Charm Decays
and
Quantum Coherence

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Charm Decays and Quantum Coherence

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Rare/Forbidden Decays
Not supposed to see them according to the SM.
Then look for them, just in case!

(semi-) Leptonic Decays
Extractions of $|V_{cx}|$, and decay constants.

Hadronic Decays
i.e. BF of $D_s$ decays (often, important inputs in B decays).
Charm productions in continuum (i.e., $\psi(3770) \rightarrow \bar{D}D$ line shape).

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Charm Decays and Quantum Coherence

Use the quantum-correlated Charm mesons
- usually produced at mass threshold $\rightarrow$ relevant experiments: CLEO-c/BESIII.
- can extract $D\bar{D}$ mixing parameters.
- can also help the $\gamma/\phi_3$ measurement (i.e., via the GGSZ method).
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Charm Decays

I will briefly go through some of the recent experimental results in these three topics today

Quantum Coherence

Use the quantum-correlated Charm mesons
- usually produced at mass threshold $\rightarrow$ relevant experiments: CLEO-c/BESIII.
- can extract $D\bar{D}$ mixing parameters.
- can also help the $\gamma/\phi_3$ measurement (i.e., via the GGSZ method).
Outline

1. Rare/Forbidden searches
   - Experimental limits are starting to reach $\text{BF} \sim 10^{-9}$ and starting to overlap some non-SM predictions.
   - Will go through two recent results on FCNC transitions.

2. Leptonic and semi-leptonic decays.
   - Access to CKM matrix elements, $V_{cd(s)}$.
   - Will go through recent results on $D^0$ and $D^+$ decays.

3. Quantum-Correlated Charm analyses
   - Provide access to the mixing parameters.
   - Can also contribute the $\gamma/\phi_3$ measurement.
   - Will go through the recent measurements of $\delta_{K\pi}$, $\gamma_{cp}$, as well as the $c_i$ and $s_i$. 
All results are from Modern Heavy Flavor Factories

Belle Detector
- SC solenoid 1.5T
- CsI(Tl) 16X0
- TOF counter
- 3.5 GeV e+
- Central Drift Chamber
  small cell + He/C2H6
- Si vtx. det.
  3 yr. DSSD
- μ/KA detection
  14/15 yr. RPC+Fe

BABAR Detector

CLEO-c
- SC Quadrupole Pylon
- Rare Earth Quadrupole
- Iron Polypiece
- Iron Polepiece
- SC Solenoid
- Barrel Calorimeter
- Endcap Calorimeter
- Drift Chamber
- Inner Drift Chamber
  Beam pipe
- Wire tracker (no Si)
  TOF + dE/dx for PID
  Si End.: RPC muon
**Why Rare Charm Decay?**

- Charm is unique. FCNC transitions are highly suppressed in the SM.
  - mediated by the lighter down-quark sector.
  - more effective GIM suppression here than in B decays.

- That is, the “SM noise” is much lower in Charm! (non-existent at current experimental limits)
  Of course, this does not mean the signature of new Physics (NP) is larger in Charm, however.

- Observing or even NOT observing rare decays help to constrain effects from NP.
\[ D^0 \rightarrow \mu^+\mu^- \]

- Expect small short distance contribution: \( B(D^0 \rightarrow \mu^+\mu^-) \sim 10^{-18} \).
- The long distance might be dominated by the two photon intermediate state;
  \( B(D^0 \rightarrow \mu^+\mu^-) \sim 2.7 \times 10^{-5} \times B(D^0 \rightarrow \gamma\gamma) \) (PRD66, 014009 (2002)).
  If we take \( B(D^0 \rightarrow \gamma\gamma) < 2.2 \times 10^{-6} \) @90% C.L. (PRD85, 091107 (2012)), then \( B(D^0 \rightarrow \mu^+\mu^-) < \sim 6 \times 10^{-11} \).

- LHCb (PLB725, 15 (2013))
  - \( B(D^0 \rightarrow \mu^+\mu^-) < 6.2 \times 10^{-9} \) @ 90% C.L.
  - Still some room to reach the prediction
  - Some BSM predict BF \( \sim 10^{-10} \)
    (R-parity violation, PRD66, 014009 (2002);
    Warped extra dimensions, PRD90, 014035 (2014))

- More data at LHCb would be very interesting!
\( D^0 \rightarrow \gamma\gamma \)

- Forbidden by the tree level.
- Short distance: \( BF \sim 10^{-11} \) (PRD66, 014009)
- Long distance: (VMD, HQχPT): \( BF \sim 10^{-8} \) [(PRD66, 014009 (2002)), (PRD64, 074008 (2001))]
- MSSM could enhance the rate up to \( \sim 10^{-6} \) (c \( \rightarrow \) u \( \gamma \) via gluino exchange) (PLB500, 304 (2001)).
- BaBar (PRD85, 091107(R) (2012)):
  - Reconstruct through \( D^{*+} \rightarrow D^0(\rightarrow \gamma\gamma)\pi^+ \), normalized by \( D^{*+} \rightarrow D^0(\rightarrow K_S\pi^0)\pi^+ \).
  - Peaking background from \( D^0 \rightarrow \pi^0\pi^0 \).
  - \( B(D^0 \rightarrow \gamma\gamma) < 2.2 \times 10^{-6} \) @ 90% C.L.
Experimental status in Charm decays - I (from HFAG 2014)

• LHCb ULs are now reaching ~$10^{-7}$-$10^{-8}$ level.
• B factories are doing well, reaching $10^{-6}$-$10^{-7}$ level. Looking forward to the Belle II!
Experimental status - II

BF(D⁺ → X)

BF(Dˢ⁺ → X)

E687 E653 E791
BaBar Focus CLEO
LHCb D0

E653 E791 CLEO
BaBar Focus LHCb
Experimental status - III

• So far, no surprises.
• LHCb upgrade and Belle II are on the horizon.
• Should be able to see some of the listed rare decays soon
  ... or we may see a surprise!?
Recent results in leptonic and semi-leptonic decays of Charm mesons
Leptonic decays $D_{(s)}^+ \rightarrow \ell + \nu_{\ell}$

\[
\Gamma(D^+ \rightarrow \ell^+ \nu_{\ell}) = \frac{f_D^2 |V_{cd}|^2 G_F^2}{8\pi m_\ell m_D^2} \left(1 - \frac{m_\ell^2}{m_D^2}\right)^2
\]

- With the knowledge of $|V_{cd(s)}|$, extract the decay constant, $f_{D(s)}$ → compare to the Lattice QCD → validate the Lattice QCD calculations in $f_{B(s)}$.

- Or vice versa: Taking the calculated $f_{D(s)}$, extract $|V_{cd(s)}|$ to help to over-constrain the CKM unitarity.

- Also interesting is:
  \[
  \Gamma(D^+ \rightarrow \tau^+ \nu_\tau) : \Gamma(D^+ \rightarrow \mu^+ \nu_\mu) : \Gamma(D^+ \rightarrow e^+ \nu_e) = 2.67 : 1 : 2.35 \times 10^{-5}
  \]
  comes with the minimal uncertainties.
  (masses of the meson and the lepton)
  But $D^+ \rightarrow \tau^+ \nu_\tau$ has not been seen, yet.
  BF<$1.2 \times 10^{-3}$ @90% CL: CLEO PRD78,052003 (2008)

Notice: this UL is $\sim 3.14 \times \text{BF}(D^+ \rightarrow \mu^+ \nu_\mu)$. Could BESIII see this?
\[ D^+ \rightarrow \mu^+ \nu_\mu \]

- BESIII (PRD89, 051104(R) (2014)) : 2.9 fb\(^{-1}\) at \( E_{cm} = 3.773 \) GeV.
- Measured \( B(D^+ \rightarrow \mu^+ \nu_\mu) = (3.71\pm0.19\pm0.06)\times10^{-4} \)
  The most precise measurement to date.
  - With \( |V_{cd}| \) of CKM-fitter input, \( f_{D^+} = (203.2\pm5.3\pm1.8) \) MeV
  - With \( f_{D^+} \) of LQCD input (PRL100, 062002 (2008))
    \[ |V_{cd}| = 0.2210\pm0.0058\pm0.0047. \]
- Statistically limited.
  More data would be welcome.
- BESIII plans to take
  \( \sim 10 \) fb\(^{-1}\) in the future!
Comparison of $B(D^+ \rightarrow \mu^+ \nu_\mu)$ and $f_{D^+}$

Good consistencies are seen among the previous experimental results.
Comparison of $B(D_s^+ \rightarrow (\mu^+/\tau^+) \nu_\mu$)

- Similar results in the Charmed strange meson.
- Good overall consistency in BFs.
Comparison of $f_{D_s}$

- Reasonable consistencies.

- BESIII plans to have a dedicated $D_s$ data taking in the near future.
Semi-Leptonic decays $D_{(s)}^+ \rightarrow P + l + \nu_l$

$\frac{d\Gamma(D \rightarrow K(\pi)e\nu)}{dq^2} = \frac{G_F^2 |V_{cs(d)}|^2 P_K^3(q^2) f_+(q^2)}{24\pi^3}$

$q^2 = (p_1 + p_\nu)^2 \Rightarrow M_{inv}^{lepton pair}$

- Essentially measure $|V_{cd(s)}| \times |f(q^2)|$.
- Input $|V_{cs(d)}| \rightarrow \text{extract} |f(q^2)| \rightarrow$ compared to the LQCD. Validating the FF calculations of LQCD here is important i.e., the measurement of $|V_{ub}|$ via $B \rightarrow \pi l \nu$ has a large dependence on the “theoretical input (its FF)” from LQCD.
- Or vice versa:
  Input $|f(q^2)|$ from LQCD $\rightarrow \text{extract} |V_{cs(d)}|$ $\rightarrow$ constrain the CKM unitarity.
\(D^0 \rightarrow K/\pi\ e^+\ \nu_e\)

- BESIII: 2.9 fb\(^{-1}\) at \(E_{cm} = 3.773\) GeV.
- \(U_{\text{miss}} \sim 0\) if the missing particle is a neutrino.
- The resultant BFs are consistent with the previous measurements (see the next slide).

Most precise to date.
Comparison of $B(D^0 \rightarrow (K/\pi)^- e^+ \nu_e)$

\[ \frac{\Gamma(K^- e^+ \nu) / \Gamma_{total}}{\Gamma(\pi^- e^+ \nu) / \Gamma_{total}} \]

\begin{align*}
(3.4\pm 0.5 \pm 0.4)\% & \quad \text{MARK-III (1989)} \\
(3.82\pm 0.40 \pm 0.27)\% & \quad \text{BES-II (2004)} \\
(3.45\pm 0.10 \pm 0.19)\% & \quad \text{BELLE (2006)} \\
(3.50\pm 0.03 \pm 0.04)\% & \quad \text{CLEO-c (2009)} \\
(3.50\pm 0.014 \pm 0.033)\% & \quad \text{BESIII Preliminary} \\
(3.53\pm 0.27 \pm 0.43 \pm 0.05)\% & \quad \text{E691 (1989)} \\
(3.49\pm 0.23 \pm 0.23 \pm 0.05)\% & \quad \text{CLEO (1991)} \\
(3.80\pm 0.10 \pm 0.17 \pm 0.05)\% & \quad \text{CLEO2 (1993)} \\
(3.60\pm 0.03 \pm 0.05 \pm 0.05)\% & \quad \text{BaBar (2007)} \\
(3.55\pm 0.05)\% & \quad \text{PDG13} \\
\end{align*}

$B[D^0 \rightarrow K^- e^+ \nu]$  \hspace{1cm} $B[D^0 \rightarrow \pi^- e^+ \nu]$
Comparison of form factors

- Points: BESIII
- Curves: Fermilab Lattice, MILC, and HPQCD (PRL94, 011601 (2005))
  Fermilab Lattice and MILC (PRD80, 034026 (2009))
  Based on the BK model (PLB478, 417 (2000))
- Consistent with each other.
- Would be nice to have an even larger sample to probe the higher $q^2$ bins.
Quantum Coherence in $e^+e^-$ annihilation near Charm mass threshold
The decay rate of a correlated state

At $E_{cm} \sim M(\psi(3770))$, a pair of $D^0 \bar{D}^0$ is produced via

$$e^+e^- \rightarrow \gamma^* (\rightarrow \psi(3770)) \rightarrow D^0 \bar{D}^0.$$ 

This obeys the following selection rules on the produced pair of D mesons.

- The two produced neutral mesons must have opposite CP (i.e., see Goldhaber and Rosner, PRD15, 1254 (1977).

For instance,

- $D^0 \rightarrow$ CP+ final states (such as $K^+K^-$) AND
- $\bar{D}^0 \rightarrow$ CP+ final states (such as $\pi^+\pi^-$) does NOT happen.

And (CP-, CP-) combo does not happen either.

- $D^0 \rightarrow$ CP- final states (such as $K_S\pi^0$) are maximally enhanced (doubled).

That is, the measured $BF_{\text{eff}}(D^0 \rightarrow K_S\pi^0)$ is twice as $BF(D^0 \rightarrow K_S\pi^0)$ with no such coherence effect on the parent D.
The decay rates in mixed CP final states

- $D^0 \rightarrow$ CP+ final states (such as $K^+K^-$) AND
  $\bar{D}^0 \rightarrow$ generically (not look at its decay experimentally).
  This decay rate (e.g., $D^0 \rightarrow K^+K^-$) is not affected.

- $D^0 \rightarrow$ Flavored final states (CF+DCSD, such as $K^-\pi^+$) AND
  $\bar{D}^0 \rightarrow$ CP± final states.

The rates are still affected due to the interference between CF and DCS.

-> extract $\delta_{K\pi}$, where $\langle K^-\pi^+|\bar{D}^0\rangle/\langle K^-\pi^+|D^0\rangle = -r \cdot e^{-i\delta}$.

For multi-body (such $K_\pi\pi^+\pi^-$), one can obtain the $\delta$, averaged over each bin of a Dalitz distribution.
The decay rates in semi-leptonic decays

- On the other hand, for the case of semi-leptonic decay, such as $D^0 \rightarrow K^- e^+ \nu$ (only the CF mode!) AND $\bar{D}^0 \rightarrow CP\pm$ final states, there is no interference. Its decay rate does not depend on the CP content of its parent $D$. Yet, the total width of its parent $D$ depends on CP.

For instance,

$$N(D^0 \rightarrow K^- e^+ \nu; \bar{D}^0 \rightarrow CP\pm)/N(\bar{D}^0 \rightarrow CP\pm) = B_{eff}(D^0 \rightarrow K^- e^+ \nu)$$

$$= B(D^0 \rightarrow K^- e^+ \nu) \times \Gamma/\Gamma_{CP\pm}$$

$$\approx B(D^0 \rightarrow K^- e^+ \nu) \times (1\pm y) \text{ (neglecting terms with } y^2 \text{ or higher).}$$

→ can extract the $y$ via semi-leptonic tags.
The latest measurement of $\delta_{K\pi}$ from BESIII

- In $D^0 \rightarrow K\pi$ decays, its CF and DCSD interfere. The ratio of the two amplitudes is $\langle K^-\pi^+|\bar{D}^0 \rangle / \langle K^-\pi^+|D^0 \rangle = -r \cdot e^{-i\delta}$.

- Neglecting higher orders in the mixing parameters (e.g., $y^2$), one can arrive at the following relation:

$$A_{CP \rightarrow K\pi} = r \cdot \cos \delta_{K\pi} + [D\text{-mixing correction (y and } R_{WS})]$$

where $A_{CP \rightarrow K\pi} = \text{CP-tagged rate asymmetry}$

$$= [B(D_2 \rightarrow K^-\pi^+)-B(D_1 \rightarrow K^-\pi^+)]/B(D_2 \rightarrow K^-\pi^+)+B(D_1 \rightarrow K^-\pi^+)].$$

- $B(D_{1,2} \rightarrow K\pi)$ can be measured by tagging one $D$ (tag side) with exclusive CP-eigenstates which then defines the eigenvalue of the other $D$. 

\[\text{CP tag at threshold.}\]
D → CP states
(no requirement on how the other D decays)

\[ M_{BC} = \sqrt{E_{beam}^2 - \vec{p}_D^2} \]

- PLB734, 227 (2014)
$D_{1,2} \rightarrow K\pi$, $D_{2,1} \rightarrow CP$ states

- Example fit for the case of $(K\pi, K_\pi\pi^0)$
- PLB734, 227 (2014)

- Measured $A_{CP \rightarrow K\pi} = (12.77 \pm 1.31 ({\text{stat.}}) \pm 0.33 - 0.31 ({\text{syst.}})) \%$.

- With external inputs from HFAG2013 and PDG (for $\gamma$ and $R_{WS}$)
  $\cos \delta_{K\pi} = 1.03 \pm 0.12 ({\text{stat.}}) \pm 0.04 ({\text{syst.}}) \pm 0.01 ({\text{external}})$.

- This result is consistent with and more precise than the recent CLEO-c result (PRD86, 112001 (2012)):
  $\cos \delta_{K\pi} = 1.15^{+0.19}_{-0.17} ({\text{stat.}}) ^{+0.00}_{-0.08} ({\text{syst.}})$. 
Could also determine the mixing parameter, $y_{CP}$

- $y_{CP}$ is defined as:

$$2 \cdot y_{CP} = (|q/p| + |p/q|) \cdot y \cdot \cos \phi - (|q/p| - |p/q|) \cdot x \cdot \sin \phi,$$

where $p$ and $q$ are mixing parameters, and $\phi = \arg(q/p)$ is the weak phase difference of the mixing amplitudes.

Notice: for no CPV case, $p = q = 1/\sqrt{2}$ and $y_{CP} \equiv y$.

- From the fact that semileptonic BF of $D_{1,2}$, $B(D_{CP \pm} \rightarrow l)$, gets modified by a factor of $1 \pm y_{CP}$, and neglecting terms with $y^2$ (or higher), one can arrive at

$$y_{CP} \approx \frac{1}{4} \left( \frac{B_{DCP^{-} \rightarrow l}}{B_{DCP^{+} \rightarrow l}} - \frac{B_{DCP^{+} \rightarrow l}}{B_{DCP^{-} \rightarrow l}} \right)$$
Extracting $\gamma_{CP}$ in BESIII data

BESIII preliminary result;

$\gamma_{CP} = [-1.6\pm1.3{\text{(stat.)}}\pm0.6{\text{(syst.)}}]\%$.

Most precise result based on QC Charm mesons. Having a larger sample would be a help.
Comparison with other measurements

- Our result is consistent with the world average (HFAG2013; this preliminary result is not included in the average).
- Also consistent with the latest result from CLEO-c (PRD86, 112001 (2012));
  \[ y_{CP} = (4.2 \pm 2.0 \pm 1.0)\% \]
  (not listed in the figure).
Can also contribute to the measurement of $\gamma/\phi_3$

- B factories can measure $\gamma/\phi_3$ through $B \rightarrow D K$.
- The latest comes from the LHCb (arXiv:1408.2748) via the GGSZ method in $D^0 \rightarrow K_S \pi \pi^-$ and $K_S K^+ K^-$. 
- Measured $\gamma = (62^{+15}_{-14})^\circ$
  (along with $r_B = 0.080^{+0.019}_{-0.021}$ and $\delta_B = (134^{+14}_{-15})^\circ$).
  A single most precise measurement of $\gamma$ to date.
- They needed inputs, $c_i$ and $s_i$:
  cosine and sine of the strong-phase difference between the $D^0$ and $\bar{D}^0$ decay, averaged in each Dalitz bin, $i$.
- Took the CLEO-c (statistically limited) results (PRD82, 112006, (2010)).
- BESIII has recently repeated this CLEO-c analysis based on their data which is $\sim 3.5 \times$ larger than that of CLEO.
Relations between $c_i$, $s_i$, and yields in Dalitz bins

- One could derive the following relations between efficiency-corrected yields in the $i^{th}$ Dalitz bins and $c_i$ ($s_i$) (see backups more details and PRD82, 112006 (2010)).

  - For the case of $D \rightarrow \text{CP states AND } D \rightarrow K_S \pi^+\pi^-$:
    \[
    \text{Yields in } i^{th} \text{ bin } \propto \pm c_i
    \]

  - For the case of $D \rightarrow K_S \pi^+\pi^- \text{ AND } D \rightarrow K_S \pi^+\pi^-$:
    \[
    \text{Yields in } i^{th} \text{ and } j^{th} \text{ bins of the two Dalitz plots } \propto c_i c_j + s_i s_j
    \]

- Simultaneously fit to these “Yields in each bin” to extract $c_i$ and $s_i$.

- One could also gain statistical power by employing $K_L \pi^+\pi^-$. 
For the case of “CP tag vs $K_S\pi^+\pi^-$”

- Data is using the full 2.9 fb$^{-1} \psi(3770)$ dataset
- Results presented here will be using Optimal Binning scheme.
Preliminary result

- Only statistical uncertainties are shown in the optimal binning scheme (which dominate in most of the bins).
- Consistent results with the previous CLEO-c measurement, but statistically superior.
- What this result could do to the $\gamma/\phi_3$ is, if we take the Belle’s Dalitz result (PRD85, 112014 (2012)),
  $\gamma$ (in degrees) = $77.3^{+15.1}_{-14.9}$ (stat.) $\pm 4.2$ (syst.) $\pm 4.3 (c_i/s_i) \rightarrow \pm 2.5 (c_i/s_i)$
  We expect the uncertainly would be reduced by $\sim 40\%$
- Very important inputs for the future analyses by LHCb and Belle II, where the statistical sensitivity starts to reach $\sim 1\sim 2$ degrees.
Summary

- Searches for rare/forbidden Charm decays are finally becoming interesting (exciting) with LHCb upgrade and Belle II on the horizon.

- Leptonic and semi-leptonic decays in Charm provide access to $|V_{cx}|$ and complementary to the B Physics. Having even larger Charm samples at BESIII improves the current results further.

- Quantum-correlated $D^0\bar{D}^0$ in $e^+e^-$ annihilations near threshold:
  - provides an unique way to measure the Charm mixing parameters.
  - also can provide precise measurements on $c_i$ and $s_i$. 
Backups
**D^0\bar{D}^0** mixing

- Observation of D\bar{D} mixing, first seen by the B factories (HFAG: arXiv 1207.1158) and now observed by LHCb: PRL110, 101802 (2013).

- D\bar{D} mixing is conventionally described by two parameters:

\[ x = \frac{2(M_1-M_2)}{\Gamma_1+\Gamma_2}, \quad y = \frac{(\Gamma_1-\Gamma_2)}{(\Gamma_1+\Gamma_2)}, \]

where \( M_{1,2} \) and \( \Gamma_{1,2} \) are the masses and widths of the neutral D meson mass eigenstates.

(Flavor eigenstates, D^0/\bar{D}^0, are not the same as mass eigenstates, D_1/D_2)

Or \( x' = x \cdot \cos \delta_{K\pi} + y \cdot \sin \delta_{K\pi}, \quad y' = y \cdot \cos \delta_{K\pi} - x \cdot \sin \delta_{K\pi}. \)

- \( \delta_{K\pi} \) is the strong phase difference between the doubly Cabibbo suppressed (DCS) decay, \( \bar{D}^0 \to K\pi^+ \) and the Cabibbo favored (CF) decay, \( D^0 \to K\pi^+ \) or \( \langle K\pi^+ | \bar{D}^0 \rangle / \langle K\pi^+ | D^0 \rangle = -r \cdot e^{-i\delta}. \)

So one can connect \((x,y)\) with \((x',y')\) via \( \delta_{K\pi}. \)

- For this part of my talk, I present preliminary results on \( \delta_{K\pi} \) and \( y \) using the quantum correlation between the produced D^0 and \( \bar{D}^0 \) pair in data taken at BESIII. This will then improve the determination of the mixing params, \((x,y)\).
Reconstructing events with a neutrino

- Reconstruct the all decay particles, except the neutrino.
- At CLEO-c and BESIII, where they take the data close to the mass threshold (i.e., $e^+e^- \rightarrow (\psi(3770)) \rightarrow D\bar{D}$), one can reconstruct one of the $D$ mesons fully (tag side), while the other $D$ is reconstructed, except the neutrino (signal side). The existence of neutrino can be inferred by a missing variable such as:

$$M_{\text{miss}}^2 = (E_{\text{beam}} - E_{\mu^+})^2 - (-\vec{p}_{D_{\text{tag}}} - \vec{p}_\mu)^2$$

for the case of $D^+ \rightarrow \mu^+ \nu_\mu$. $M_{\text{missing}}^2 \sim 0$ for the signal events.
Measuring $B(D_{CP\pm} \rightarrow K^-\pi^+)$

- Double-Tag technique:

$$B(D_{CP\pm} \rightarrow K\pi) = \frac{B(D_{CP\pm} \rightarrow CP^\mp \text{ states}) \times B(D_{CP\pm} \rightarrow K\pi)}{B(D_{CP\pm} \rightarrow CP^\mp \text{ states})}.$$  

So they need to measure;

- Yields (BF) when one D decays a CP final state
  while the other D decays generically

- Yields (BF) when one D decays a CP final state
  while the other D decays into $K\pi$.

- CP states they employ (8 modes):

| $CP^+$ | $K^+K^-$, $\pi^+\pi^-$, $K^0_S\pi^0\pi^0$, $\pi^0\pi^0$, $\rho^0\pi^0$ |
| $CP^-$ | $K^0_S\pi^0$, $K^0_S\eta$, $K^0_S\omega$ |

where we reconstruct $K_S \rightarrow \pi^+\pi^-$, $\pi^0/\eta \rightarrow \gamma\gamma$, $\omega \rightarrow \pi^+\pi^-\pi^0$, $\rho \rightarrow \pi^+\pi^-\pi^0$.

- Notice that most of systematics on the tag side get canceled in $B(D_{CP\pm} \rightarrow K\pi)$.

The remaining systematics (reconstruction/simulation) of $K\pi$ are also canceled in the determination of $A_{CP \rightarrow K\pi}$.
The selection rule can be seen in data

<table>
<thead>
<tr>
<th>Mode</th>
<th>CP+</th>
<th>CP-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yield(tag $KK$)</td>
<td>efficiency(%)</td>
</tr>
<tr>
<td>CP+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_S^0\pi^0\pi^0$</td>
<td>$8 \pm 3(\ast)$</td>
<td>$11.80 \pm 0.11$</td>
</tr>
<tr>
<td>CP+</td>
<td>$\rho\pi^0$</td>
<td>$13 \pm 8(\ast)$</td>
</tr>
<tr>
<td>CP-</td>
<td>$K_S^0\omega$</td>
<td>$158 \pm 13$</td>
</tr>
</tbody>
</table>

* Consistent with zero.
* Consider as one of the systematics.


Yields of $K\mu\nu$ in double tags ($n_{K\mu\nu,CP\mp}$) (reconstruct CP-final states from one D decay, with \textquote{\textquoteright}K\mu\nu\textquoteright{} from the other D)

- $K\pi\pi^0$ shapes and sizes are fixed based on control samples of actual data.
- The control samples are obtained by the same CP states and $K\pi\pi^0$, while ignoring the two photons from $\pi^0$ decays to calculate $U_{\text{miss}}$.
See the next slide for detail.

- **Signal shape:** MC shape, convoluted with an asymmetric Gaussian.
- **Background:** A 1st order polynomial. $K\pi\pi^0$ (dominant).
Fixing the $K\pi\pi^0$ shape

- Obtain $E_{\text{extra}} \equiv$ Sum of the all un-used energies deposited in EM calorimeter.
- $E_{\text{extra}}$ tends to be larger if it is $K\pi\pi^0$ due to the ignored extra photons from $\pi^0$ decay and is small if it is $K\mu\nu$.
- We actually do require $E_{\text{extra}} < 0.2$ GeV to select $K\mu\nu$ signal candidates.

![Fix shape](image)

- Fit to $U_{\text{miss}}$ in $E_{\text{extra}} > 0.5$ GeV where $K\mu\nu$ peak is suppressed.
- The fitted shape $\equiv$ MC shape, convoluted with a Gaussian.

$(K\pi\pi^0$ yields in data in $E_{\text{extra}} < 0.2$ GeV) = $R \times (K\pi\pi^0$ yields in data in $E_{\text{extra}} > 0.5$ GeV), where $R = (K\pi\pi^0$ yields in MC in $E_{\text{extra}} < 0.2$ GeV)/(MC in $E_{\text{extra}} > 0.5$ GeV).
Can also contribute to the measurement of $\gamma/\phi_3$

- Extract the $\gamma$ through the measurement of the interference between $b \rightarrow c$ and $b \rightarrow u$ when both $D^0$ and $\bar{D}^0$ decay to the same final state, $f(D)$.

$$A_{B\pm} \propto A_D + r_B e^{i(\delta_B \pm \gamma)} A_{\bar{D}}$$ (where $r_B$ is $|\langle B^- \rightarrow \bar{D}^0 K^- \rangle|/|\langle B^- \rightarrow D^0 K^- \rangle|$). $\delta_B$ is the strong phase difference).
Can also contribute to the measurement of $\gamma/\phi_3$

- This is one of the popular binning scheme, "Optimal binning", where bins are adjusted to maximize the sensitivity to $\gamma/\phi_3$ (CLEO: PRD82, 112006 (2010)).
- BESIII has recently repeated this analysis based on their data which is $\sim3.5\times$ larger than that of CLEO.
$c_i$ and $s_i$ in $D^0 \rightarrow K_{S,L} \pi^+\pi^-$ Dalitz analysis

Result of splitting the Dalitz phase space into 8 equally spaced phase bins based on the BaBar 2008 Model.

Starting with the equally spaced bins, bins are adjusted to optimize the sensitivity to $\gamma$. A secondary adjustment smooths binned areas smaller than detector resolution.

Similar to the “optimal binning” except the expected background is taken into account before optimizing for $\gamma$ sensitivity.

Source: CLEO Collaboration, Physical Review D, vol 82, pp. 112006 - 112035
**Equation on calculating $c_i$**

For the CP tag modes, one can show that the total bin yields are related to $c_i$ by

$$M_i^\pm = \frac{S_i}{2S_f} \left( K_i \pm 2c_i\sqrt{K_iK_{-i} + K_{-i}} \right)$$

- $M_i^+(M_i^-)$ yields in each bin of Dalitz plot for CP even(odd) modes.
- $S_i$ ($S_{-i}$) number of single tags for CP even(odd) modes.
- $S_f$ number of single tags for flavor modes.
- $K_i$ ($K_{-i}$), yields in each bin of Dalitz plot in flavor modes.

**Single Tag modes**

<table>
<thead>
<tr>
<th>Type</th>
<th>Tag List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-Flavored</td>
<td>$K^-\pi^+, K^-\pi^+\pi^0, K^-\pi^+\pi^+\pi^-$</td>
</tr>
<tr>
<td>$S^+$</td>
<td>$K^+K^-, \pi^+\pi^-, K_S\pi^0\pi^0, K_L\pi^0$</td>
</tr>
<tr>
<td>$S^-$</td>
<td>$K_S\pi^0, K_S\eta(\rightarrow \gamma\gamma), K_S\eta(\rightarrow \pi^+\pi^-\pi^0), K_S\omega, K_S\eta'$</td>
</tr>
</tbody>
</table>
Calculating both $c_i$ and $s_i$

Using $D^0 \rightarrow K_s\pi^+\pi^-$ vs $\bar{D}^0 \rightarrow K_s\pi^+\pi^-$ we can calculate both $c_i$ and $s_i$:

$$M_{i,j} = \frac{N_{D,\bar{D}}}{2S_f^2} \left( K_i K_{-j} + K_{-i} K_j - 2 \sqrt{K_i K_{-j} K_{-i} K_j (c_i c_j + s_i s_j)} \right)$$

- $M_{i,j}$ yields in bin $i$ of first Dalitz plot and bin $j$ of second Dalitz plot.
- $S_f$ number of single tags for flavor modes.
- $N_{D,\bar{D}}$ total number of $D^0 \bar{D}^0$ events.
- $K_i(K_{-i})$, yields in each bin of Dalitz plot in flavor modes.