

Sept 11, 2014  
muon g-2  
experimental  
hall



**$e^+e^-$  hadronic cross section and  
muon g-2**

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PIC 2014

September 17, 2014

# Outline

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- What is  $g$  and how do we predict its value
- How we measure it for muons
- Where things stand and where they are going
- Most numbers are from a snowmass white paper edited by Lee Roberts arXiv:1211.2198

# What is g?

Gyromagnetic ratio = magnetic dipole moment / angular momentum

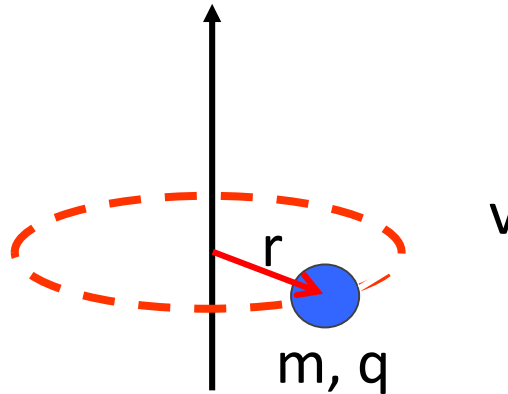
Magnetic dipole moment

$$\mu = IA$$

$$I = \frac{qv}{2\pi r}; A = \pi r^2$$

$$\mu = \frac{1}{2} qvr$$

**g-factor  
nominally 1**



Angular momentum

$$L = rmv$$

Classical gyromagnetic ratio

$$\gamma = \frac{q}{2m}$$

Quantum

$$\vec{m} = g \frac{e\hbar}{2m} \frac{\vec{S}}{\hbar}$$

g-factor

Bohr magneton

# Counting degrees of freedom

Byproduct of the Dirac equation is extra degrees of freedom of the electron associated with spin

$$\left( \beta mc^2 + \sum_{k=1}^3 \alpha_k p_k c \right) \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}$$

$$y = \begin{matrix} \hat{e} & \gamma_1 & \hat{u} \\ \hat{e} & \gamma_2 & \hat{u} \\ \hat{e} & \gamma_3 & \hat{u} \\ \hat{e} & \gamma_4 & \hat{u} \end{matrix}$$

If we take the non-relativistic limit and try and recover the Pauli equation we get an extra factor of 2

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{p^2}{2m} - \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi$$

(Bjorken, Drell)

With the extra degrees of freedom,  $g = 2$

# Self energy

Also need to include the corrections due to self interactions of the muon with its own field



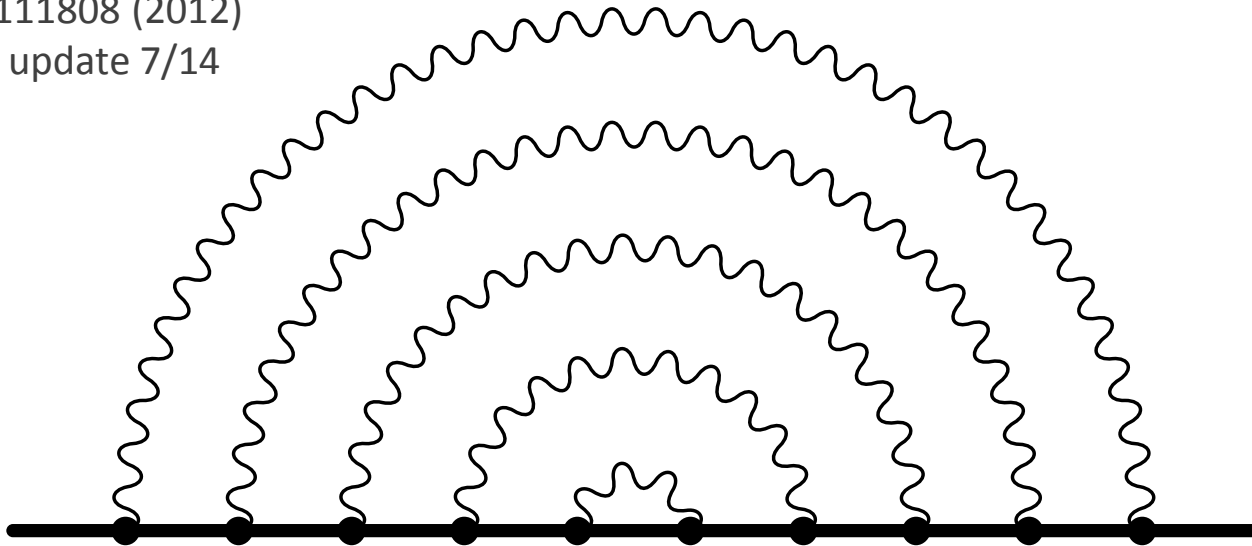
$$\frac{g - 2}{2} = \frac{a}{2p} \gg 0.1\%$$

(Schwinger term)

Predicting  $g$  now becomes a question of determining radiative corrections to the required precision

# QED out to 10<sup>th</sup> order

Aoyama, Hayakawa, Kinoshita, Nio  
PRL 109, 111808 (2012)  
+ prelim update 7/14



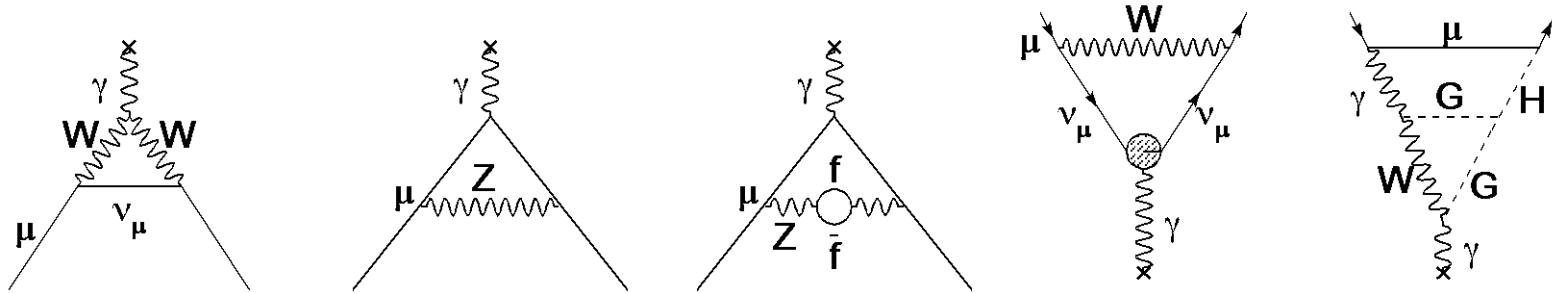
Calculated analytically to 6<sup>th</sup> order (72) diagrams  
Calculated numerically to 10<sup>th</sup> order (12672) diagrams  
Largest 12<sup>th</sup> order terms estimated

$$\frac{(g - 2)_m}{2} (QED) = 0.00116584718951(80)$$

Uncertainty dominated  
by fine structure  
constant

# Electroweak contribution

(G = longitudinal component of gauge boson)



Calculated analytically to 2<sup>nd</sup> order and estimated out to 4<sup>th</sup> order  
 Recently updated to include measured value of the Higgs mass

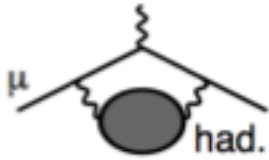
$$\frac{(g - 2)_m}{2} (EW) = 0.0000000001536(10)$$

Gnendiger, Stockinger, Stockinger-Kim  
 PRD 88, 053005 (2013)

This is 10<sup>-9</sup> and the leading term is 10<sup>-3</sup> so we call this a ppm correction

Very convenient way of thinking about different contributions:  
 New physics with weak scale masses and weak scale couplings naively  
 gives a ppm level correction to muon g-2

# Leading hadronic contribution



Hadronic vacuum polarization

$$\text{had. blob} = \int \frac{ds}{\pi(s-q^2)} \text{Im} \text{had. blob}$$

Use analyticity to convert into a dispersion relation

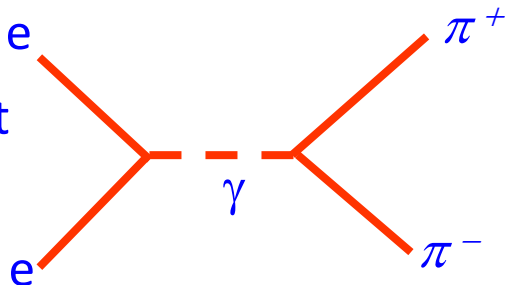
$$2 \text{Im} \text{had. blob} = \sum_{\text{had.}} \int d\Phi \left| \text{had. blob} \right|^2$$

Use optical theorem in reverse to convert to a cross section

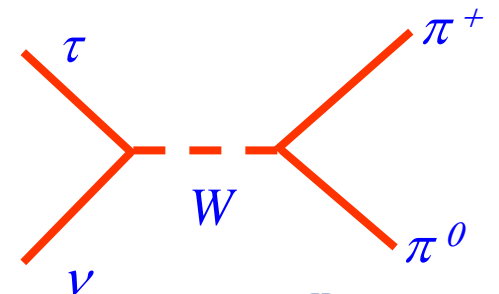
Figs from T. Teubner

$$a_{\mu}^{\text{had;LO}} = \frac{1}{3} \frac{m_{\mu}^2}{m_e^2} \int \frac{ds}{s^2} K(s) R(s), \quad \text{where } R \equiv \frac{\sigma_{\text{tot}}(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)},$$

Dominant term:



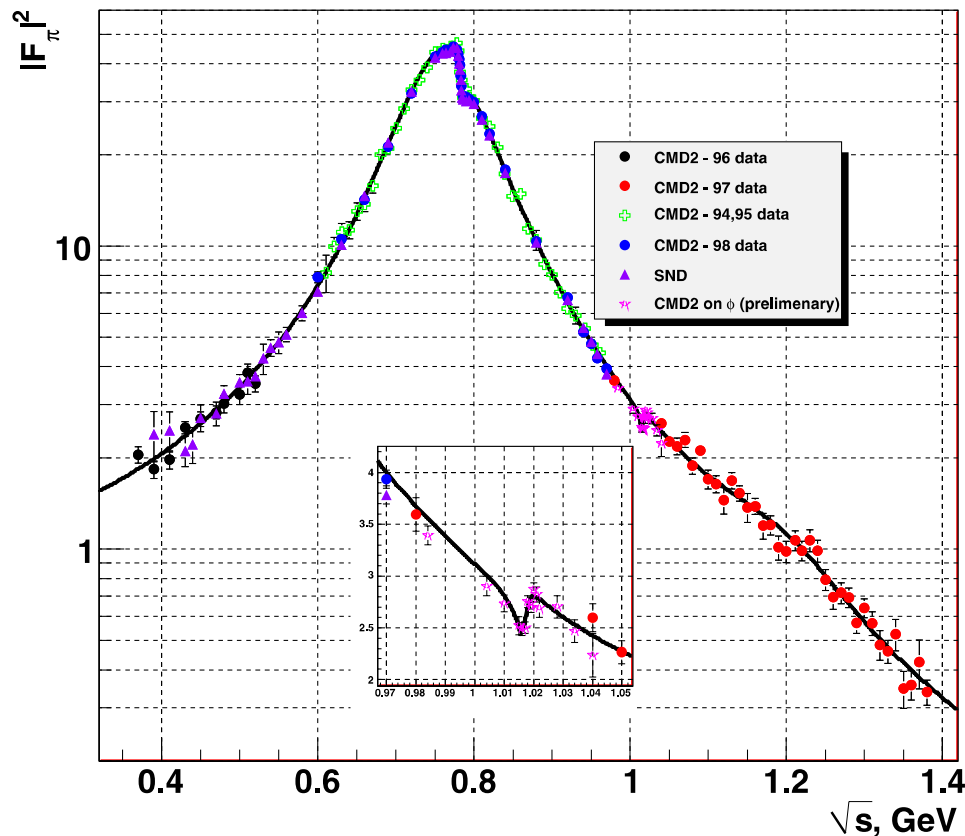
Use CVC and isospin to convert to  $m(\pi^+ \pi^0)$  in  $\tau$  decays





# R-scan data

Most relevant R-scan data for muon  $g-2$  comes from the SND and CMD-II detectors at the Novosibirsk VEPP-2M collider

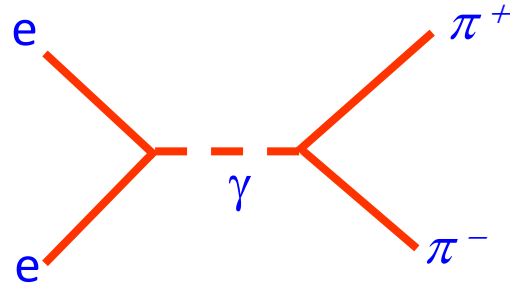


Scans from 1992-2000  
~1% determination of  
the hadronic  
contribution to muon  $g-2$

This is a major effort

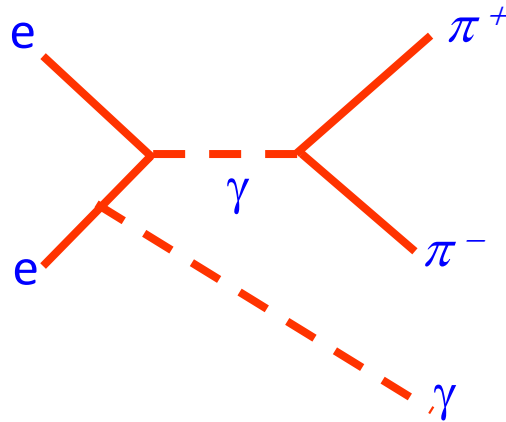
# Radiative return

R-scan: vary beam energy to scan



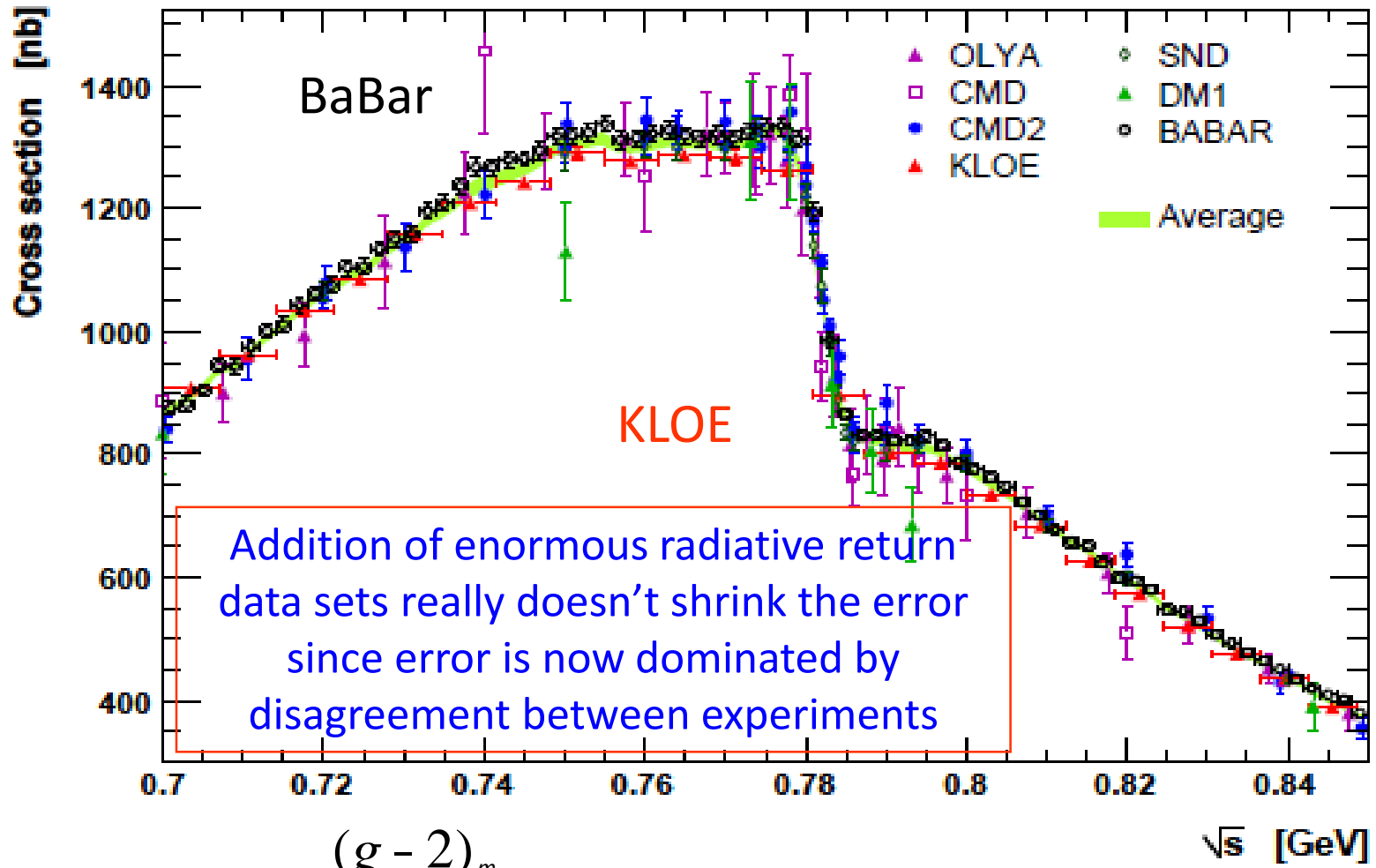
Long and dedicated run time

Radiative return: sit on a resonance and probe lower energies through ISR



Ideal for the era of high luminosity factories sitting at the  $\phi$ ,  $\tau/c$ , and  $Y(4S)$  resonances where it becomes a parasitic measurement

# R-scan + radiative return

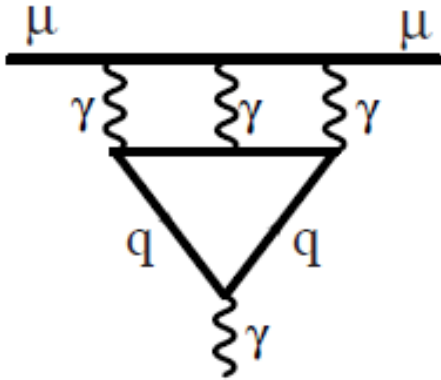


$$\frac{(g-2)_m(HVP)}{2} = 0.00000006923(42)$$

$\sqrt{s}$  [GeV]

# Higher order QCD

Most relevant term is hadronic light by light scattering



Current knowledge is based on combinations of several model dependent calculations with error derived from the spread in the results

$$\frac{(g-2)_m}{2} (HLbL) = 0.00000000105(26)$$

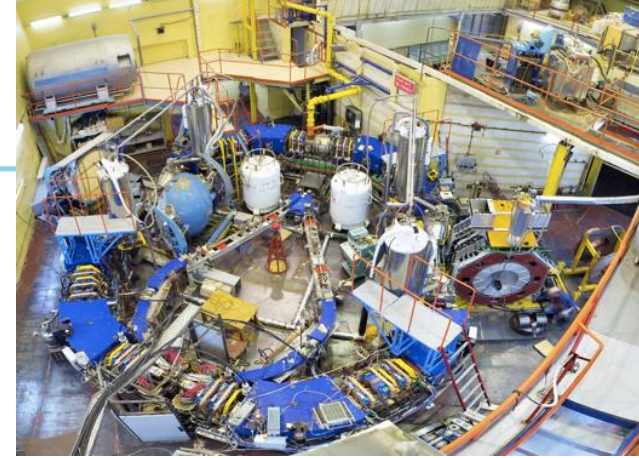
# Current problems

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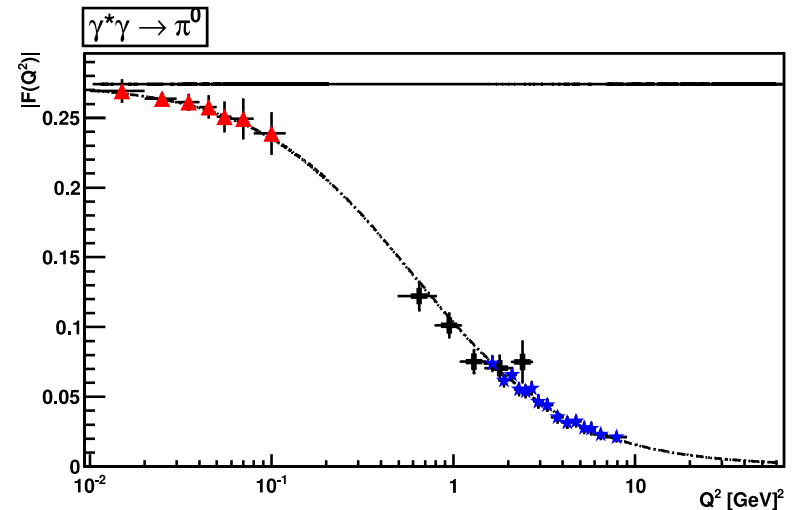
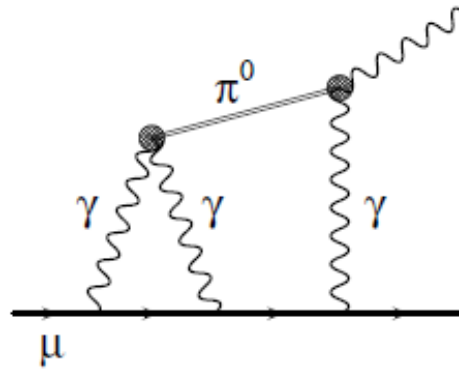
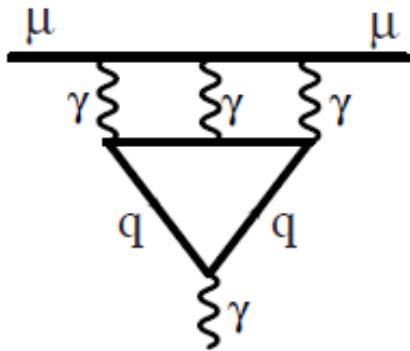
- Two most precise data-based determinations of leading order QCD contributions do not agree.
- The  $e^+e^-$  determination does not agree with the  $\tau$  determination
  - Growing evidence that this is due to unaccounted for isospin breaking effects but jury is still out
- It is difficult to quantify the error in the hadronic light-by-light contribution
  - Many people are worried that it is underestimated
- Each of these effects is roughly the size of the quoted uncertainty and cloud the interpretation of the comparison between data and prediction
- Without a program to address these, many people feel an upgraded muon g-2 experiment doesn't make sense

# The Program I: New R-scan data

- New Novosibirsk R-scan
  - Upgraded higher luminosity machine
  - Major detector upgrades
  - Data taking began in 2009 and already have data sets on tape comparable to BaBar
  - After complete R-scan up to 2 GeV, machine will sit at  $N \bar{N}$  threshold and collect radiative return data
- Radiative return measurements now integral part of all the factory programs (BES III, Belle II, KLOE)
  - Not to mention enormous  $\tau$  data sets
- Now have 2 high statistics measurements, by the end of the decade we expect 8
- Projection is for a factor of 2 reduction in the uncertainty on muon  $g-2$



# The Program II: Data driven light-by-light



- New detectors installed in KLOE-II to measure outgoing e+e- in two photon collisions
- Can measure transition form factors down to unprecedented  $q^2$
- This data can be used to verify the models used to calculate hadronic light-by-light
- Recent workshop held in Mianz produced a draft roadmap for a data driven approach to hLbL (arXiv:1407.4021)
- Projections for future improvement do not assume a reduction in uncertainty. Only a more robust uncertainty.

# The Program III: Lattice QCD

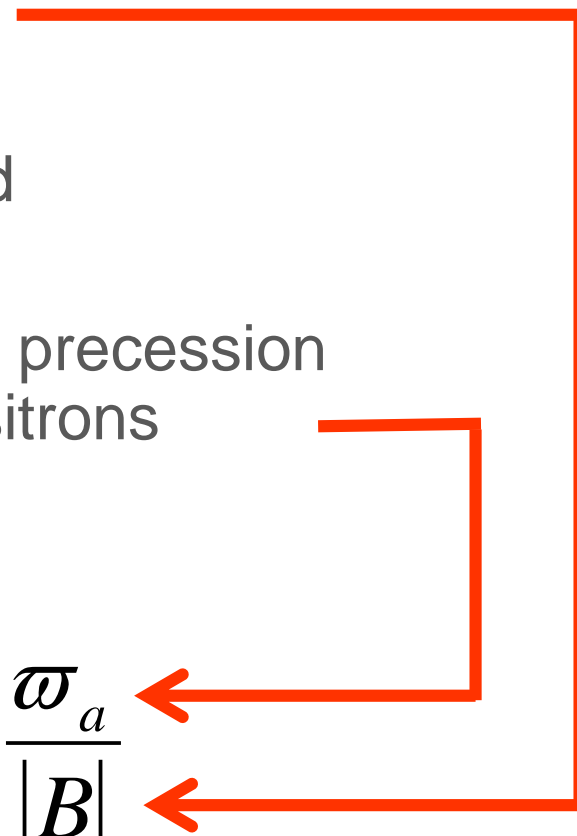
- First principles calculation of hadronic contributions becoming a fairly significant thrust in lattice QCD
  - 11 papers presented at Lattice-2014
    - HVP and hLbL, alternate techniques, fitting biases, strange and charm quark contributions, disconnected diagrams.....

Lattice	precision	timescale	benchmark
HVP	1-2%	Few years	$\tau \rightarrow e^+e^-$ discrepancy
HVP	sub-%	This decade	Competitive w/ $e^+e^-$
hLbL	any	soon	Course Verification of models
hLbL	~30%	3-5 years	Competitive with models
hLbL	~10%	Ultimate goal	Replace models



# Measuring muon g-2

- Produce polarized muons and inject them into a storage ring with vertical B field
- B field is mapped using NMR probes
- Muon spin precesses around the B field
- Positrons decay along spin direction so precession frequency is measured by counting positrons

$$a_{\mu} = \frac{(g - 2)_{\mu}}{2} = \frac{m_{\mu}}{e} \times \frac{\omega_a}{|B|}$$


# Magic momentum

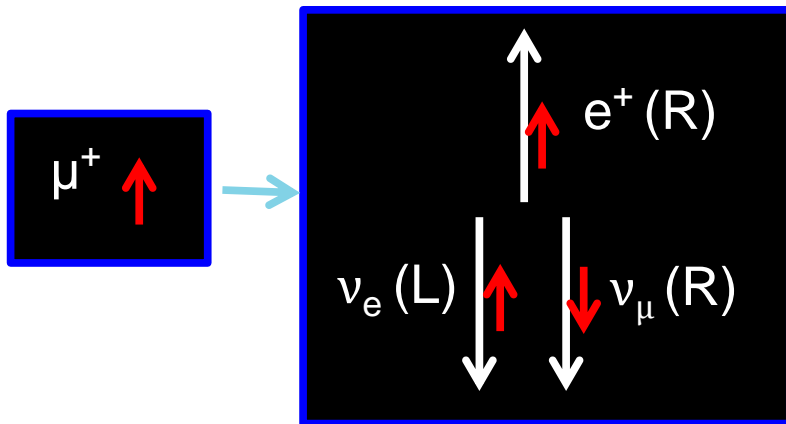
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- Need to focus the muons to store them.
  - Done using electrostatic quadropoles
- Adds a motional B field term to the precession frequency

$$\vec{\omega}_a = -\frac{q}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right].$$

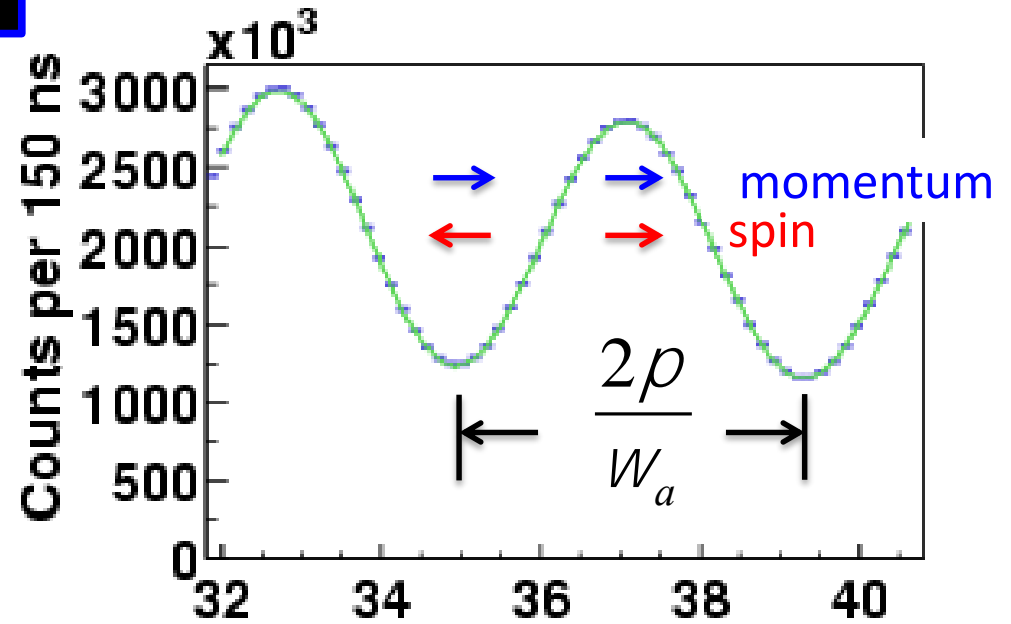
- For  $(g-2)/2 = 0.1\%$  and  $\gamma = 29.3$ , the above term cancels
- CERN II, III, Brookhaven and Fermilab experiments are all magic momentum experiments with  $p = 3.094$  GeV

# Frequency measurement



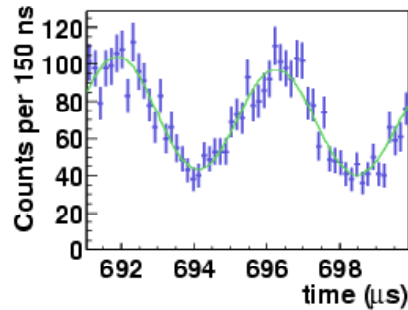
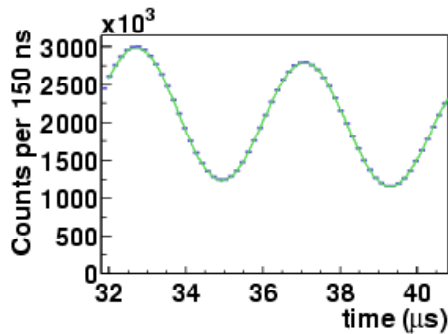
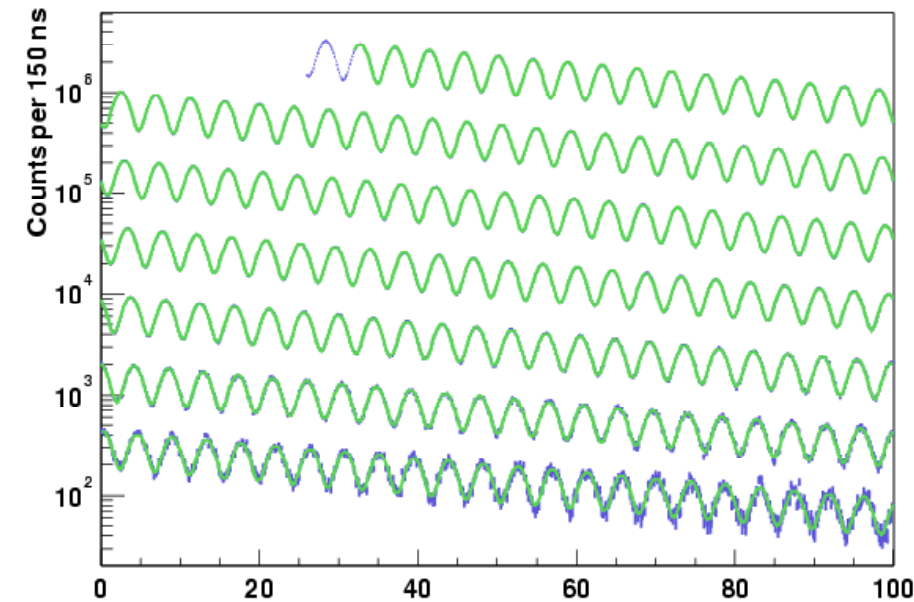
Weak decay so  
positron direction follows muon spin

Highest energy positrons  
occur when muon spin  
and momentum are  
aligned



# high energy positrons versus time

# Brookhaven result



$$\frac{(g-2)_m}{2} (BNL) = 0.00116592089(63)$$

0.54 ppm  
uncertainty

$$\frac{(g-2)_m}{2} (SM) = 0.00116591802(49)$$

0.42 ppm  
uncertainty

$$diff = (287 \pm 80) \cdot 10^{-11}$$

2.5 ppm difference

Big effect,  
needs  
confirmation

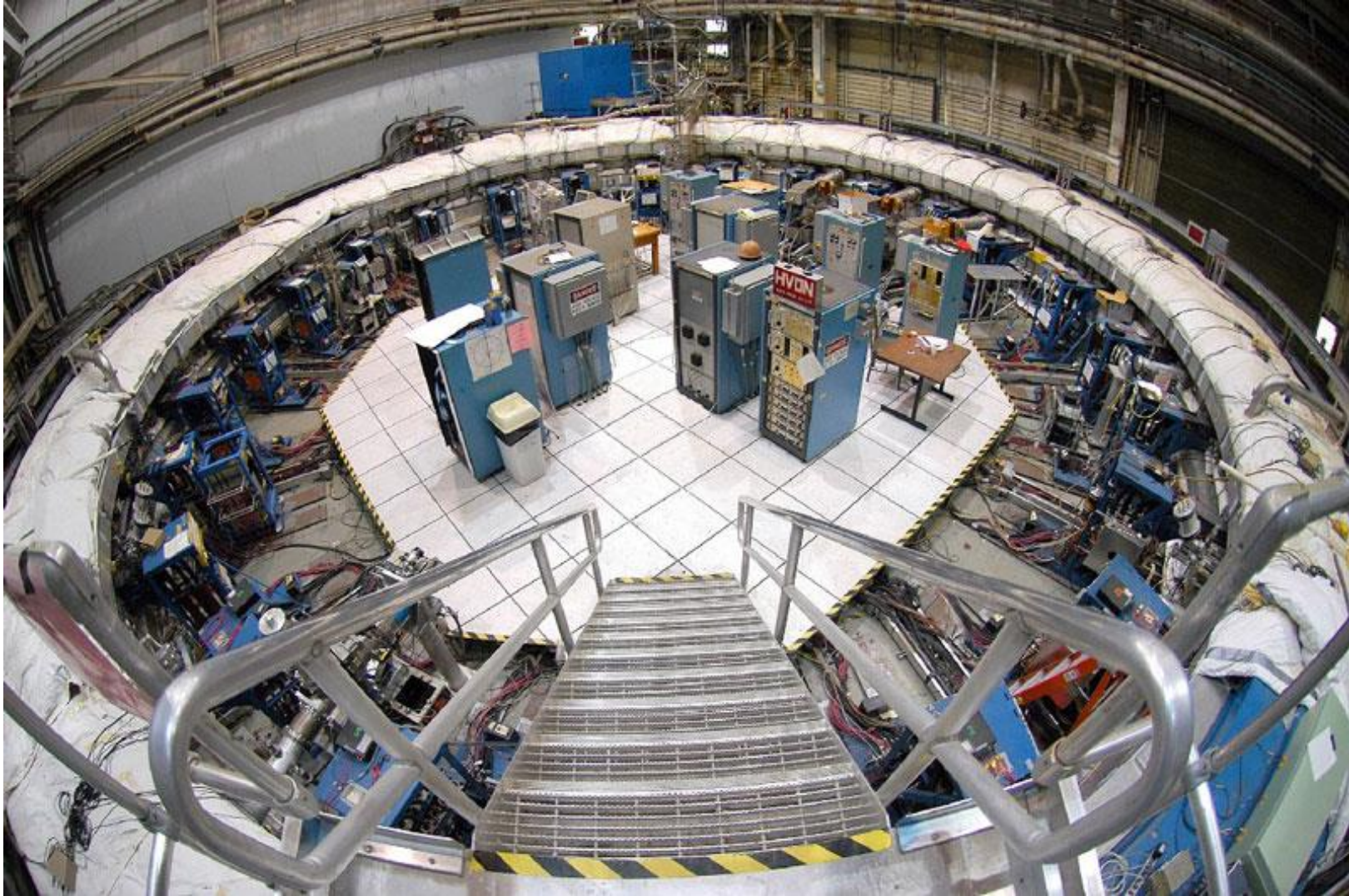
# The Program IV: A new experiment at Fermilab

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- Philosophy:
  - Re-use the BNL storage ring
    - It is one continuous conductor and has sub-ppm level uniformity averaged around the ring
  - Move the ring to Fermilab
    - Higher rate, higher polarization, higher purity than at BNL
    - Factor of 20 increase in statistics per year
  - Rebuild (almost) all instrumentation from scratch
    - Use of modern detector technology reduces systematic uncertainties to keep pace with the reduced statistical uncertainty
  - Goal: 140 ppb

# Disassembly

storage ring at the end of the last experiment



# Disassembly



Summer 2011

# The big move



B. Casey, muon g-2

9/17/24



# reassembly



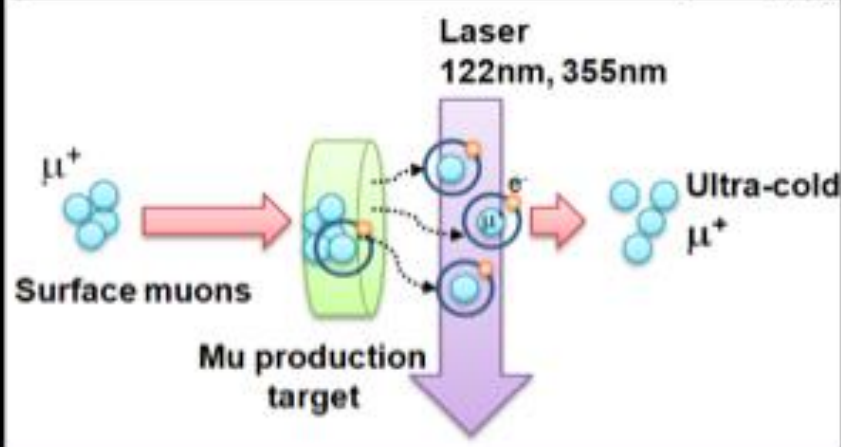
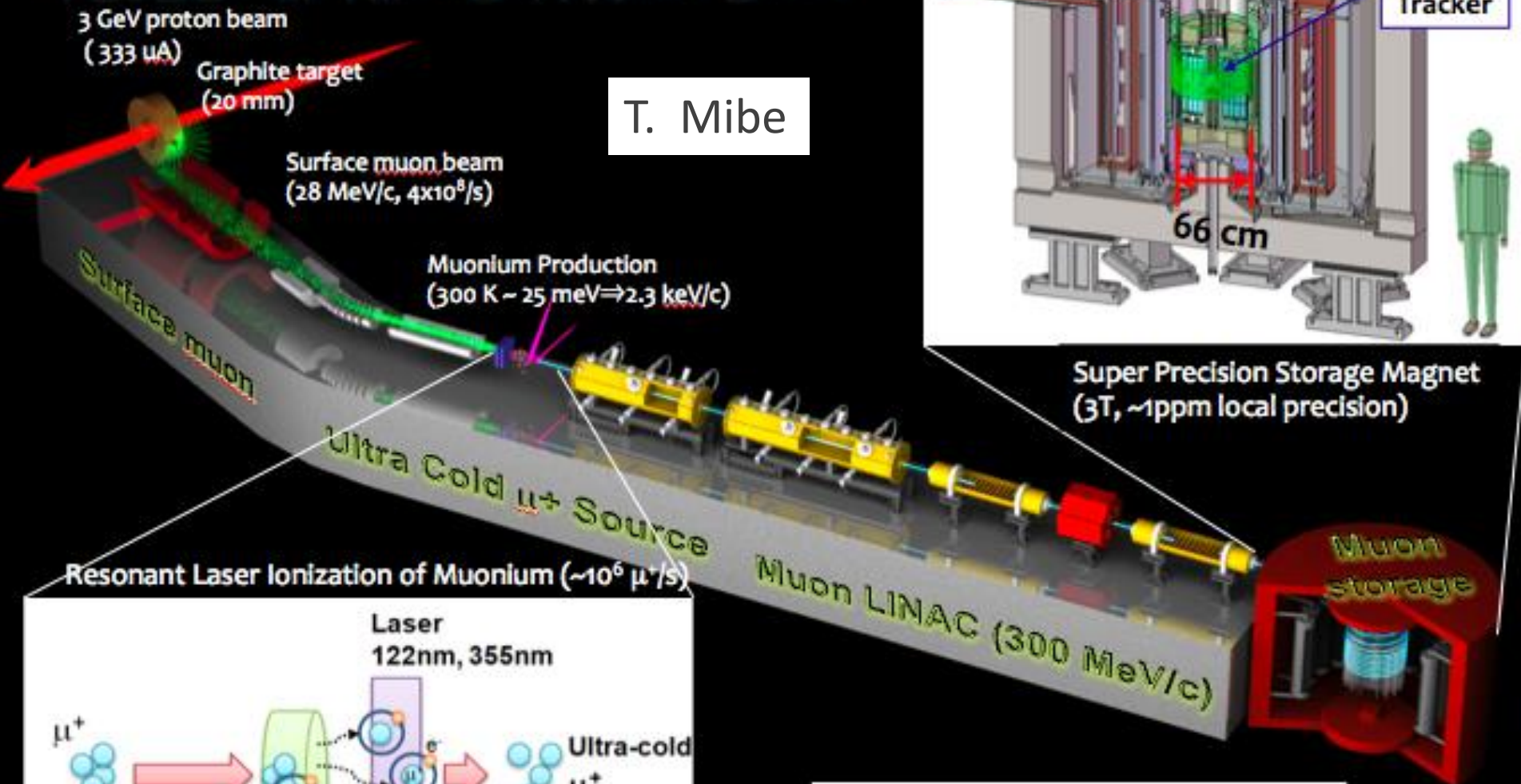
# Milestones

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- Submitted proposal to Fermilab Nov 2009
- Ring disassembly began Summer 2011
- Ring shipped to Fermilab Summer 2013
- Ring moved into new building Summer 2014
- Ring cold Spring 2015 and shimming begins
- Detectors installed and accelerator work complete in 2016
- First large data set in 2017
- Significant results in 2018

# New Muon g-2/EDM Experiment at J-PARC with Ultra-Cold Muon Beam

T. Mibe



$\Delta(g-2) = 0.1\text{ppm}$   
 $\text{EDM} \sim 10^{-21} \text{ e} \cdot \text{cm}$

# The Program at the end of the decade

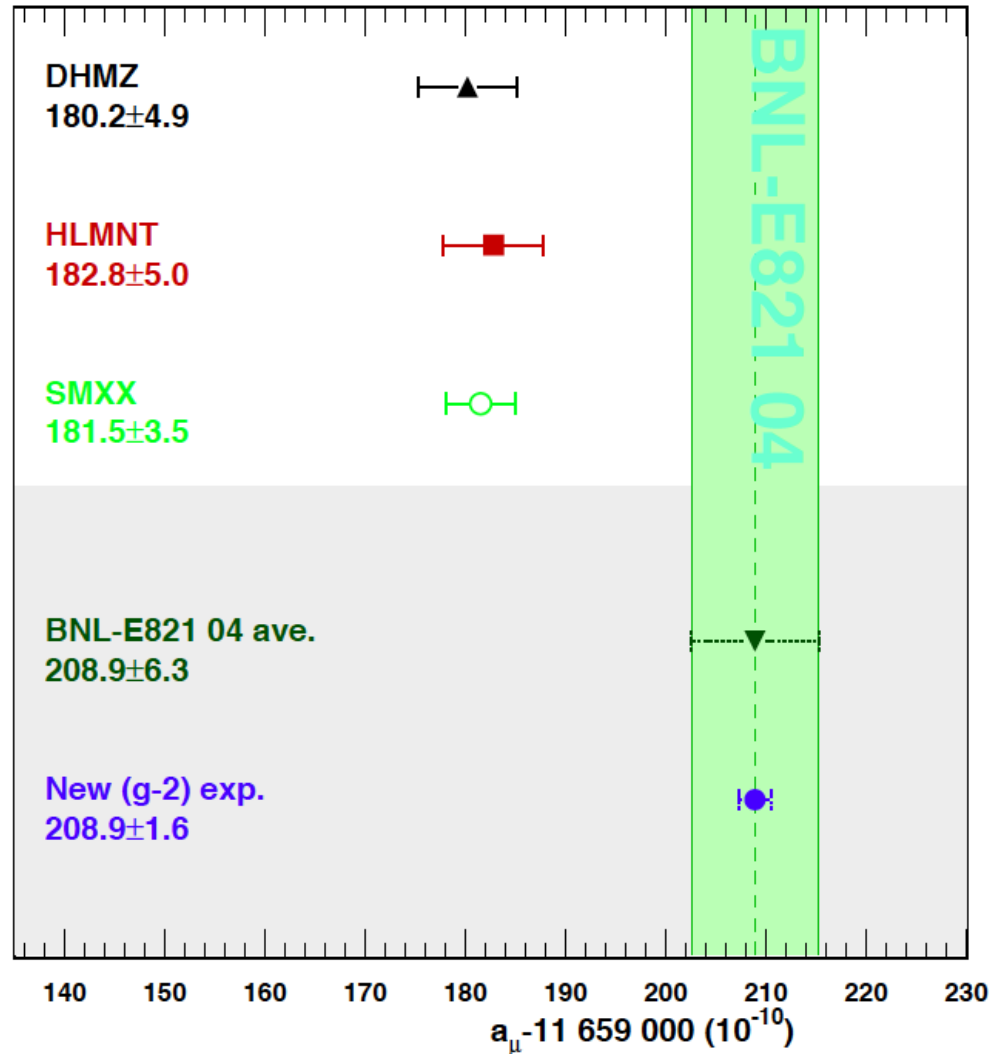
Error	[20]	[21]	Future
$\delta a_\mu^{\text{SM}}$	49	50	35
$\delta a_\mu^{\text{HLO}}$	42	43	26
$\delta a_\mu^{\text{HLbL}}$	26	26	25
$\delta(a_\mu^{\text{EXP}} - a_\mu^{\text{SM}})$	80	80	40

## Minimum outcome

Have an independent measurement of muon g-2  
 Have a much more robust understanding of the uncertainty in the prediction

## Maximum outcome

If discrepancies in prediction are resolved and experimental value is confirmed, we will have an 8 sigma result



# Conclusions

- There is a worldwide program underway to drastically improve our understanding of muon g-2
- Results on all fronts are expected this decade
- Extremely challenging but also extremely exciting and hopefully extremely rewarding

