Application of Compact Vector Summation to Reroute Sequence Planning in Telecommunication Networks

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Abstract

Our method of Compact Vector Summation(CVS) has been used in [1] to solve the Reroute Sequence Planning problem (RSP-problem) in telecommunication network. Here we give a short account of such application.

A network consists of the set V of vertices and the set E of edges connecting certain pairs from V (not necessary *all* pairs), $|E| = m$ (the number of edges equals m). Consider a system of paths $P = (P_1, ..., P_k)$, the path P_i connecting the vertices s_i and t_i, $i = 1, ..., k$. By a path connecting s_i and t_i, we mean a broken line consisting of edges with the starting point at s_i and endpoint at t_i. (It is worthy to remark that the problem can be stated formally, entirely in terms of abstract graph theory.)

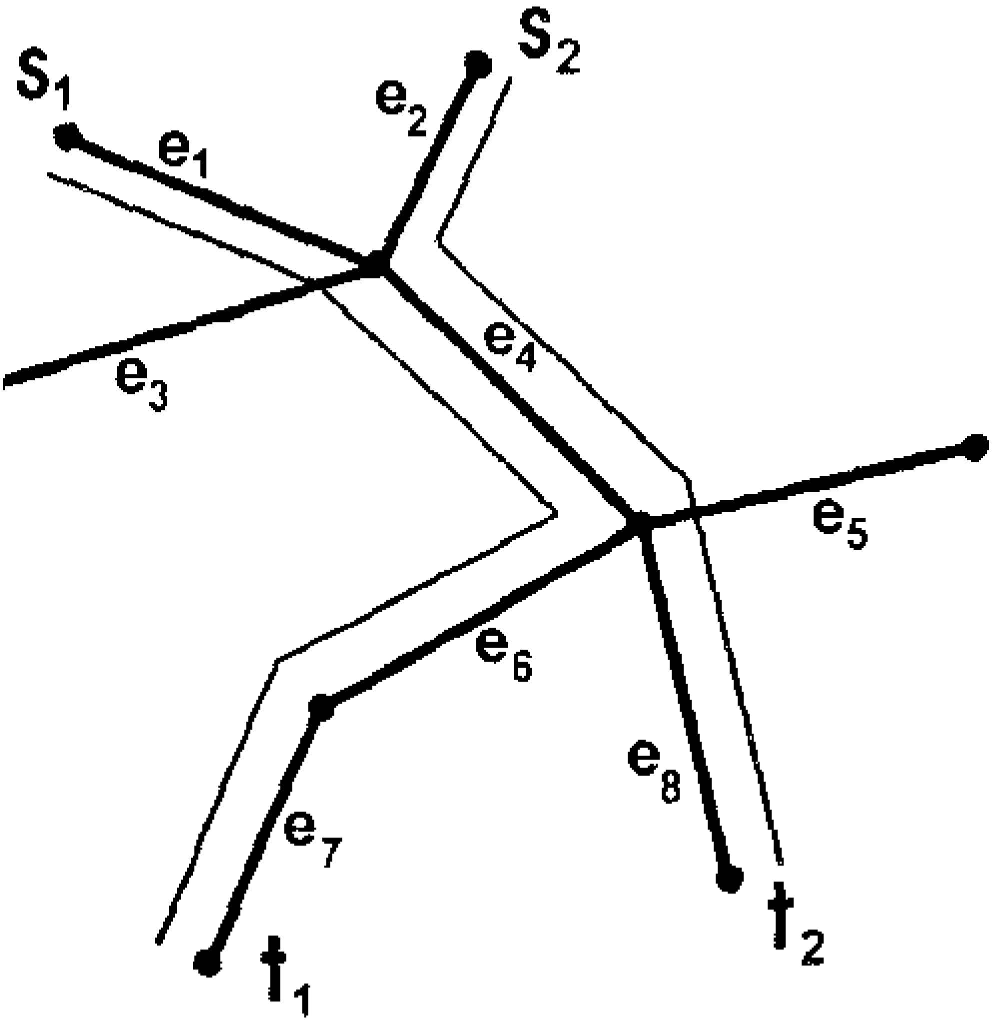

Fig.1.

Fig. 1 shows an example of a network consisting of 8 vertices and 8 edges. Consider a system $P=(P_1, P_2)$, where the path P_1 consists of edges e_1, e_2, e_6, e_7, while P_2 consists of edges e_2, e_4, e_8. For a general network each of the paths (say P_j) of the system $P=(P_1,...,P_k)$ imposes a demand d_j, on the capacity for each of the edges of the path P_j. If an edge e belongs to several paths, then the total demand $u(e)$ imposed on this edge is the sum of corresponding demands. For example, in Fig. 1 the total capacity demand on the edge e_4 (which is common for P_1 and P_2) is d_1+d_2. The demand on the edges e_1, e_6, e_7 is d_1, while the demand on e_2, e_8 is d_2. In order the network to function, the total demand $u(e)$ for each of the edges e should not exceed the given capacity $c(e)$.

In reroute sequence planning (RSP) the system $P=(P_1,...,P_k)$ of paths should be replaced by the system $Q=(Q_1,...,Q_k)$ connecting the same vertices s_i and t_i , $i=1,...,k$ imposing the same demands $d_1,...,d_k$. In a reroute sequence procedure at each step only one of $P's$ is being replaced by the corresponding path of $Q's$. RSP consists in finding a sequence $n_1,...,n_k$ such that after each of successive replacements P_{n_1} by Q_{n_1} , …, P_{n_k} by Q_{n_k} , the total demand $u(e)$ does not exceed the capacity $c(e)$ for each $e \in E$. Denote by $x_i(e)$ the change in demand for the edge $e \in E$ when P_i is replaced by Q_i (i.e. the total demand after the replacement minus the total demand before the replacement). We have:

$$x_i(e) = \begin{cases} d_i \ \mathit{if} \ e \notin P_i,\ e \in Q_i \\ -d_i ,\ \mathit{if} \ e \in P_i , e \notin Q_i \\ 0 \ \mathit{otherwise} \end{cases} .$$

The vectors $x_1,...,x_k$ belong to the space $\mathbf{R}^n$, where n, the dimension of the space , is the number of edges. Denote by $L_0(e)$ the starting total demand on the edge $e \in E$ and assume we have chosen a sequence $n_1,...,n_k$ of replacements P_{n_1} by Q_{n_1} , …, P_{n_k} by Q_{n_k}. Then the demand on the edge $e \in E$, after the l-th step is

$$L_0(e) + x_{n_1}(e) + ... + x_{n_l}(e).$$

This implies that the demand on the edge e at each step does not exceed

$$L_0(e) + \max_{1\le l\le k} \max_{e\in E} | x_{n_1}(e) + ... + x_{n_l}(e) | =$$
$$L_0(e) + \max_{1\le l\le k} \| x_{n_1}(e) + ... + x_{n_l}(e) \|_\infty ,$$

where $\|\cdot\|_\infty$ denotes the maximum norm. Thus we have come to the problem of finding a permutation $\sigma: \{1,...,k\} \to \{1,...,k\}$ minimizing

$$r(\pi) = \max_{1\le l\le k} \| x_{n_1}(e) + ... + x_{n_l}(e) \| ,$$

where the permutation π is defined by the sequence $n_1,...,n_k$.

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