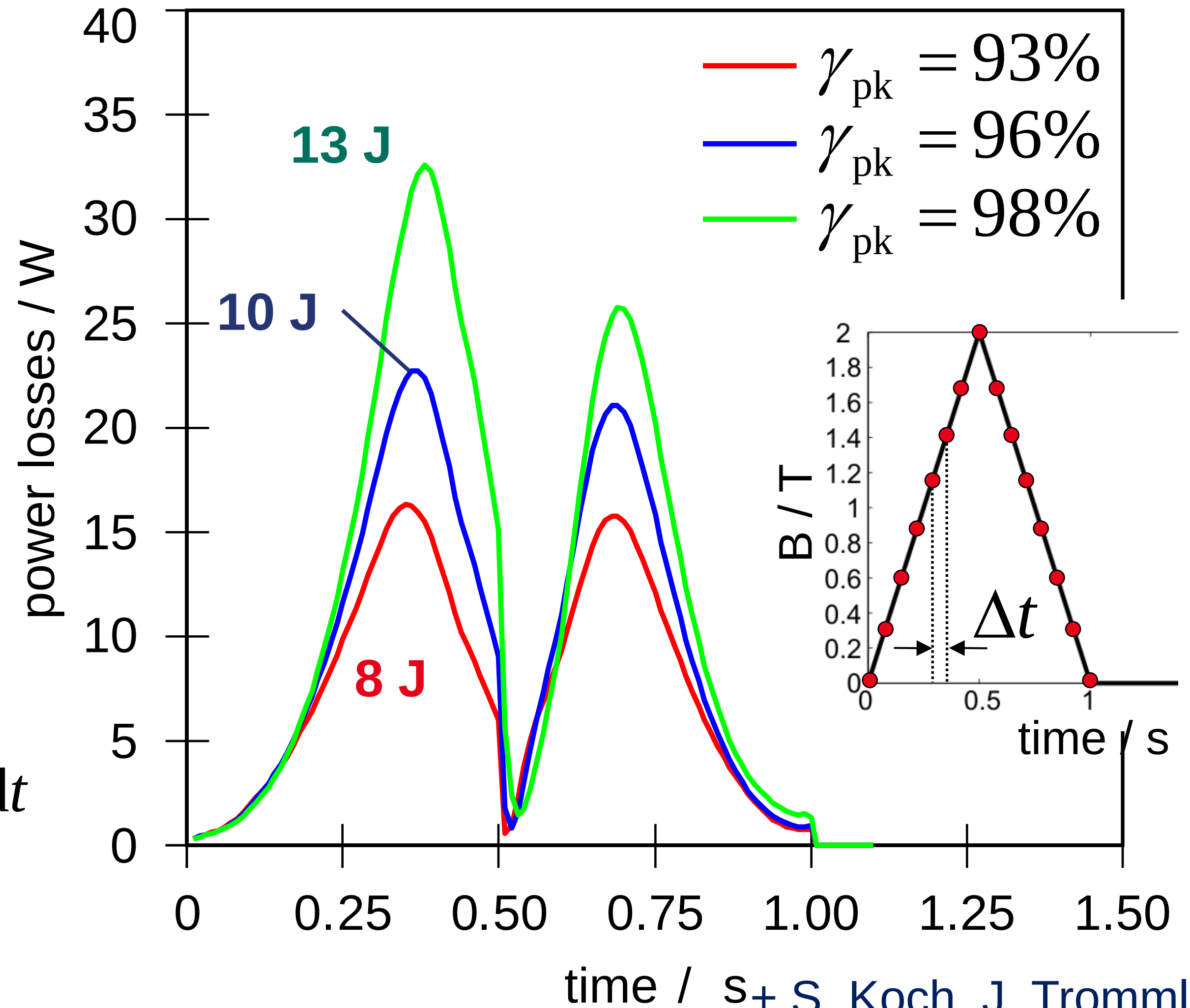
Results: Eddy-Current Losses

eddy-current losses over one cycle

for different stacking factors γ_{pk}



loss energy: $W = \int_0^T P \mathrm{d}t$



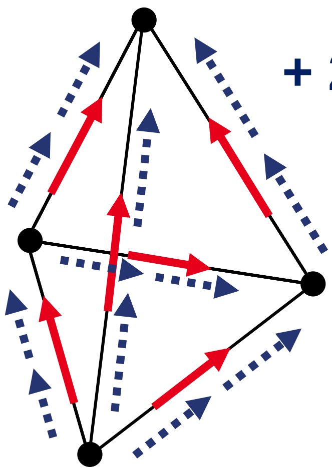
+ S. Koch, J. Trommler

Results: Loss Energy

- discretization:
  - increase number of elements
  - increase order of approximation

$$\vec{A} \approx \vec{A}_{\text{FE}} = \sum_j a_j \vec{w}_j^{\text{tv}}$$

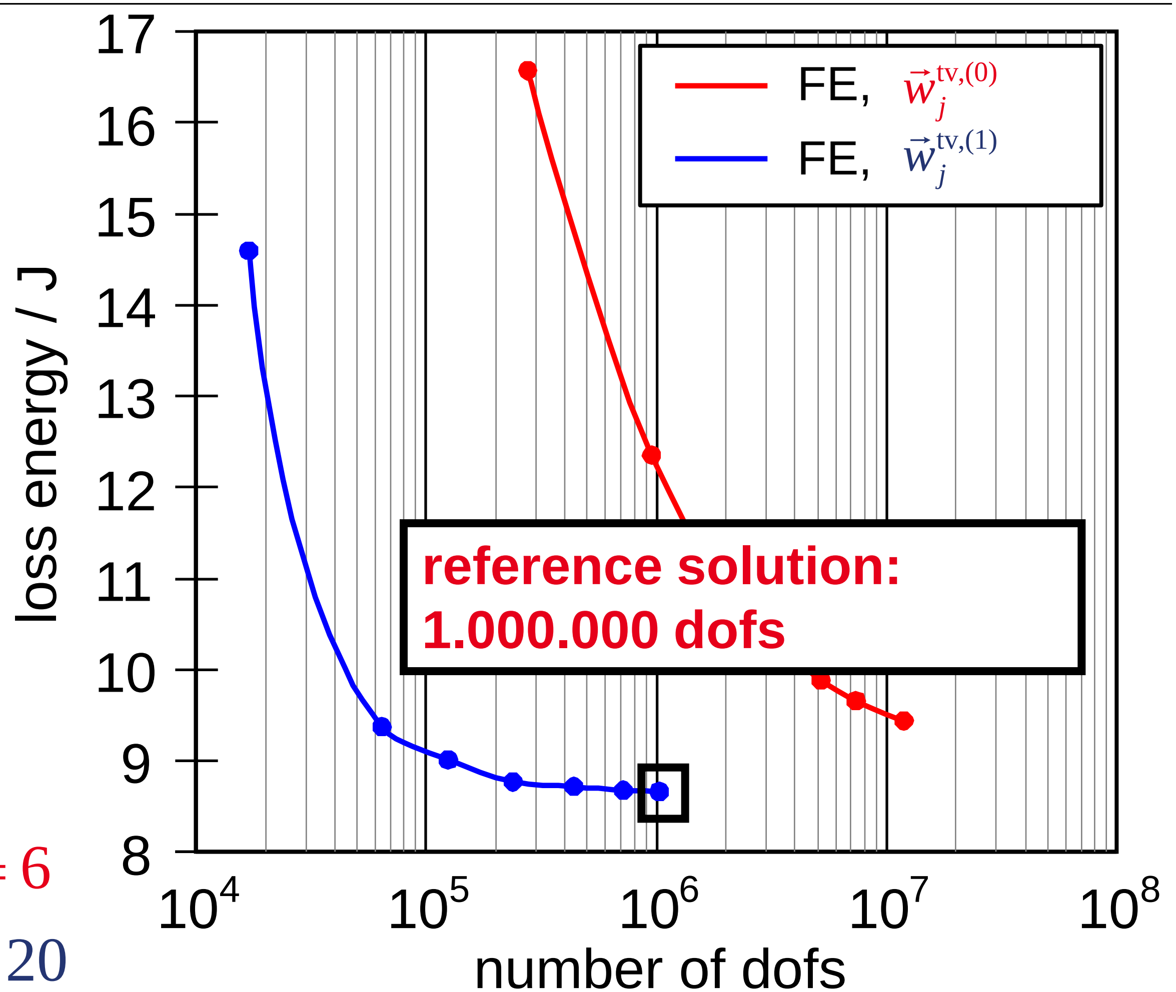
degrees of freedom:



+ 2 per face

$\vec{w}_j^{\text{tv},(0)}$ $n_{\text{dof}} = 6$

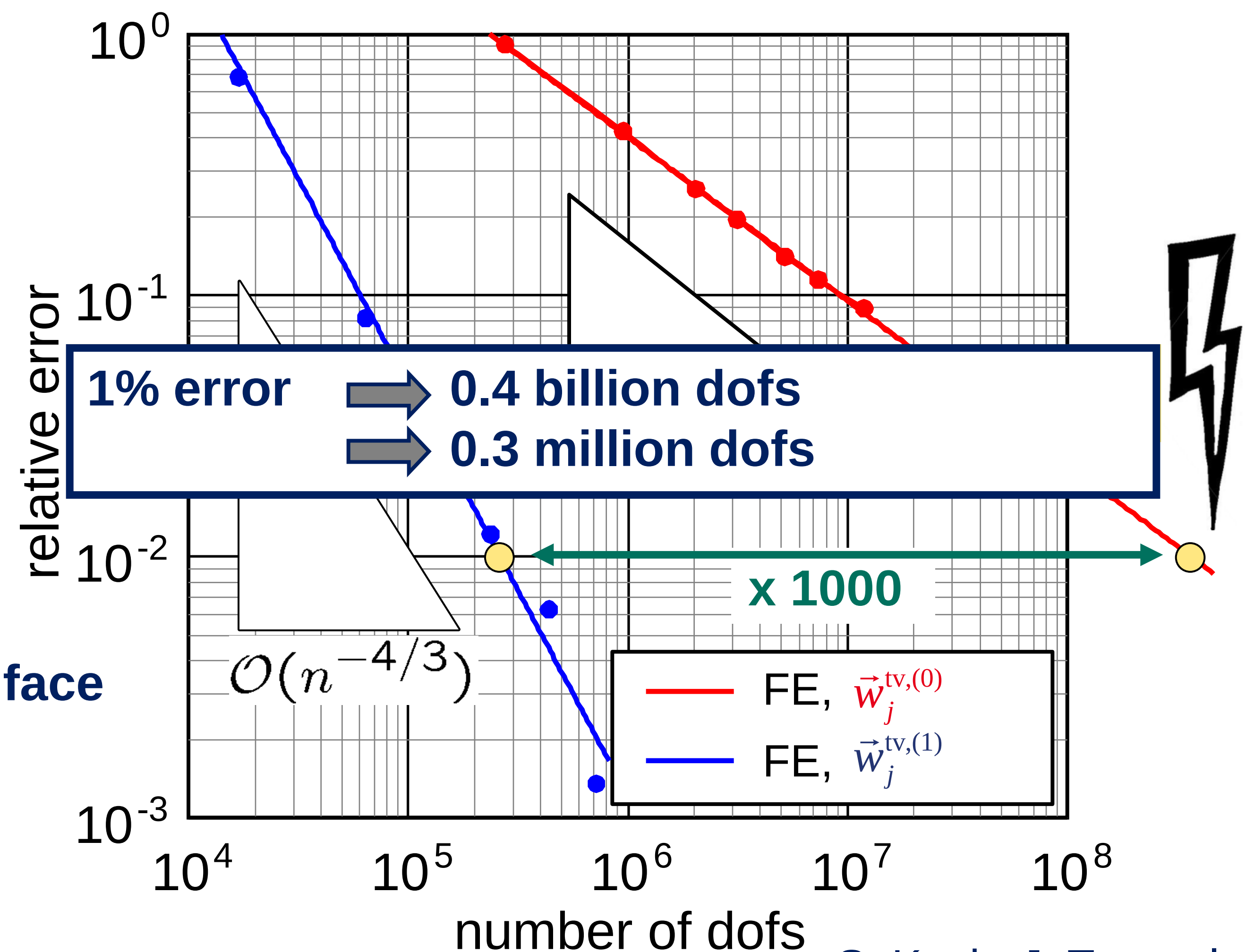
$\vec{w}_j^{\text{tv},(1)}$ $n_{\text{dof}} = 20$



+ S. Koch, J. Trommler

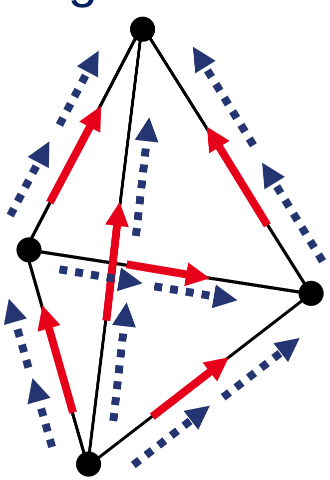
Convergence: Loss Energy

- **relative error with respect to reference solution**



1% error ➡ **0.4 billion dofs**
➡ **0.3 million dofs**

degrees of freedom:



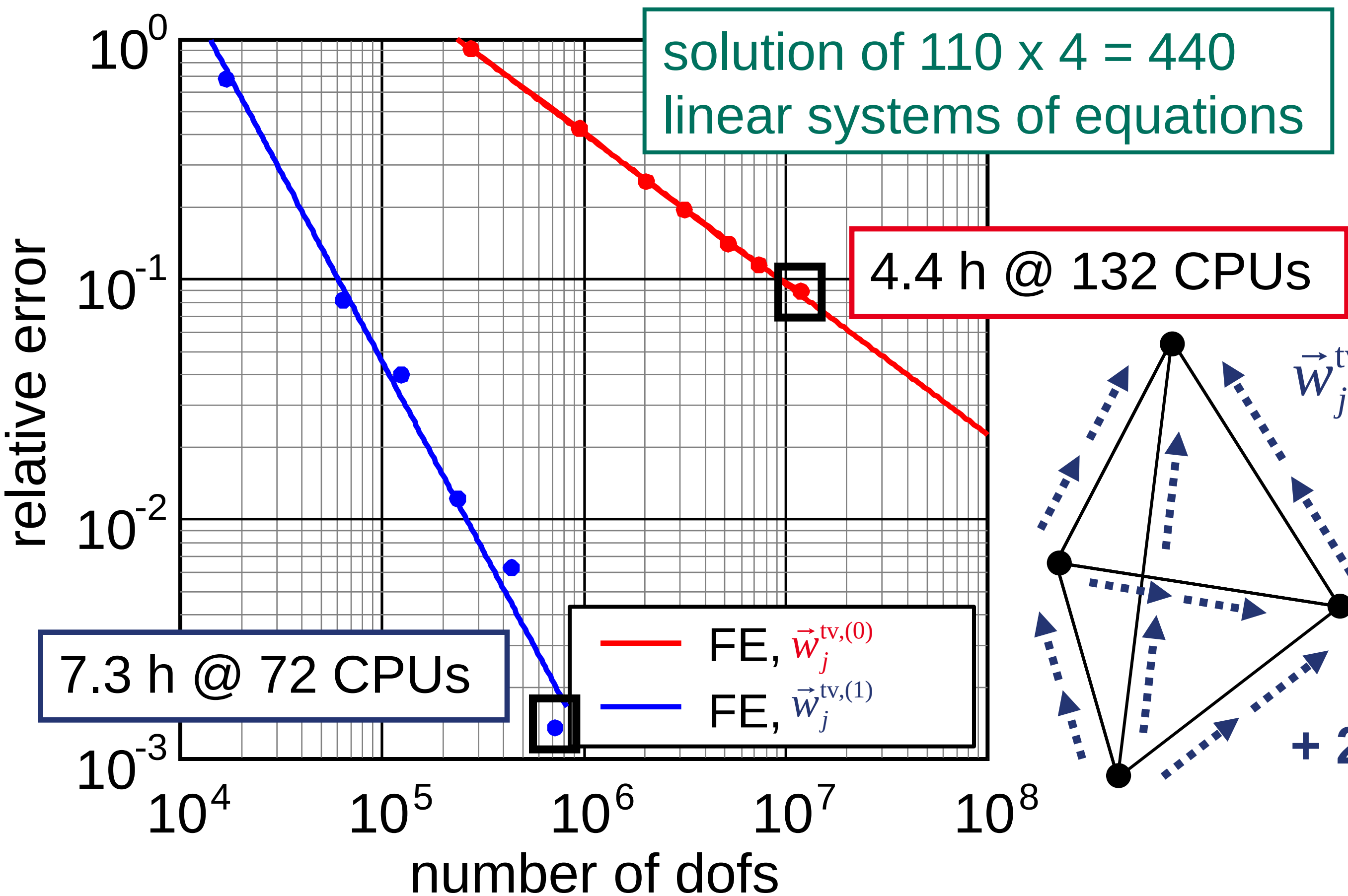
+ 2 per face

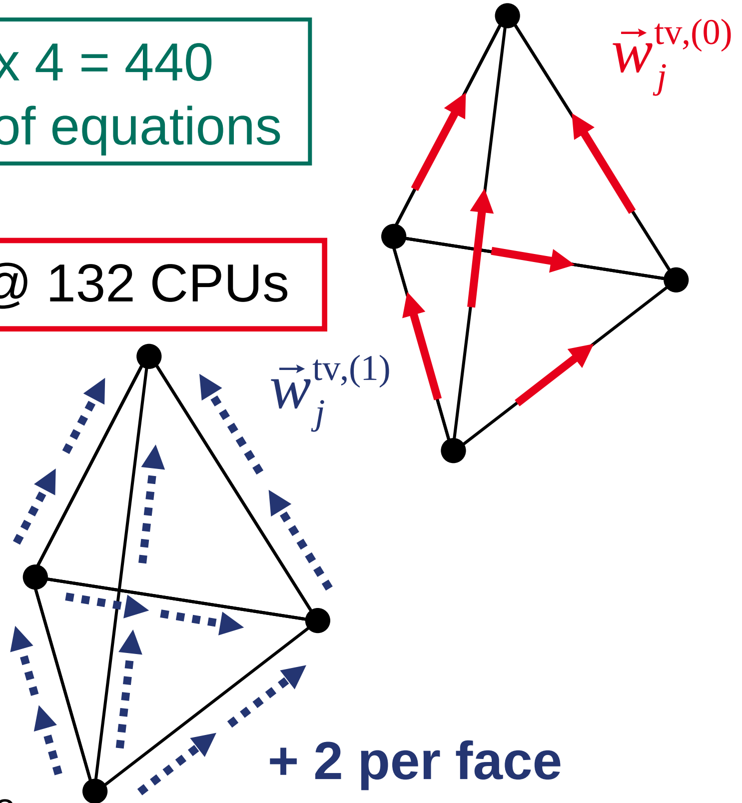
$\vec{w}_j^{\mathrm{tv},(0)}$

$\vec{w}_j^{\mathrm{tv},(1)}$

+ S. Koch, J. Trommler

Comparison: Shape Functions

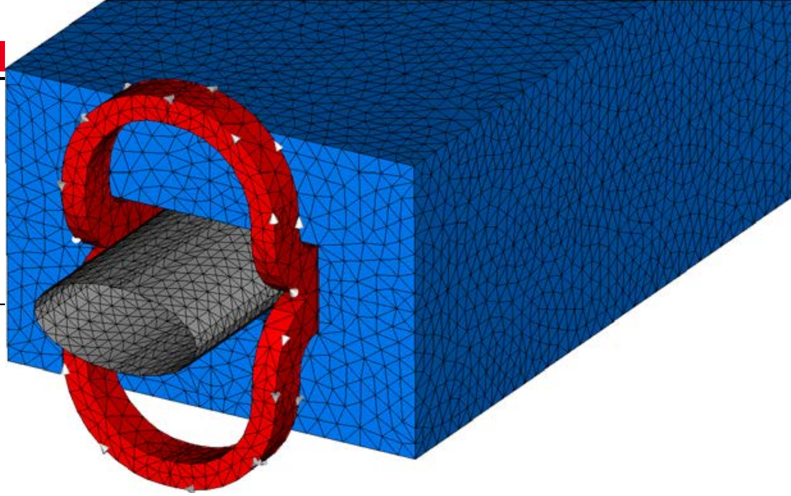




+ 2 per face

+ S. Koch, J. Trommler

Overview



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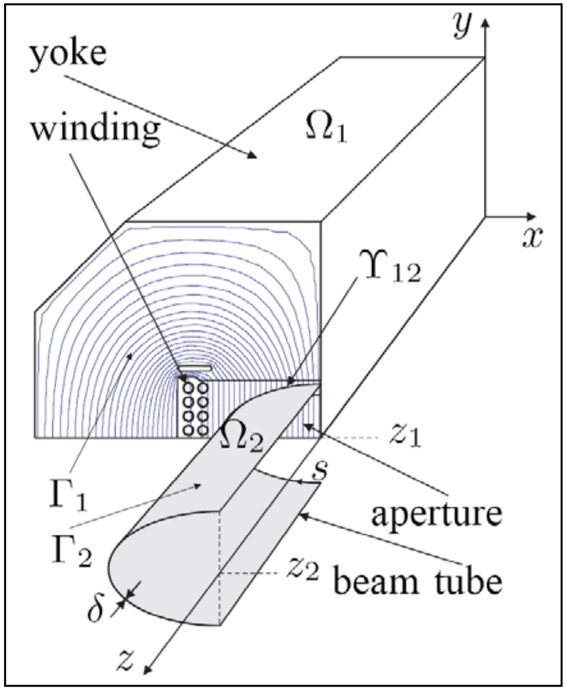
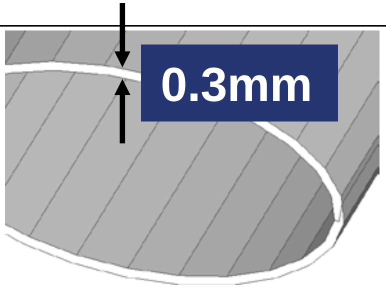
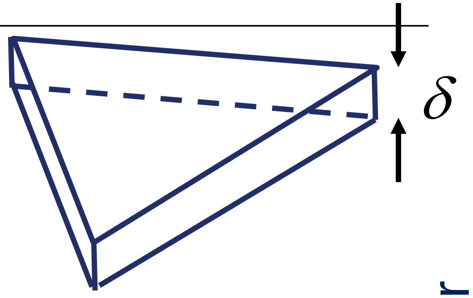


Beam tube : thin-shell model





beam tube

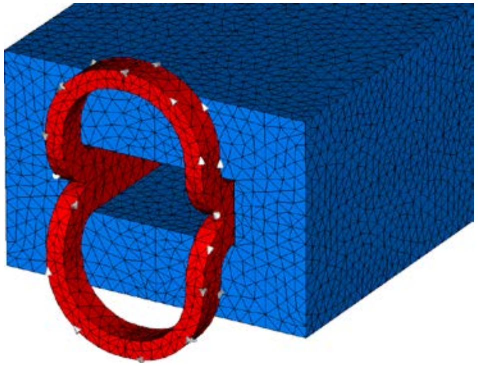
- eddy currents

shell elements

additional matrix contributions $\mathbf{K}_\delta$ and $\mathbf{M}_\delta$

assembling into system matrix by $\mathbf{Q}$

$$\mathbf{K}_\nu + \sigma\mathbf{M}_\sigma + \mathbf{Q}^{\mathrm{T}}\left(\mathbf{K}_\delta + \alpha\mathbf{M}_\delta\right)\mathbf{Q} = \mathbf{K}_{\mathrm{full}} + \alpha\mathbf{M}_{\mathrm{full}}$$



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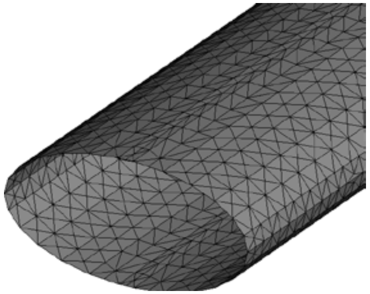

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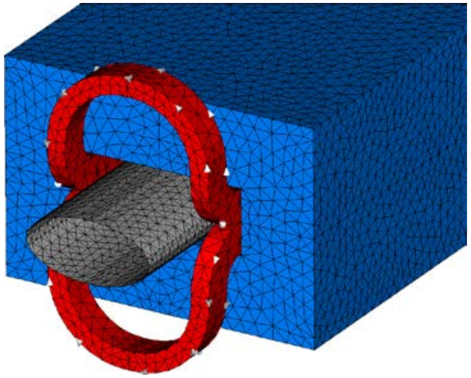
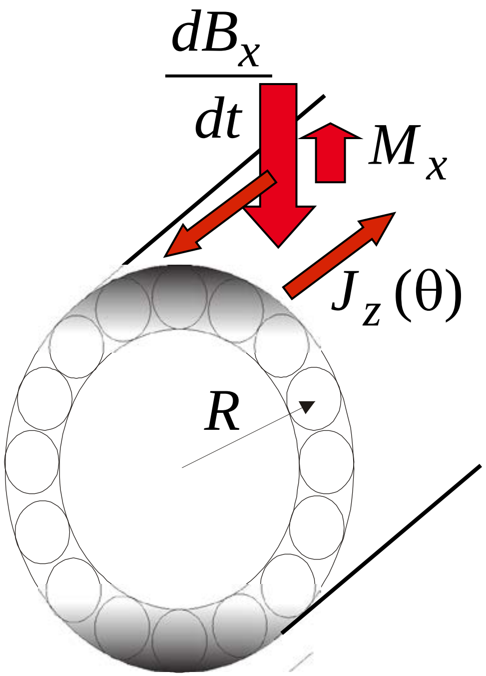


\+ Jens Trommler

Cable eddy current : homogenisation



time-varying magnetic field $\frac{dB_x}{dt}$

↓

induced electric field E_θ

↓

induced current density J_θ

↓

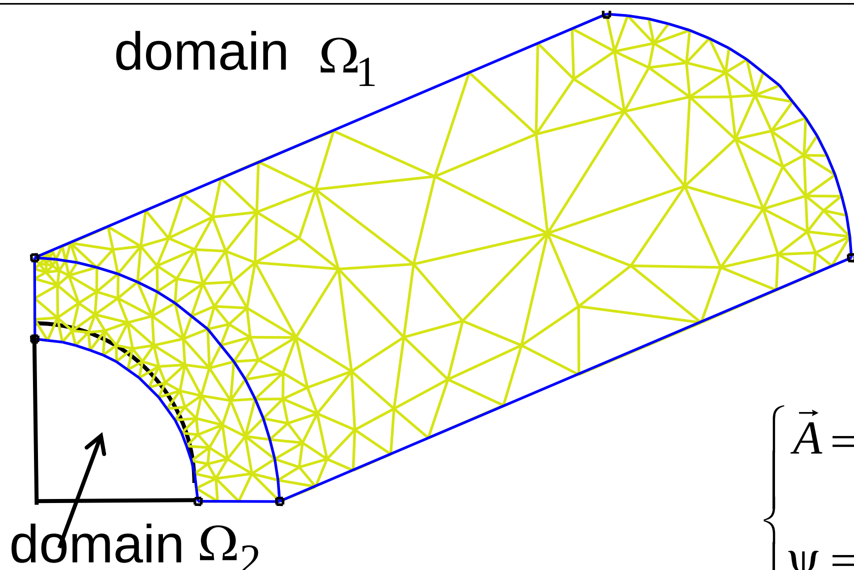
induced magnetisation M_x

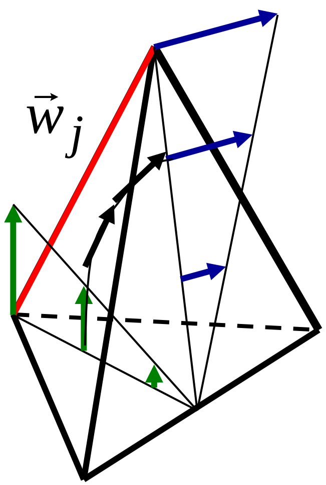
(magnetisation)

additional magnetisation

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times \left(\nu_0 \overline{\overline{\tau}}_{\mathrm{cb}} \nabla \times \frac{\partial \vec{A}}{\partial t} \right) = \vec{J}_{\mathrm{s}}$$

Mixed formulation





magnetoquasistatic formulation

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_s \quad \text{in } \Omega_1$$

$$-\nabla \cdot \left(\mu \nabla \psi \right) = 0 \quad \text{in } \Omega_2$$

$$\begin{cases} \vec{A} = \sum_j u_j \vec{w}_j & \text{in } \Omega_1 \\ \psi = \sum_q v_q N_q & \text{in } \Omega_2 \end{cases}$$

discretization

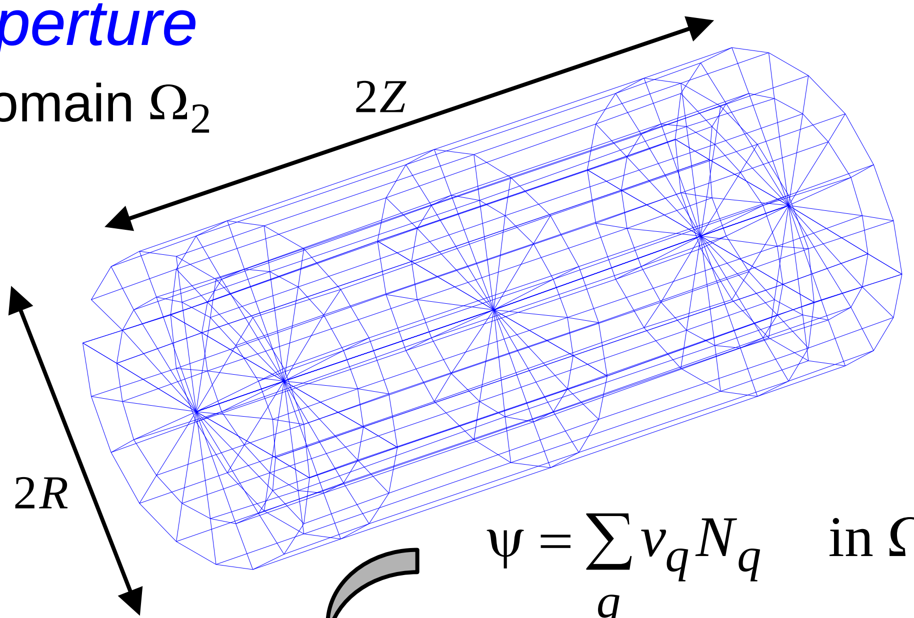
system of equations

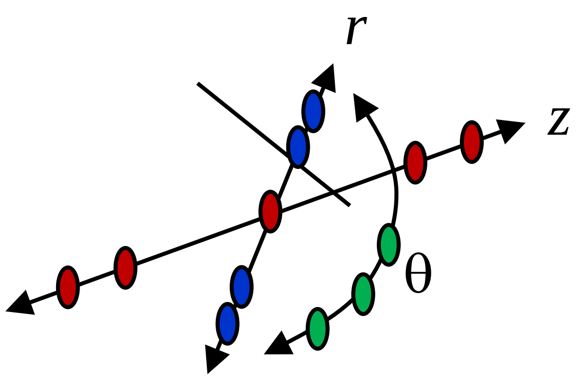
$$\begin{bmatrix} \mathbf{K} + \mathbf{M}\frac{d}{dt} & \mathbf{B}^T \\ \mathbf{B}\frac{d}{dt} & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}$$

Hybrid discretisation

aperture

domain Ω_2





Legendre distribution in *r*
equidistant distribution in *θ*
Legendre distribution in *z*

$$\psi = \sum_q v_q N_q \quad \text{in } \Omega_2$$

$$N_q(r,\theta,z) = N_{q_1,q_2,\lambda}(r,\theta,z) = P_{q_1}\left(\frac{r}{R}\right) e^{-j\lambda_q\theta} P_{q_2}\left(\frac{z}{Z}\right)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{D}_r^T & \mathbf{D}_\theta^T & \mathbf{D}_z^T \end{bmatrix} \mu_0 \begin{bmatrix} \mathbf{D}_r & \mathbf{D}_\theta & \mathbf{D}_z \end{bmatrix}^T$$

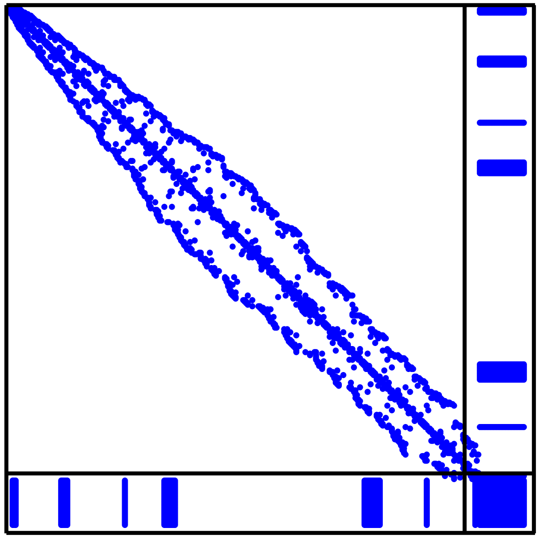
+FIT : Dehler, Weiland (1994)
in 2D : HDG, Clemens, Weiland (2003)

Specialised solver

ingredients:

- Krylov subspace solver (CG or MINRES)
- Schur complements
- domain decomposition preconditioner (additive or multiplicative Schwarz)
- algebraic multigrid (AMG) for FE part
- FFT for spectral discretisation

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & -\mathbf{G} \end{bmatrix} =$$

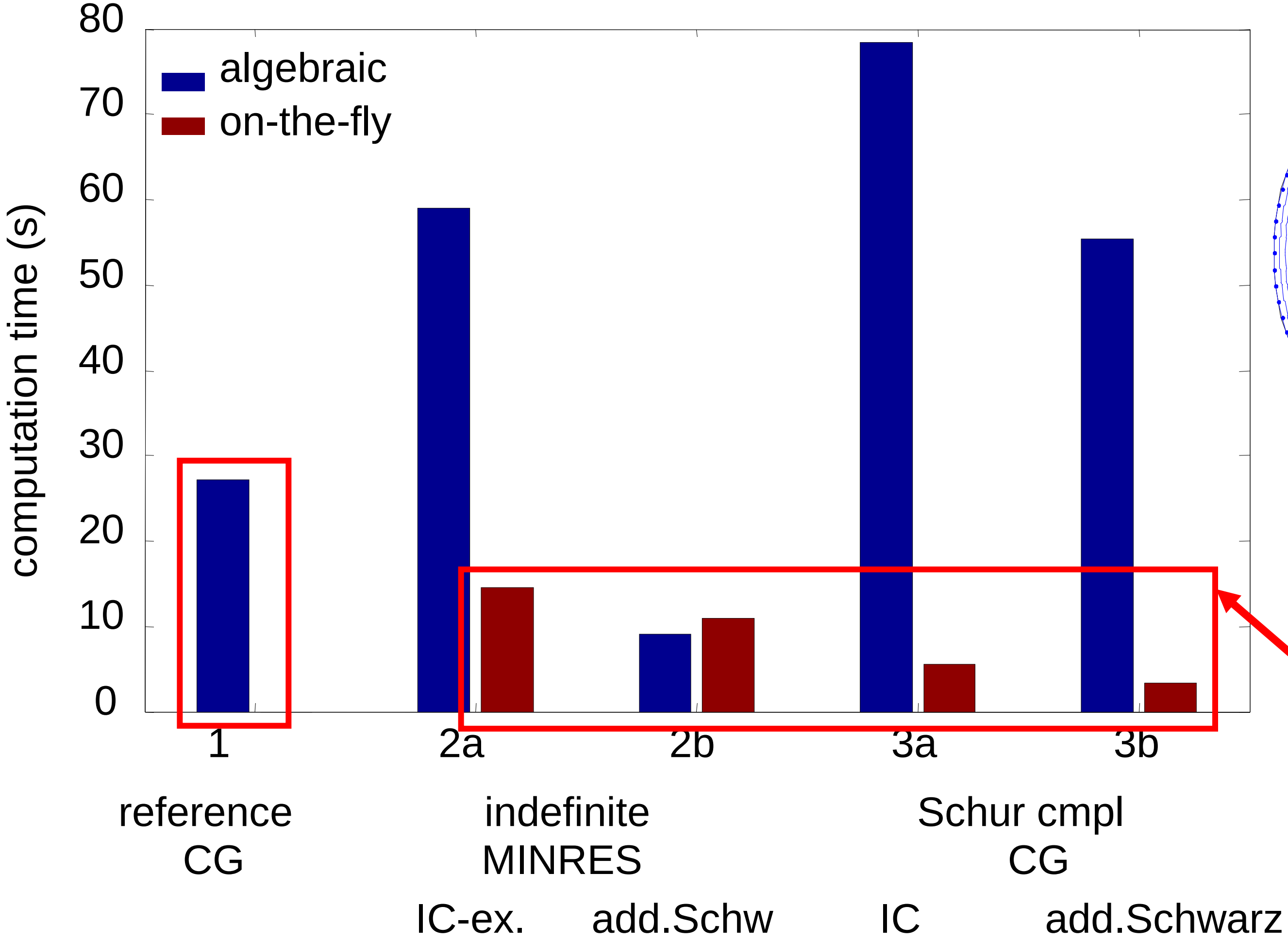


(+) $\mathbf{B}$ and $\mathbf{G}$ applied by (partially) dense algebraic matrices

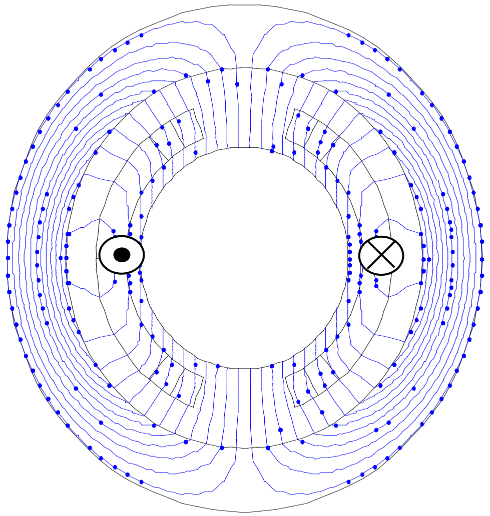
(*) $\mathbf{B}$ and $\mathbf{G}$ carried out „on the fly“

selection by index sets
interpolation by sparse matrices
2D Fast Fourier Transforms

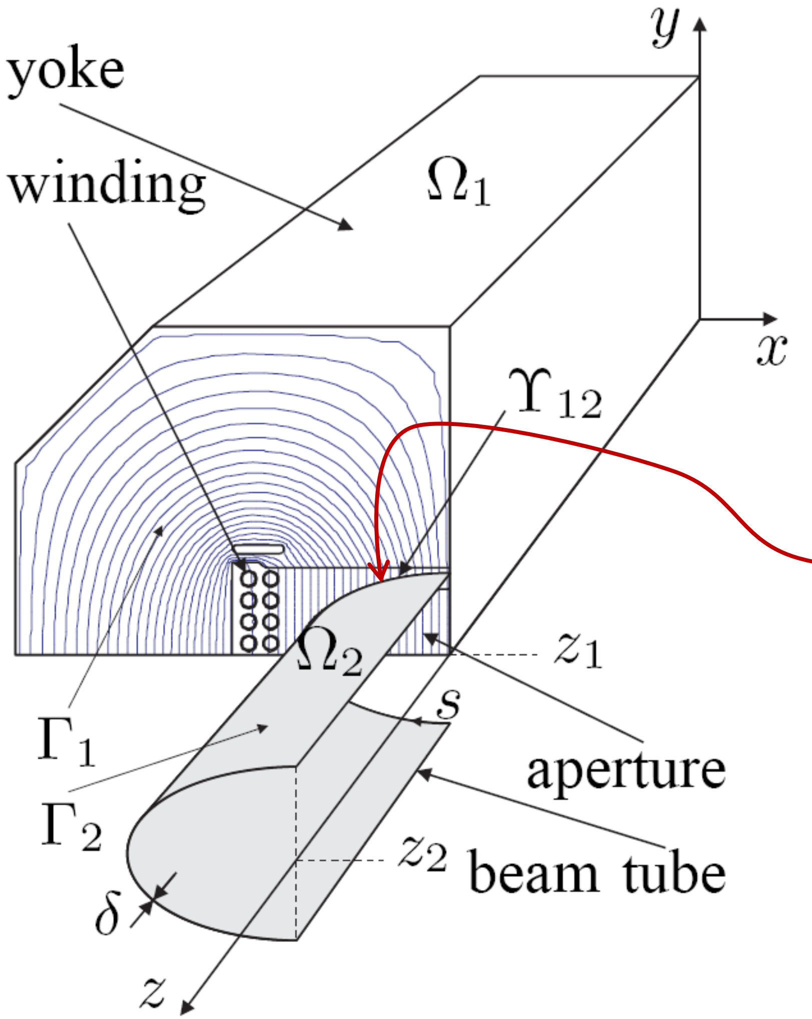
Numerical tests



test model



Hybrid discretisation



standard 2D model

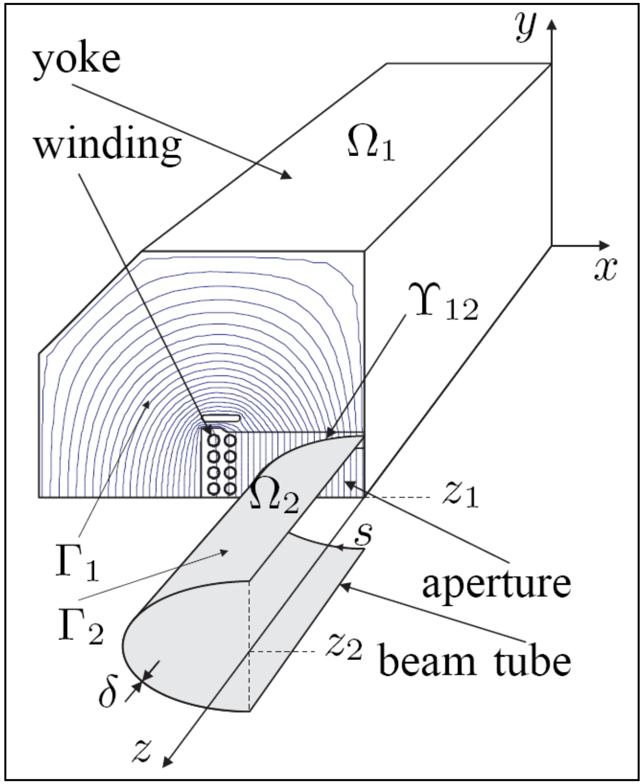
cross-section Γ_1 of volume Ω_1

common interface Υ_{12}

additional 2D model for the beam-tube end

cross-section Γ_2 of volume Ω_2

Coupled formulation



magnetoquasistatic formulation

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$$-\nabla\cdot\left(\sigma\frac{\partial\vec{A}}{\partial t}\right)-\nabla\cdot\left(\sigma\nabla\varphi\right)=0\quad\text{in }\Omega_2$$

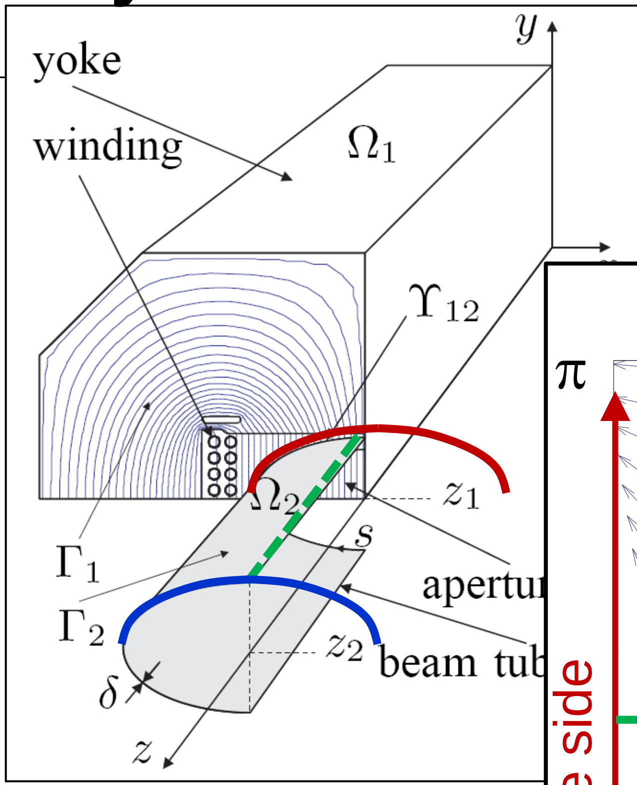
FE shape functions

$$\begin{cases}\vec{w}_j=\dfrac{1}{z_1}N_j(x,y)\,\vec{e}_z & \text{in }\Omega_1\\ P_{\tilde{q}}=M_{\tilde{q}}(s,z_1)\dfrac{z}{z_1} & \text{in }\Omega_1\\ P_q=M_q(s,z) & \text{in }\Omega_2\end{cases}$$

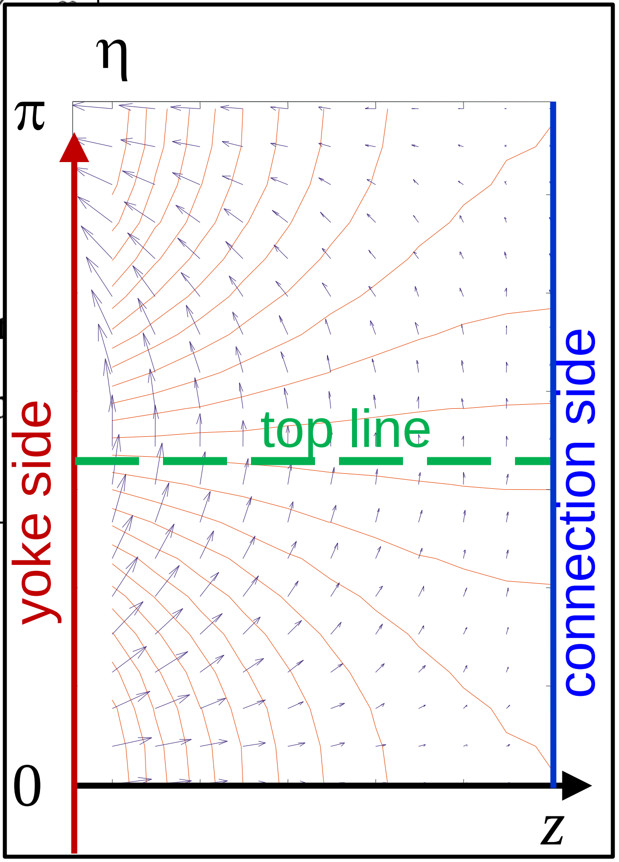
system

$$\begin{bmatrix}\mathbf{K}+\mathbf{M}\dfrac{d}{dt} & \mathbf{B}^T\\ \mathbf{B}\dfrac{d}{dt} & \mathbf{G}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ \mathbf{v}\end{bmatrix}=\begin{bmatrix}\mathbf{f}\\ 0\end{bmatrix}$$

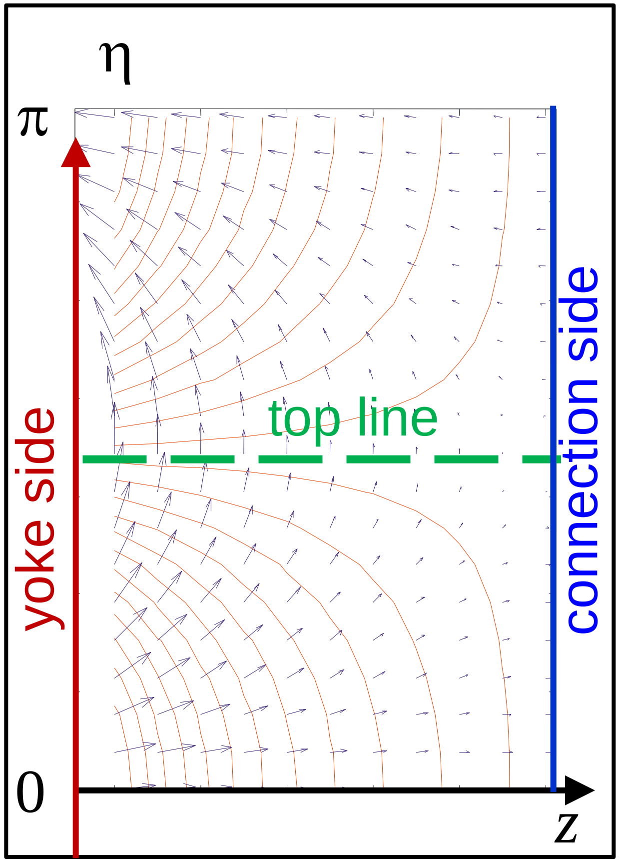
Eddy-current closing paths



connected beam tube



disconnected beam tube

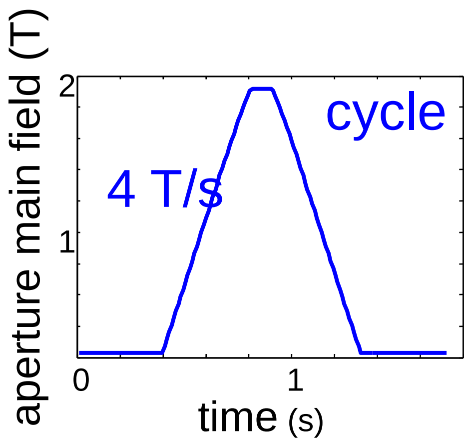


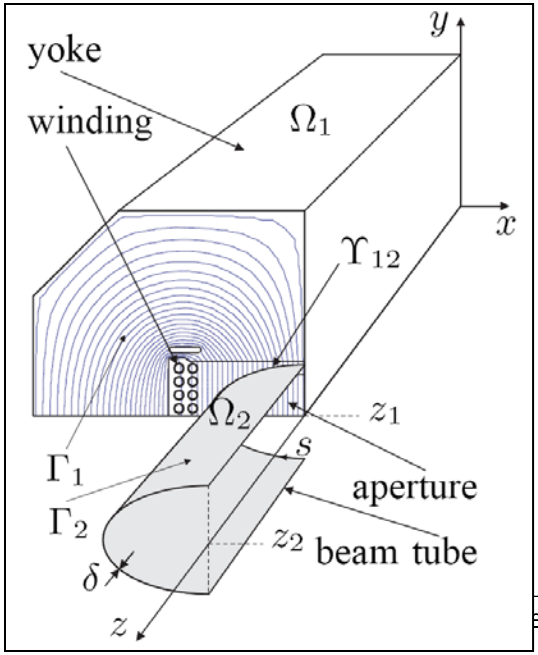
yoke side

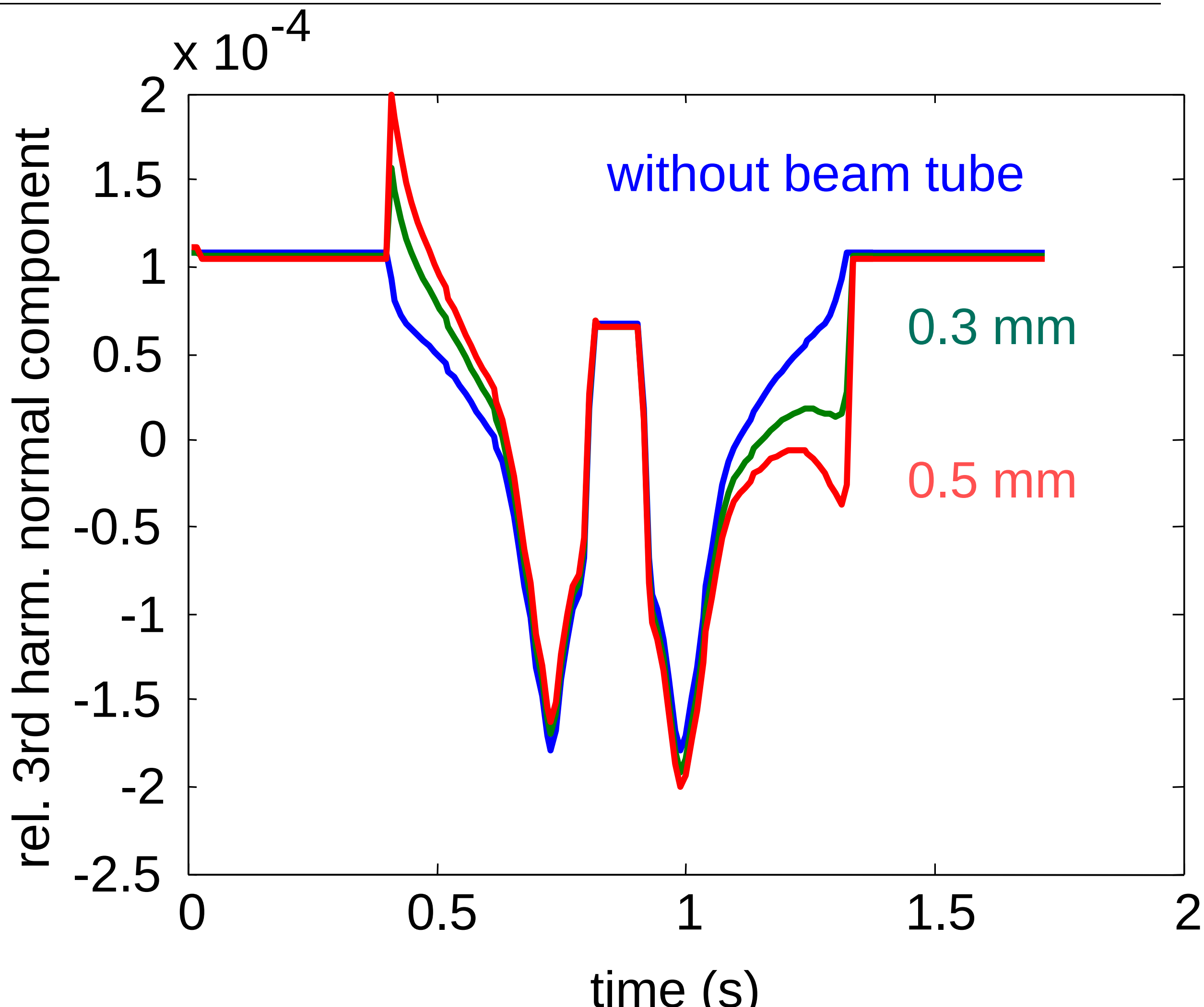
top line

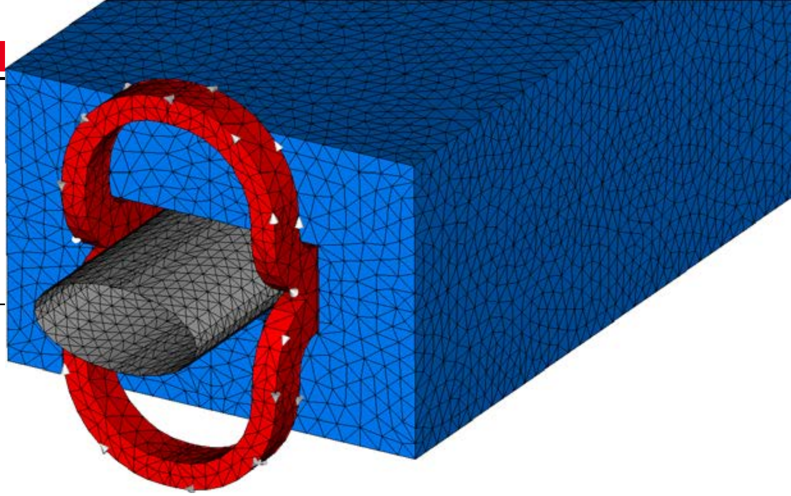
connection side

Sextupole component









Conclusions

- nonlinear 3D transient magnetic simulation feasible with of-the-shell software
- challenges
- dedicated methods and software