Thermo-electro-dymanic simulations on SC wires & cables

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- CUDI
  - Electrical model
  - Thermal model
  - Solving algorithm
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CUDI

In the LHC design phase (~1990) a good electro-dynamic model of a Rutherford cable was needed in order to understand the field errors caused by inter-strand coupling currents, while ramping up and down the magnets.

Later, the model was extended to also investigate non-uniform current distribution, so-called boundary-induced coupling currents, and magnetization/snap-back/decay.

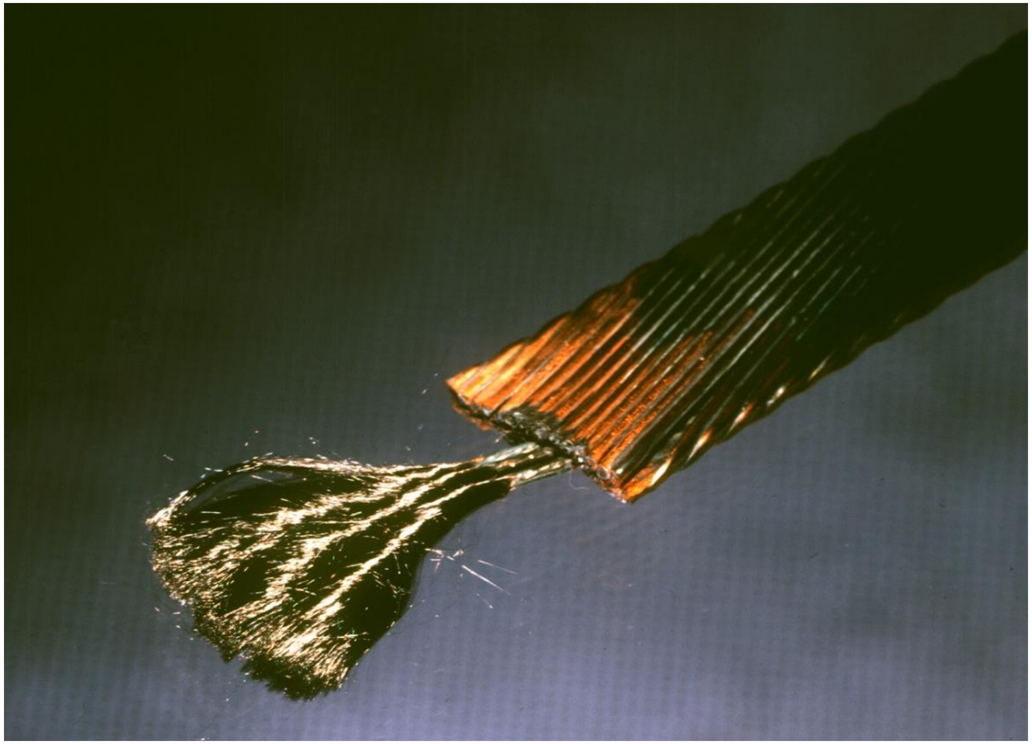
Around 2005 the model was extended with a thermal part, and also used for stability (MQE) calculations.

The code is written in Fortran 90 and has a LabView-based user input.

Nowadays it is the only existing code to calculate the electro-dynamic and thermal behaviour across the width and along the length of any type of Rutherford cable, subject to global and/or local variations in field, transport current, and heat release.

Rutherford cable

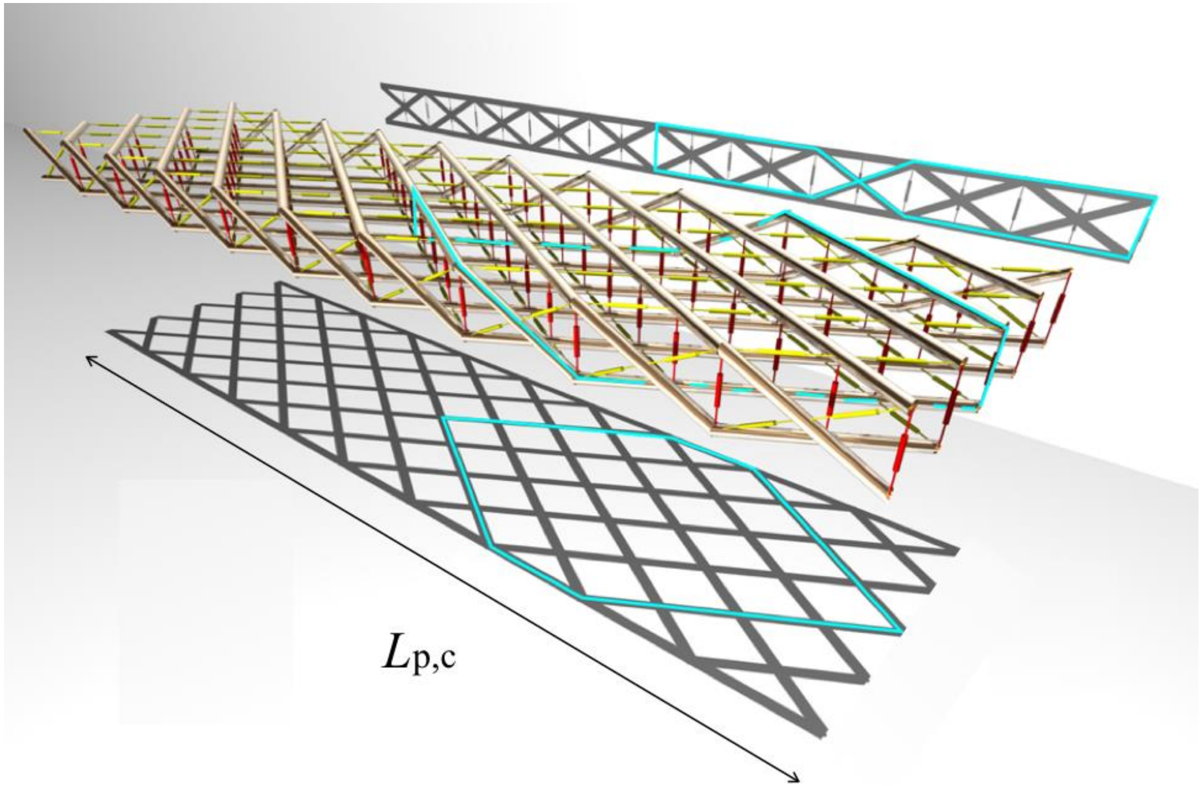
A Rutherford cable is a fully transposed multi-strand compacted cable with a rectangular, or slightly keystoned, cross-section.



CUDI: Electrical model

Typically:

- 20-40 strands/wires (N_S)
- Width of about 8-20 mm (w)
- Thickness of about 1-3 mm (t)
- Cable pitch of 0.1 m (L_P)
- Compaction of 85-95%



CUDI: Electrical model

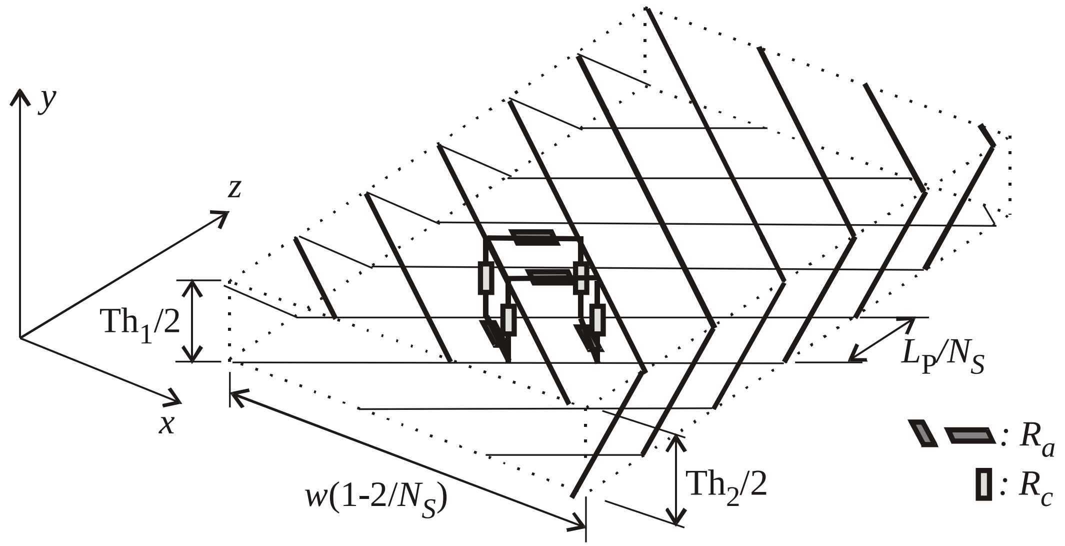
One twist pitch has N_S bands. Discretization length = L_P/N_S (usually a few mm).

Each band has:

- $2N_S$ unknown currents between adjacent strands C_A
- N_S-1 unknown currents between crossing strands C_C
- $2N_S$-2 unknown strand currents C_S

So a cable with length L_{cab} has (L_{cab}/L_P)*(5NS-3) unknows.

Imagine L_{cab}=10 m, L_P=0.1 m, N_S=30. This gives 14700 unknowns (Q).



CUDI: Electrical model

The Q unknowns are solved by setting up Q equations based on Kirchhoff's laws, and solving a Q^2 matrix.

In the nodes: $C_a + C_c + C_s = 0$

In a loop:

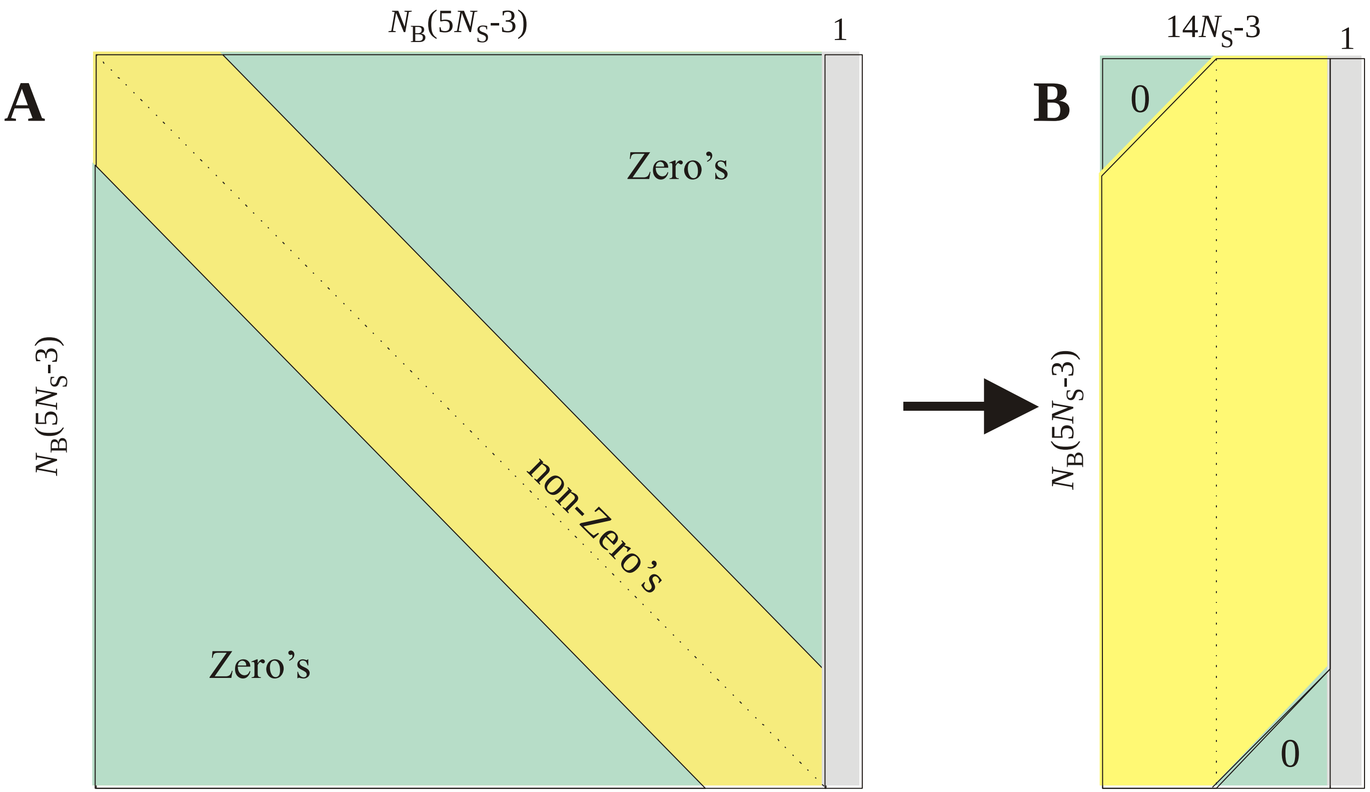
$$\sum C_a R_a + \sum C_c R_c + \sum C_s R_s + \sum V_{ind} = A \times \frac{\partial B}{\partial t}$$

$$\sum C_a R_a + \sum C_c R_c + \sum C_s R_s + \sum M \frac{C_s^{<m>}}{\Delta t} = A \times \frac{\partial B}{\partial t} + \sum M \frac{C_s^{<m-1>}}{\Delta t}$$

Since R_s has a strongly non-linear current dependency, $\sim(C_s/I_C)^n$, iteration is needed as soon as one or more strand currents approach I_C.

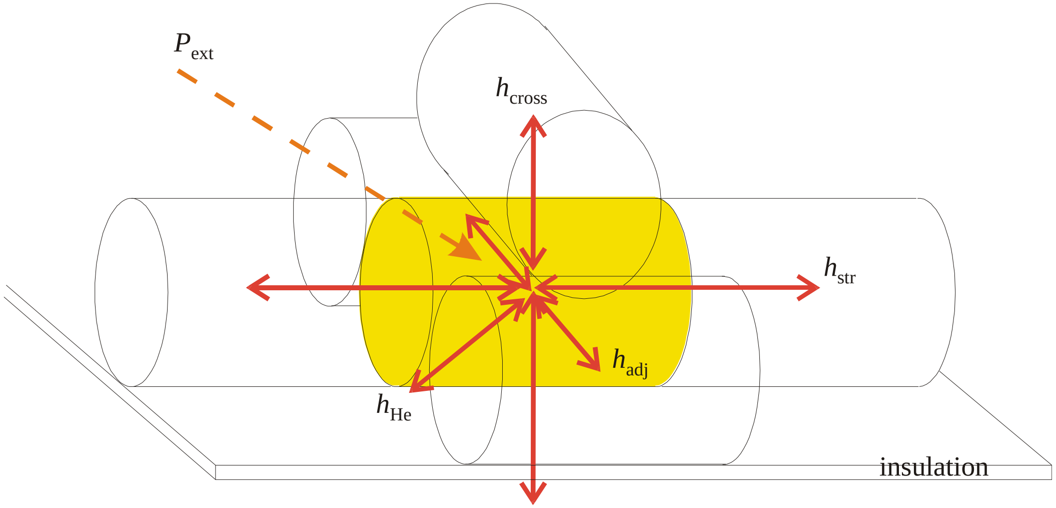
CUDI: Solving the electrical model

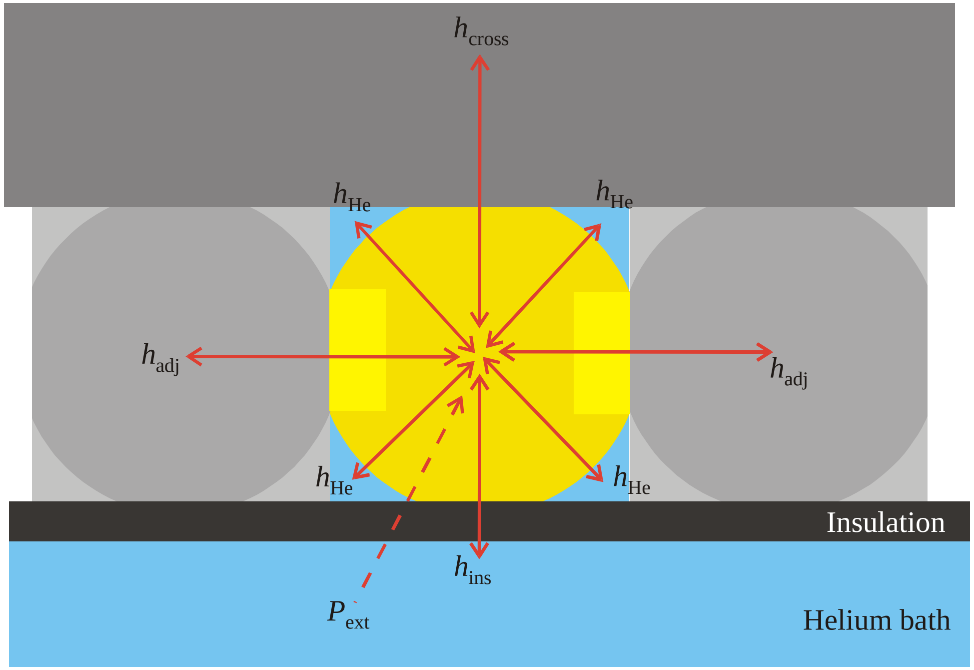
For steady-state condition there is “band reduction” since most of the matrix contains zero (because M=0).



Transient conditions are calculated by only considering mutual inductances over limited lengths, so still profiting from “band reduction”. Sometimes it is faster to move the entire inductive term to the right hand side of the matrix and solve the matrix iteratively.

CUDI: Thermal model





CUDI: Thermal model

Heat balance for strand section:

$$V_s C_{P,s} \frac{\Delta T_s}{\Delta t} = F_{IS}(P_a + P_c) + P_s + P_{IF,s} + P_{ext} - h_{str} - h_{adj} - h_{cross} - h_{He} - h_{ins}$$

Heat balance for helium volume adjacent to strand section:

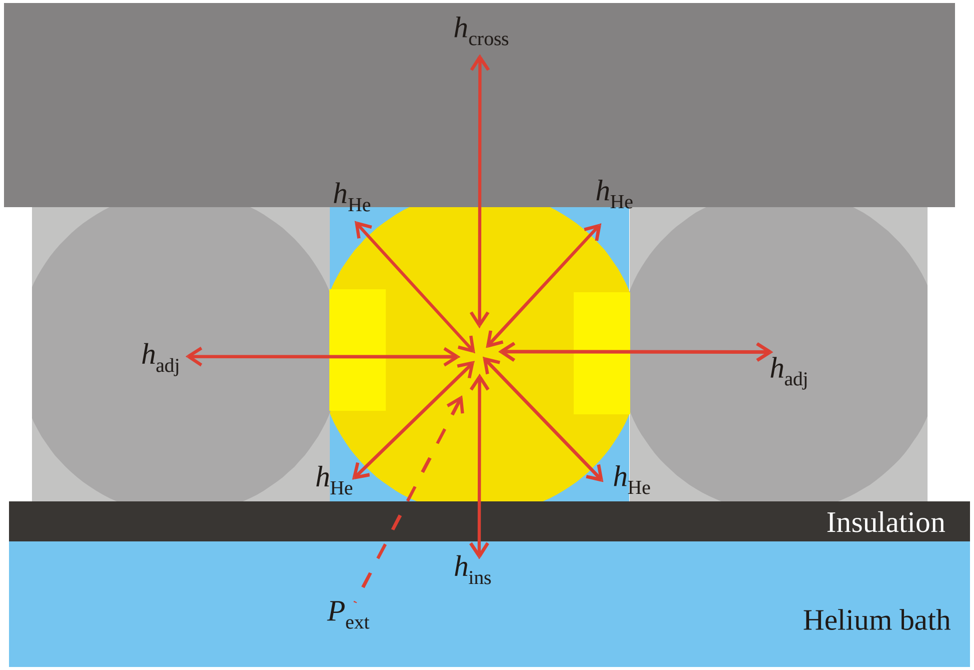
$$V_{He} C_{V,He} \frac{\Delta T_{He,s}}{\Delta t} = (1 - F_{IS})(P_a + P_c) + h_{He}$$

$$h_{Str} = F_{h,str}(k_{mat}S_{mat} = k_{SC}S_{SC})\left[\frac{T_s - T_{s-n1}}{d_1} + \frac{T_s - T_{s-n2}}{d_2}\right]$$

$$h_{adj} = F_{a1}(T_s - T_{s-a1}) + F_{a2}(T_s - T_{s-a2})$$

$$h_{cross} = F_{cross}(T_s - T_{s-cr})$$

$$h_{ins} = k_{ins}(T_s - T_{bath})\frac{F_{ins}A_{ins}}{Th_{ins}}$$



CUDI: Thermal model

The heat transfer from the strand surface to normal or superfluid helium (h_{He}) is complicated, and usually passes through several cooling regimes, such as:

- Kapitza cooling
- Film boiling to HeII
- Natural convection
- Nucleate boiling
- Film boiling to HeI

Usually the heat transfer in a given regime is limited to a maximum value h_{max}.

The user can himself decide which regimes should be taken into account, set the parameters in the heat transfer equations, and define h_{max} for each regime.

CUDI: Solving algorithm

- $R_s = f(I_s, T_s, B)$
- $I_s = f(R, \ldots)$
- $T_s = f(I^2R_s, h_{str}, h_{adj}, h_{cross}, h_{He}, h_{ins}, P_{ext}, P_{IF})$

So there is a very very strong coupling between the elctrical and thermal model.

To be considered:

- Explicit method, so $Y(t+\Delta t)=f(Y(t))$. This works but requires in many cases extremely small time steps.
- Implicit method, so $Y(t+\Delta t)=f(Y(t), Y(t+\Delta t))$

CUDI uses the implicit method.

CUDI: Solving algorithm

- Loop for parametric sweep
  - Iterate with different h_{ext} to calculate the Minimum Quench Energy
    - Loop through all time steps
      - Iterate until the time step is small enough
        - Iterate until the electrical module has converged
          - Iterate until the thermal module has converged
            - Loop over all the bands of the cable
              - Loop over all the strand sections of a band
                - Iterate until the temperatures of the section and the adjacent helium have converged

CUDI: Numerical problems

Stability issues:

- The normal-SC transition is strongly current dependent, $\sim(I/I_C)^n$, with n typically 30.
- The normal-SC transition is strongly temperature dependent. The electrical and thermal module are therefore strongly coupled.
- The heat capacity of metals at low temperature is extremely small.
- The heat transfer to helium is strongly temperature dependent.
- The heat transfer to superfluid helium can be very large.
- The contact resistances between strands can be very small, so that current can easily move from one strand to another.
- Adjacent strands have a strong mutual coupling.

CUDI: Physical problems

- The size and shape of the helium voids is poorly known.
- The heat transfer to helium is poorly understood, especially the transient one.
- The helium flow through the porous cable insulation is poorly known. Should we use a constant helium pressure in the voids or a constant helium mass?
- The heat transfer along the voids is poorly understood and at present not implemented.
- Many parameters vary strongly across and along the cable, such as contact resistances, strand surface condition, RRR, inter-strand heat flow, …

CUDI: final remarks

Dynamic time stepping is very important because many parameters change several orders of magnitude between 2-10 K.

Steady state and transient cases for constant temperature can usually be solved for long length cables.

Transient cases with varying temperature are limited to short lengths, typically a few twist pitches.

We do not have a model of a soldered joint between two Rutherford cables or between a Rutherford cable and a normal conducting current lead. Defining proper boundary conditions is therefore not always straightforward.

The model is in principle limited to a single cable, or a coil assuming no interaction between turns.

QP3

In the LHC design phase (~1990) several codes were developed to calculate quench behaviour of superconductors, assuming homogeneous parameters over the cross-section.

During the construction of the LHC these codes fell into oblivion, and/or their authors/users quit CERN. There was also a lack of proper documentation.

During start-up of the LHC a similar code was again required. At that time I developed QP3, basically a single-strand version of CUDI.

QP3 is mainly used to calculate quench propagation, thresholds for quench protection of busbars & splices, and thresholds for Beam Loss Monitors. The code was also used to explain the accident in the LHC in 2008 when we opened the RB circuit.

The code is written in Fortran 90 and has a LabView-based user input.

QP3

The electrical and thermal models are much simplified w.r.t. CUDI because there is only one conductor in stead of several tens.

$\Rightarrow$ no inter-strand currents

$\Rightarrow$ no inter-strand heat flows

The strand/wire resistance is calculated based on the actual (known) current and the temperature of the **past** time step. This is valid as long as the temperature difference between time steps is small. If needed, time steps are automatically reduced.

QP3: Solving algorithm

- Loop for parametric sweep
  - Iterate with different h_{ext} to calculate the Minimum Quench Energy
    - Loop through all time steps
      - Iterate until the time step is small enough
        - ~~Iterate until the electrical module has converged~~
        - Fix R_s based on previous time step (explicit method)
          - Iterate until the thermal module has converged
            - Loop over all the sections of the conductor
              - ~~Loop over all the strand sections of a band~~
                - Iterate until the temperatures of the section and the adjacent helium have converged