Approaches for the Quantification of Uncertainties in Stochastic Magnet Design

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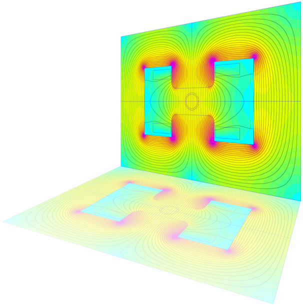
in cooperation with H. De Gersem

Outline of the Talk



UQ Procedure

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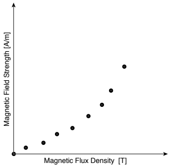
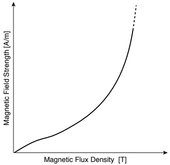
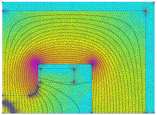
inputs → modeling → simulations → outputs



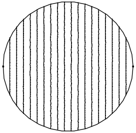
measurements,
tuples (B_i, H_i)



saturation
$H(B)$



accelerator
magnet



multipoles

UQ Procedure

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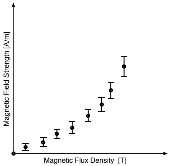
(uncertain) inputs → (stochastic) modeling → (stochastic) simulations → (stochastic) outputs



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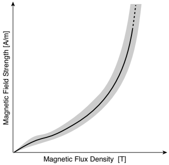

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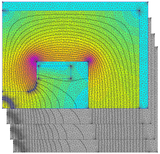
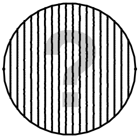


accelerator magnet



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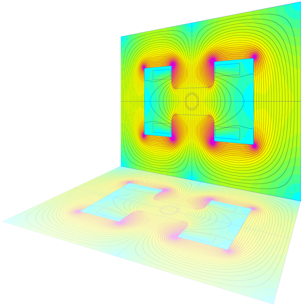

Eddy Current Problem

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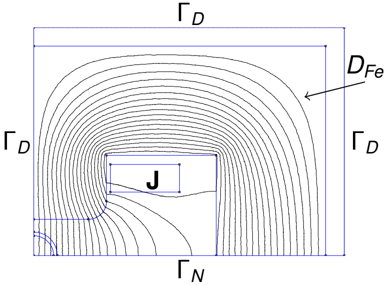
- Eddy current problem

$$\begin{cases} \nabla \times \mathbf{H} = \mathbf{J}, \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} = 0, \end{cases}$$



- Boundary: flux wall Γ_D, gate Γ_N

- Material relations $\mathbf{J} = \sigma \mathbf{E}$ and $\mathbf{H} = \nu(|\mathbf{B}|)\mathbf{B}$ with $\nu := \mu^{-1}$, $B := |\mathbf{B}|$

$$\nu(B) = \begin{cases} \nu_{\mathrm{Fe}}(B), & \text{in } D_{\mathrm{Fe}}, \\ \nu_0, & \text{in } D/D_{\mathrm{Fe}}, \end{cases}$$

- Assumptions: isotropy, anhysteretic curve

Eddy Current Equations

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- Magnetic vector potential formulation ($\mathbf{B} = \nabla \times \mathbf{A}$)

$$\sigma \frac{\partial}{\partial t}\mathbf{A} + \nabla \times (\nu(|\nabla \times \mathbf{A}|)\nabla \times \mathbf{A}) = \mathbf{J}$$

Eddy Current Equations

- Vector potential formulation (2D setting), time interval I_T

$$\begin{cases} \sigma \partial_t u - \nabla \cdot (\nu(|\nabla u|) \nabla u) = J, & \text{in } I_T \times D, \\ u = u_0 & \text{on } \{0\} \times D, \\ u = 0 & \text{on } I_T \times \Gamma, \end{cases}$$

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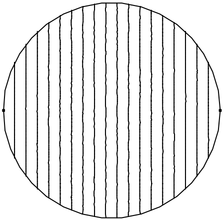
- Quantities of Interest (QOI): e.g., multipoles, inductance, ...

$$Q = \int_{I_T} \int_D q(u, \nabla u, \partial_t u)\, dxdt.$$

Multipole Expansion

- Multipoles, circular contour (r_{ref}, θ)

$$B_r\,(r_{\text{ref}}, \theta) = \sum_{p=1}^{\infty} \left[b_p(r_{\text{ref}})\sin(p\theta) + a_p(r_{\text{ref}})\cos(p\theta) \right]$$



Dipole

with pole-pair number p , normal $b_p(r_{\text{ref}})$ and skew coefficients $a_p(r_{\text{ref}})$.

- Relative high-order coefficients (b_p/b_1 and a_p/b_1, $p > 1$) below 10^{-4}.

Multipole Expansion

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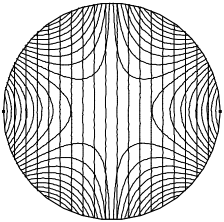
Dipole + Sextupole

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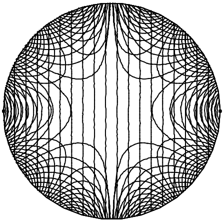
Dipole + Sextupole + Decapole

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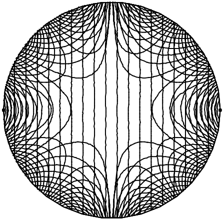

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- Relative high-order coefficients (b_p/b_1 and a_p/b_1, $p > 1$) below 10^{-4}.
- Multipole coefficients as QOI

$$Q_p = C \int_0^r \int_0^{2\pi} (u + r\partial_r u) \cos(p\theta)\, d\theta dr$$

Outline of the Talk

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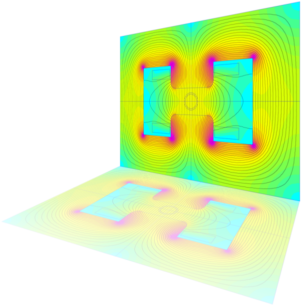

Modeling of Uncertainties by Independent Random Variables

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Probability space	(Ω, Σ, μ)	
Expectation	$\mathbb{E}\left[g\right] = \int_\Omega g(\omega)\, \mathrm{d}\mu(\omega)$	(1st moment)
Variance	$\mathrm{var}\left[g\right] = \mathbb{E}\left[\left(g(\omega) - \mathbb{E}\left[g\right]\right)^2\right]$	(2nd centered moment)
k-th moment	$\mathbb{M}^k\left[g\right] = \int_\Omega g(\omega)^k \mathrm{d}\mu(\omega)$	(k−th moment)

Random process $u(\cdot, \omega) = u(\cdot, \mathbf{Y}(\omega))$

Random variables $\mathbf{Y} = (Y_1, \dots Y_M), \quad \mathbf{Y} : \Omega \rightarrow \Pi \subset \mathbb{R}^M$

Realizations $\mathbf{Y}(\omega) = (Y_1(\omega), \dots, Y_M(\omega))$,
independent variables, e.g.
Gaussian $Y_i \sim \mathcal{N}(0, 1)$,
uniform $Y_i \sim \mathcal{U}(-1, 1)$, ...

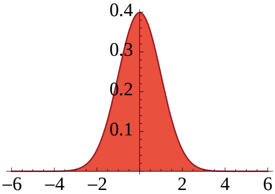

Figure: PDF of $\mathcal{N}(0, 1)$

Stochastic Eddy Current Equations

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- Weak formulation, for all $t \in I_T$, find $u \in L^2(I_T, H_0^1(D))$,

$$\int_D \sigma \partial_t uv - \int_D \nu(|\nabla u|) \nabla u \cdot \nabla v = \int_D Jv, \quad \forall v \in H_0^1(D).$$

Stochastic Eddy Current Equations

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- Weak stochastic formulation, for all $t \in I_T$, find $u \in L^2(\Omega) \otimes L^2(I_T, H_0^1(D))$,

$$\mathbb{E}\Big[\int_D \sigma \partial_t u v - \int_D \nu(\cdot, |\nabla u|) \nabla u \cdot \nabla v \Big] = \mathbb{E}\Big[\int_D J v \Big], \quad \forall v \in L^2(\Omega) \otimes H_0^1(D).$$

- Scalar material law $f(\omega, |\cdot|) = \nu(\omega, |\cdot|)|\cdot|$, random field, $f : \Omega \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$

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- Here, deterministic shape D, source current J

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- Finite dimensional noise for f, u, Q, i.e., $f(\omega, \cdot) = f(\mathbf{Y}(\omega), \cdot)$

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- Here, deterministic shape D, source current J
- Finite dimensional noise for f, u, Q, i.e., $f(\omega, \cdot) = f(\mathbf{Y}(\omega), \cdot)$
- High-dimensional parametric-*deterministic* problem $u(\mathbf{Y}, t, \mathbf{x})$

Uncertainties in the Material Model

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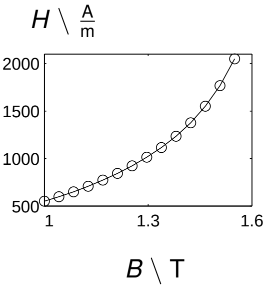
- Material relation from measurement

$$(\mathbf{B}^{\text{ms}}, \mathbf{H}^{\text{ms}}) \rightarrow H = f(B).$$



[1] R. Ramarotafika, A. Benabou, S. Clénet, IEEE Transactions on Magnetics, 2012

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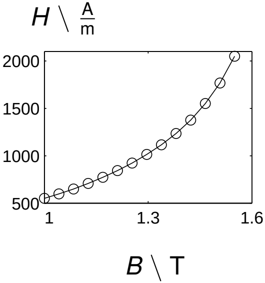
  $\alpha \leq \partial_B f(\omega, B) \leq \beta$ a.s.



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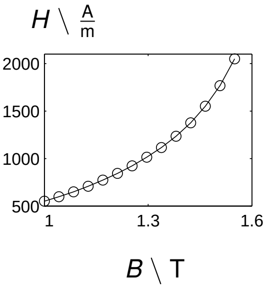

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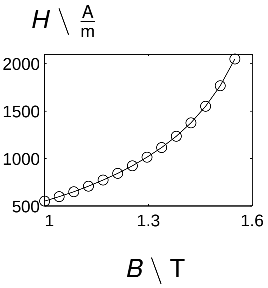

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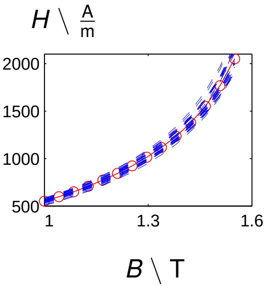

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Discretization of Random Field
$\rightarrow$ Karhunen-Loève Expansion

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- $\mathbb{E}_f$ and cov_f known
- Eigenvalue problem given by covariance

$$\lambda\varphi(x) = \int \text{cov}_f(x,y)\varphi(y)\,\text{d}y$$

Discretization with B-splines of degree p, $\mathbb{S}_N^p$

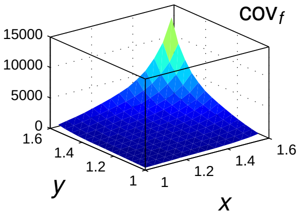
$$\lambda_N \mathbf{M}\varphi_N = \mathbf{K}\varphi_N$$

- Truncated Karhunen-Loève expansion

$$f_{M,N} \approx \mathbb{E}_f + \sum_{i=1}^{M} \sqrt{\lambda_{N,i}}\varphi_{N,i} Y_i$$

- cov_f smooth, uniform convergence

$$\|f - f_{M,N}\|_{L^\infty(\Omega, C^1(I))} \lesssim F(N,M) \rightarrow 0$$



- (scaled) fitted Gauß kernel

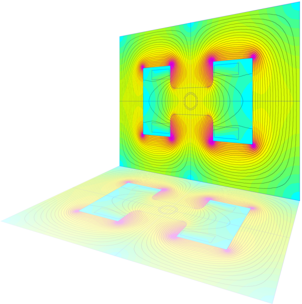
$$\text{cor}_f(x,y) = \text{e}^{-\left(\frac{x-y}{L}\right)^2}$$

Correlation length L

- Eigenvalue decay justifies

$$M \approx 2$$

Outline of the Talk



Methods for Uncertainty Quantification: First Order k-th Moment

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- First Order k-th moment method
  - *Small* perturbation $\mathbf{Y} = \overline{\mathbf{Y}} + s\tilde{\mathbf{Y}}$, approximate moments by linearization of QOI

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  - Stochastic Taylor expansion

$$Q(\mathbf{Y}) = Q(\overline{\mathbf{Y}}) + s\,\partial_Y Q(\overline{\mathbf{Y}}) + \mathcal{O}(s^2)$$

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$$\mathbb{E}[Q] \approx \overline{Q} \qquad \text{and} \qquad \text{var}[Q] \approx s^2 \sum_{i=1:M} \text{var}[\tilde{Y}_i] \left(\frac{Q(\overline{\mathbf{Y}} + h\mathbb{1}_i) - \overline{Q}}{h} \right)^2$$

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- M large: adjoint problem

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Asymptotics of Perturbation Method
Magnetostatic Analysis

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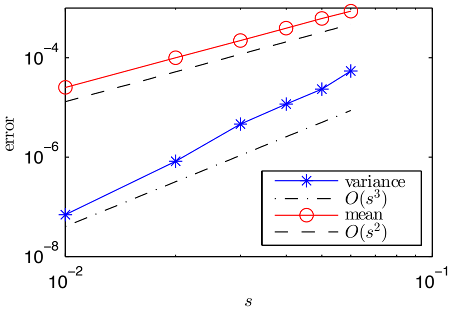
- Convergence study[1] of the perturbation method
- Error in mean and variance
- Slope 1.98 (mean), slope 3.6 (variance)



[1] U. R., S. Schöps, T. Weiland, IEEE Transactions on Magnetics, 2014

Methods for Uncertainty Quantification: Stochastic Quadrature

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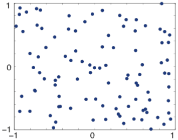
- Quadrature

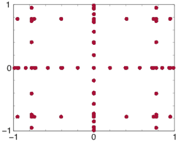
$$\mathbb{E}\left[Q(\mathbf{a})\right] \approx \sum_{k=1:N} w_k \, f\left(\mathbf{A}(\mathbf{Y}^{(k)})\right)$$

$$\mathrm{var}\left[Q(\mathbf{a})\right] \approx \sum_{k=1:N} w_k \, \left(f\left(\mathbf{A}(\mathbf{Y}^{(k)})\right) - \mathbb{E}\left[f(\mathbf{a})\right]\right)^2$$

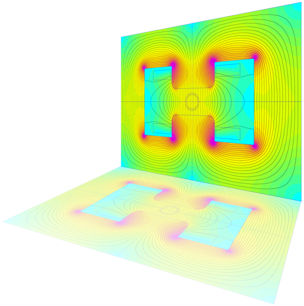
- Stochastic approach: Monte Carlo
  - large number N of samples, i.e., full simulations
  - convergence independent of number of variables
  - each simulation has equal weight $w_k = 1/N$
  - samples are stochastic, i.e., obtained by random generator



- Deterministic approach: Collocation
  - efficient for a small number of random variables
  - approximation of PDF with generalized Polynomial Chaos
  - weights w_k and samples are determined by quadrature rules e.g. Gauss-Hermite for normally distributed variables



Outline of the Talk



Standard Deviation Multipole Coefficients

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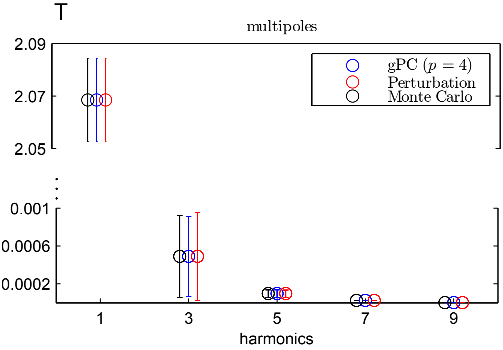
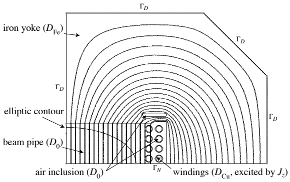
- Standard deviation of multipole coefficients (MC, Perturbation and gPC)



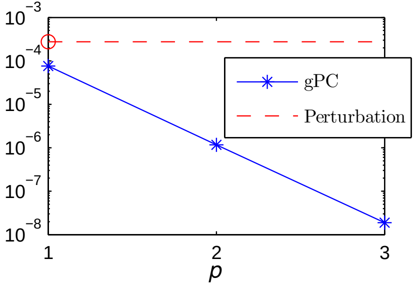


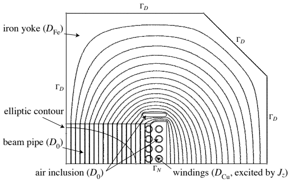
- input uncertainties up to $\approx$ 15%

Computational Cost

- Error w.r.t. polynomial chaos solution ($p = 4$)

error in std b_1





- $(p+1)^M$ eval. (gPC)
- $M+1$ eval. (Perturbation)

FEM Computation of Multipole Coefficients

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- Multipoles and UQ: high accuracy vs. cheap evaluation
- Approximation of multipole b_p involves two steps:
  - FEM solution u_h
  - Evaluate $b_{p,h} = \int_{D_{\text{obs}}} q(u_h, \nabla u_h)\, d\mathbf{x}$
- Consider error splitting: $\epsilon = \epsilon_{\text{loc}} + \epsilon_{\text{pol}}$, where local error stems from

$$\begin{aligned} -\nabla \cdot (\nu_0 \nabla u_{\text{loc}}) &= 0, \quad \text{in } D_{\text{obs}}, \\ u_{\text{loc}} &= u, \quad \text{on } \partial D_{\text{obs}} \end{aligned}$$

- Convergence order for local error and lowest order FEM

$$\left| b_p - b_{p,h} \right| = \mathcal{O}(h^2)$$

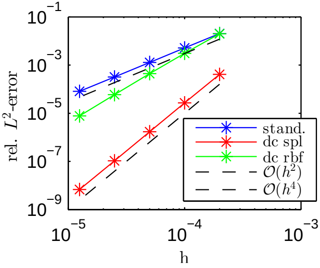
- Accelerate convergence through defect correction[1] $\rightarrow \mathcal{O}(h^4)$

[1] M. Giles, E. Süli, Acta Numerica 2002

Defect Correction

- Defect correction steps
  - Post-process the FEM solution u_h^*, e.g., spline interpolation
  - Compute defect $u_h - u_h^*$
  - One additional iterate within Newton scheme

- Simple example for Laplace equation
- Radial basis functions (rbf): unstructured meshes, but lower convergence expected
- Splines: local structured meshes, full convergence



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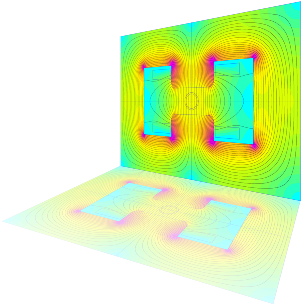

Summary and Conclusions

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- Uncertainties of eddy current problem w.r.t. material data
- KL representation of random field with shape constraints
- Uncertainty vs. high-dimensional quadrature
- Perturbation method for cheap uncertainty propagation
- Method can be nonintrusive
- Predicted asymptotic convergence in s observed

Summary and Conclusions

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Thank you for your attention

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- Federal Ministry of Education and Research (BMBF) – SIMUROM (05M2013)
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Questions? Now or via e-mail...

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Numerical Example

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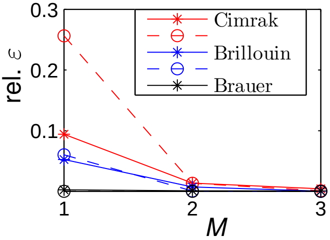
- comparison to parametric/stochastic models [1,2,3]
- e.g., stochastic Brauer model [3]

$$f(\mathbf{Y}, x) = x(Y_1\, e^{Y_2 x^2} + Y_3)$$

- model $Y_i \sim (1 + 0.05\mathcal{U}(-1,1))$
- define error $\varepsilon = \left\| f - f_{M,N} \right\|_{L^2(\Omega \times I)}$



[1] E. Rosseel, H. De Gersem, S. Vandewalle, Communications in Computational Physics, 2010
[2] I. Cimrak, SIAM Journal on Numerical Analysis, 2012
[3] J.R. Brauer, IEEE Transactions on Magnetics, 1975

Discrete Approximation and Cost

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$$\mathbb{E}[\partial_Y Q(\mathbf{Y})^2] = \int_{I_T}\int_{D^{Fe}}\int_{I_T}\int_{D^{Fe}} Cor_{\tilde{\nu}}(B(t,x),B(s,y))\alpha(t,x)\alpha(s,y)\; dxdt\; dyds$$

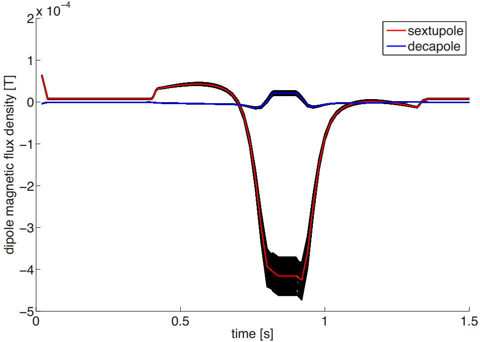
- For KL we have

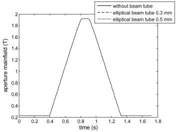
$$Cor_{\tilde{\nu}}(B(t,x),B(s,y)) \approx \sum_{i=1}^{M} \lambda_i \hat{\varphi}_i(B(t,x))\hat{\varphi}_i(B(s,y))$$

- Discretization with lowest order FEM and Backward Euler in time
- Cost Integral: Direct $\mathcal{O}(N_h^2 N_{\Delta t}^2)$, $\rightarrow$ Monte-Carlo integration

SIS100 Multipole Coefficients
Time Domain Analysis

- Time domain gPC analysis of ramping[1]





- simplified model of small material perturbation (Brauer's model)
- uncertainty of sextupole: $\max_t(\mathbb{E} \pm \text{std}) \approx 10^{-4} \pm 10^{-5}$ T

[1] H. De Gersem, S. Koch, S. Y. Shim, E. Fischer, G. Moritz, and T. Weiland, IEEE Transactions on Applied Superconductivity, 2008