Ionisation distribution near particle track in gas mixture

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Abstract

Space distribution of ionisation produced by charged particle in gas is discussed. Energy loss is described in the framework of GEANT4 PAI model. The contribution of Cerenkov photons and secondary (resonance, and quasi-free) electrons is considered for different particle energies. Calculations were made for 90% Ne + 10% CO_2 gas mixture which was proposed for ALICE TPC.

1 Motivation

According to ALICE TPC TDR simulation of ionisation produced by charged particle is based on following assumptions:

1. Primary ionisation $\bar{N}_1$ for Ne + 10% CO_2 gas mixture depends on Lorentz factor according parametrisation:

$$\bar{N}_1(\beta\gamma) = \frac{N_{prim} P_1}{\beta^{P_4}} \left\{ P_2 - \beta^{P_4} - \ln \left[P_3 + \frac{1}{(\beta\gamma)^{P_5}} \right] \right\},$$

   where N_{prim}=14.35 and P_i are fitting parameters derived from Bethe-Bloch curve for P10 (Ar + 10% CH_4) gas mixture (ALEPH data).

2. The spectrum of energy transfers E is supposed to be polynomial $E^{-2.2}$

3. Ionisation clusters (including secondary electrons) are assumed to be point-like.

These points were analysed by H. Bichsel in [1]. Here we present the ionisation analysis based on GEANT4 models [2].

2 Photo-Absorption Ionisation (PAI) Model

PAI model [3]-[5] describes the differential cross-section of ionizing collisions produced by relativistic charged particle in medium. For transverse (secondary is gamma!) and longitudinal (secondary is electron) excitations PAI-model reads, respectively, for mean number of collisions per unit trajectory length and unit energy transfer:

$$\frac{d^2\bar{N}_\perp}{\hbar d\omega\ dx} = \frac{\alpha}{\pi\hbar c}\mathrm{Im}\left\{\left[1-\frac{1}{\beta^2\epsilon_\perp}\right]\ln\left(\frac{1}{1-\beta^2\epsilon_\perp}\right)\right\} \quad \text{gamma!},$$

and

$$\frac{d^2\bar{N}_\parallel}{\hbar d\omega\, dx} = \frac{\alpha N}{\pi\hbar\beta^2\omega}\left[\sigma_\gamma(\omega)\ln\frac{2mv^2}{\hbar\omega}+\frac{1}{\omega}\int_0^\omega \sigma_\gamma(\omega')\,d\omega'\right] \quad \text{electron},$$

where α is the fine structure constant, $\beta = v/c$, $d\,x = v\,dt$, m is the electron mass, N is the atomic density, $\hbar\omega$ is the energy transfer, $\epsilon_\perp$ is the transverse dielectric permittivity, which in turn is expressed in terms of the photo-absorption (PAI) cross-section, $\sigma_\gamma(\omega)$.

The mean free path of PAI model, λ, is defined in terms of reciprocal primary ionisation, $\bar{N}_1$:

$$\lambda^{-1} = \bar{N}_1 = \int_{I_1}^{T_{max}} \left(\frac{d^2\bar{N}_{\|}}{\hbar d\omega\, dx} + \frac{d^2\bar{N}_{\perp}}{\hbar d\omega\ dx}\right)\hbar d\omega,$$

where I_1 is the first ionisation potential, and T_{max} is the maximum (kinematically) energy transfer. The integral probability for energy transfers in single collision is defined by:

$$\bar{N}_{>\hbar\omega} = \int_{\hbar\omega}^{T_{max}} \left(\frac{d^2\bar{N}_{\|}}{\hbar d\omega\, dx} + \frac{d^2\bar{N}_{\perp}}{\hbar d\omega\ dx}\right)\hbar d\omega.$$

These values reflecting GetMeanFreePath and DoIt methods of typical physical process can be parametrised in terms of particle energy (Lorentz factor), its charge and medium density.

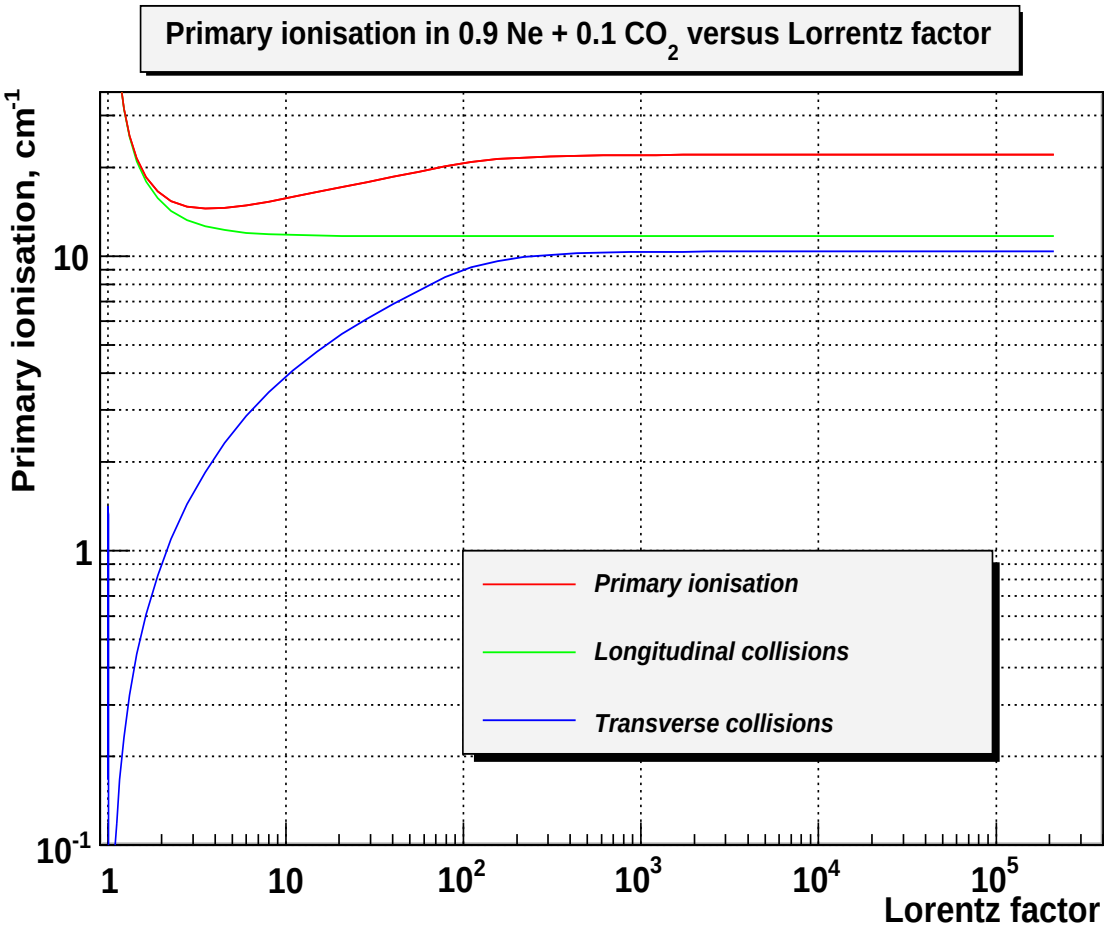
Below there are some predictions of PAI-model for noble gases and ALICE TPC gas mixture, 90% Ne + 10% CO_2. The gas mixture density was selected to correspond STP condition (1 atm, 0 °C).

Experimental and simulated (PAI-model) primary ionisation $\bar{N}_1$ in noble gases STP (1 atm, 0 °C)

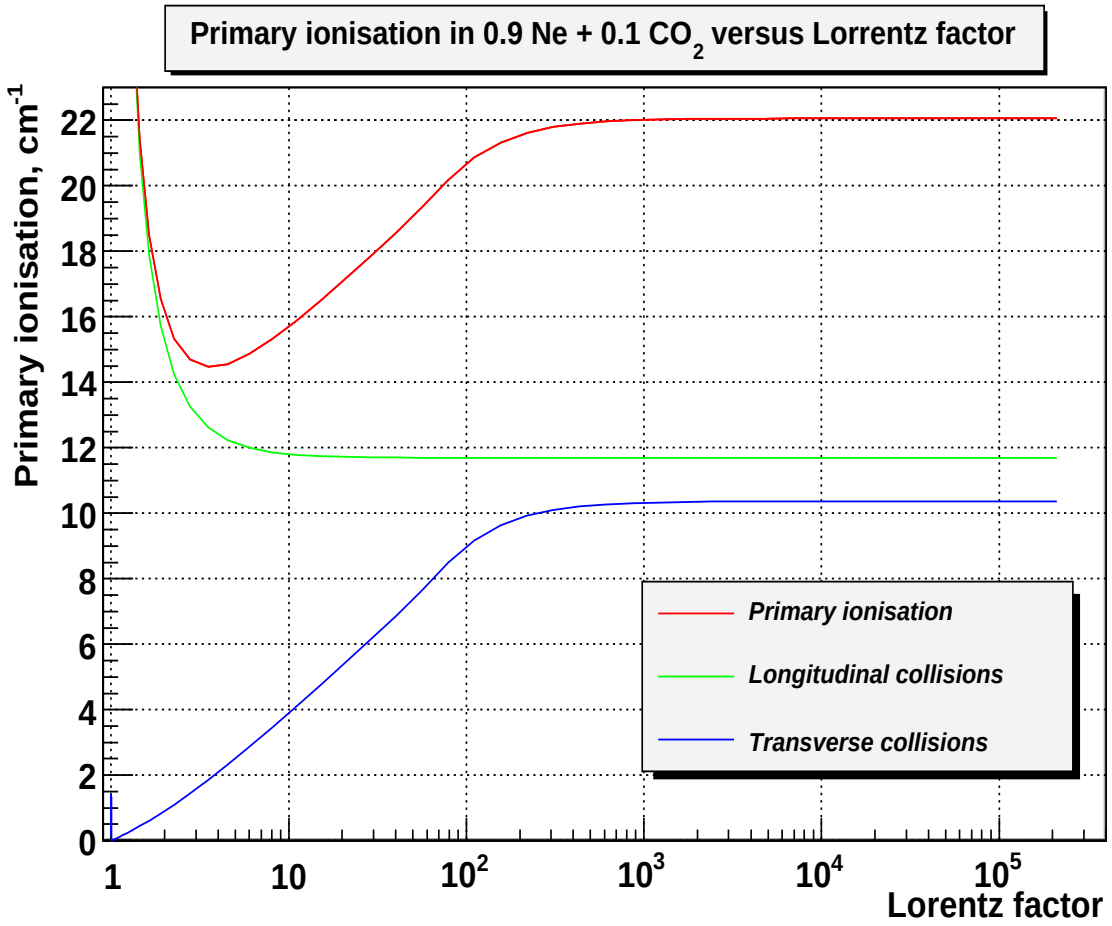
dN/dx, cm^{-1}	He	Ne	Ar	Kr	Xe
G-M counter	5.02 ± 0.12	12.4 ± 0.4	27.8 ± 0.3	-	-
streamer chamber	3.54 ± 0.11	11.7 ± 0.6	28.6 ± 0.5	-	-
spark chamber	3.65 ± 0.12	12.3 ± 0.3	28.5 ± 0.5	37.1 ± 0.6	48.0 ± 1.0
GEANT4 PAI-model	4.2	11.6	26.3	34.1 (27.4)	47.6 (20.6)

Measurements in streamer chambers are based on counting of streamers on the particle track, while in low pressure G-M counters and spark chambers the efficiency, η, is measured $1 - \eta = \exp(-\bar{N}_1)$.

The value $\bar{N}_1$=20.6 cm^{-1} for xenon ($\bar{N}_1$=27.4 cm^{-1} for kripton) corresponds to the original SANDIA parametrisation.



Transverse excitations show relativistic rise only!



Relativistic rise (RR) of primary ionization is 1.52, which is very close to RR=1.56 of truncated mean energy loss of 90% Ar + 10% CH_4 used in ALICE TPC software.

3 Ranges of Secondaries

According to PAI model relativistic charged particle produces Cerenkov photons and ionisation electrons. Cerenkov photons are described by transverse energy loss [6] (in transparent medium → Tamm-Frank formula):

$$\frac{d^2\bar{N}_{\perp}^{Cer}}{\hbar d\omega\ dx} = \frac{\alpha}{\pi\hbar c}\mathrm{Im}\left\{\left[1-\frac{1}{\beta^2\epsilon_{\perp}}\right]\ln\left(\frac{1}{1-\beta^2\epsilon_{\perp}}\right)\right\} \to \frac{\alpha}{\hbar c}\left(1-\frac{1}{\beta^2 n^2}\right)_{\beta n>1},$$

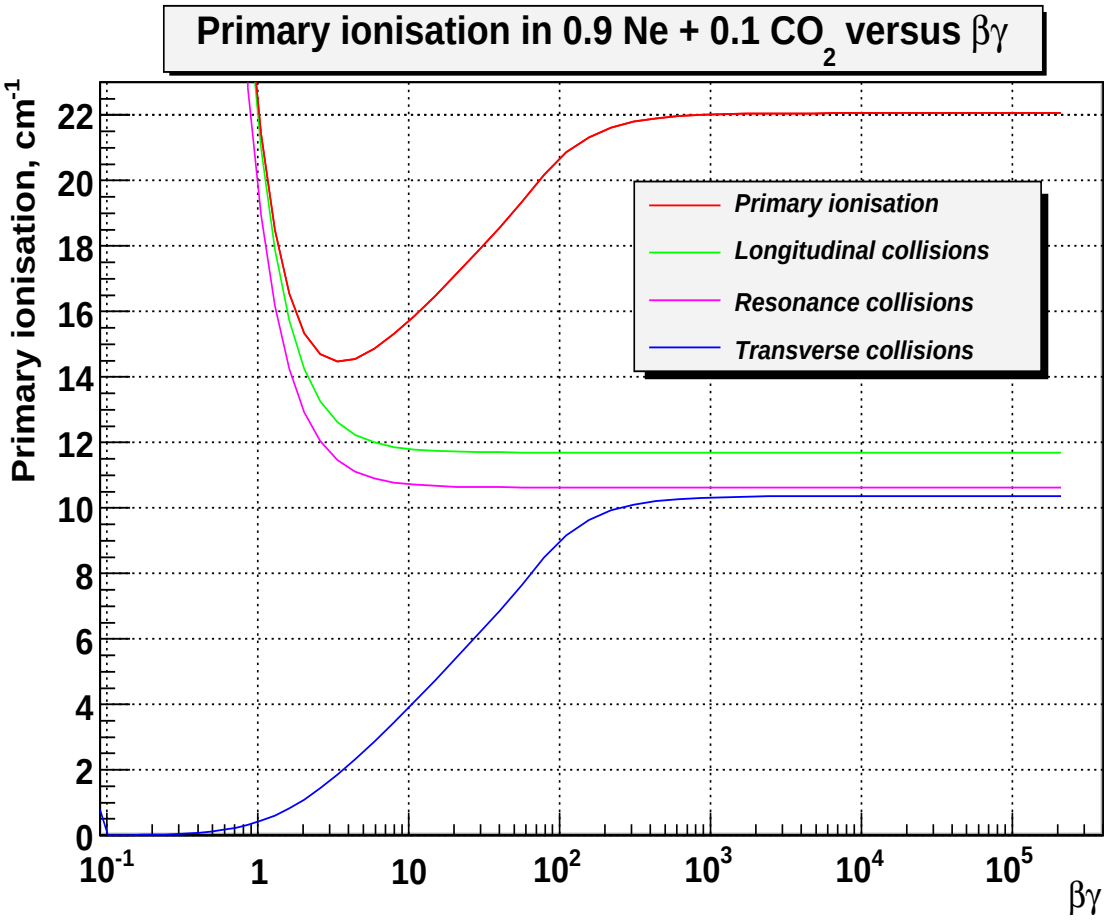
where in transparent medium refractive index, n is defined by dielectric permeability, $n^2 = \epsilon_{\perp}$. Secondary electrons are produced in longitudinal collisions. These are resonance collisions (direct excitation of atomic shell):

$$\frac{d^2\bar{N}_{\parallel}^{res}}{\hbar d\omega\, dx} = \frac{\alpha N}{\pi\hbar\beta^2}\frac{\sigma_{\gamma}(\omega)}{\omega}\ln\frac{2mv^2}{\hbar\omega},$$

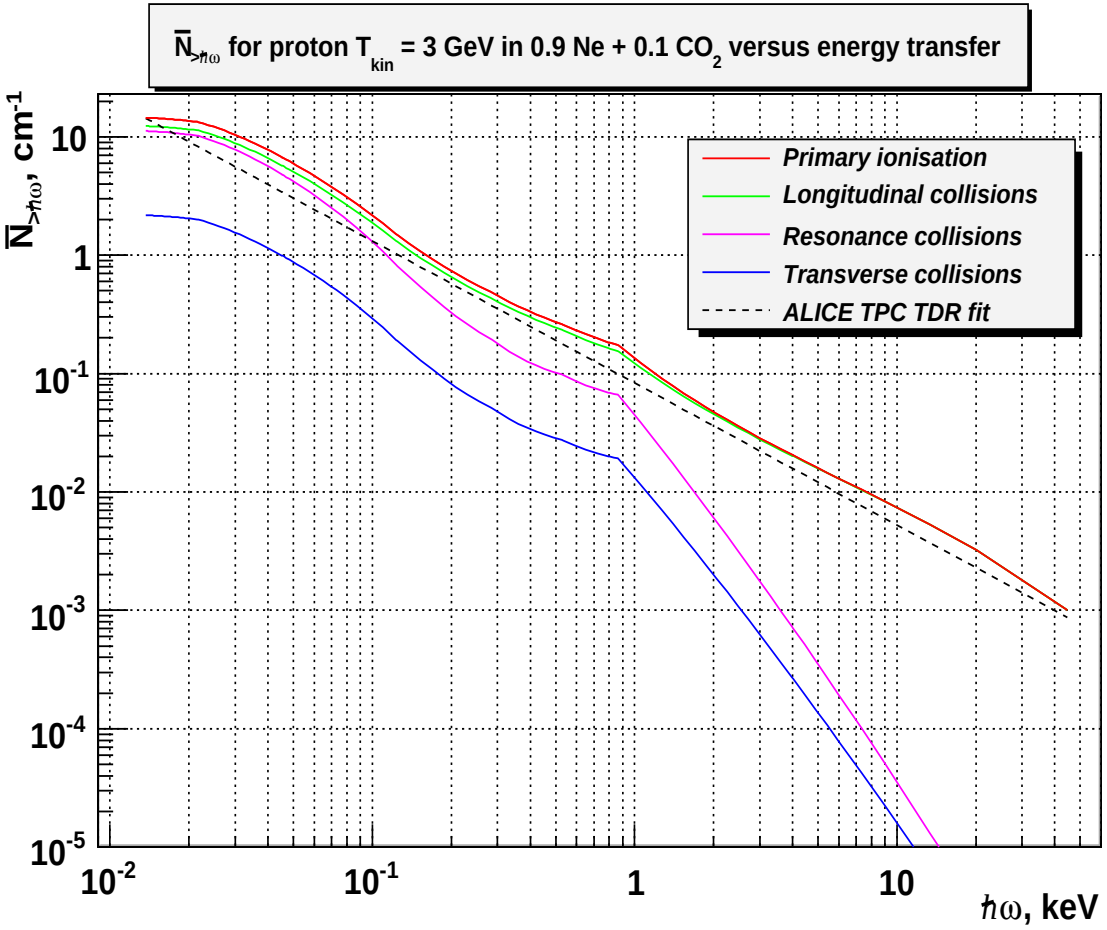
and Rutherford collisions with energy transfer more than shell binding (scattering on quasi-free electrons):

$$\frac{d^2\bar{N}_{\parallel}^{Ruth}}{\hbar d\omega\, dx} = \frac{\alpha N}{\pi\hbar\beta^2}\frac{1}{\omega^2}\int_0^{\omega}\sigma_{\gamma}(\omega')\,d\omega'.$$

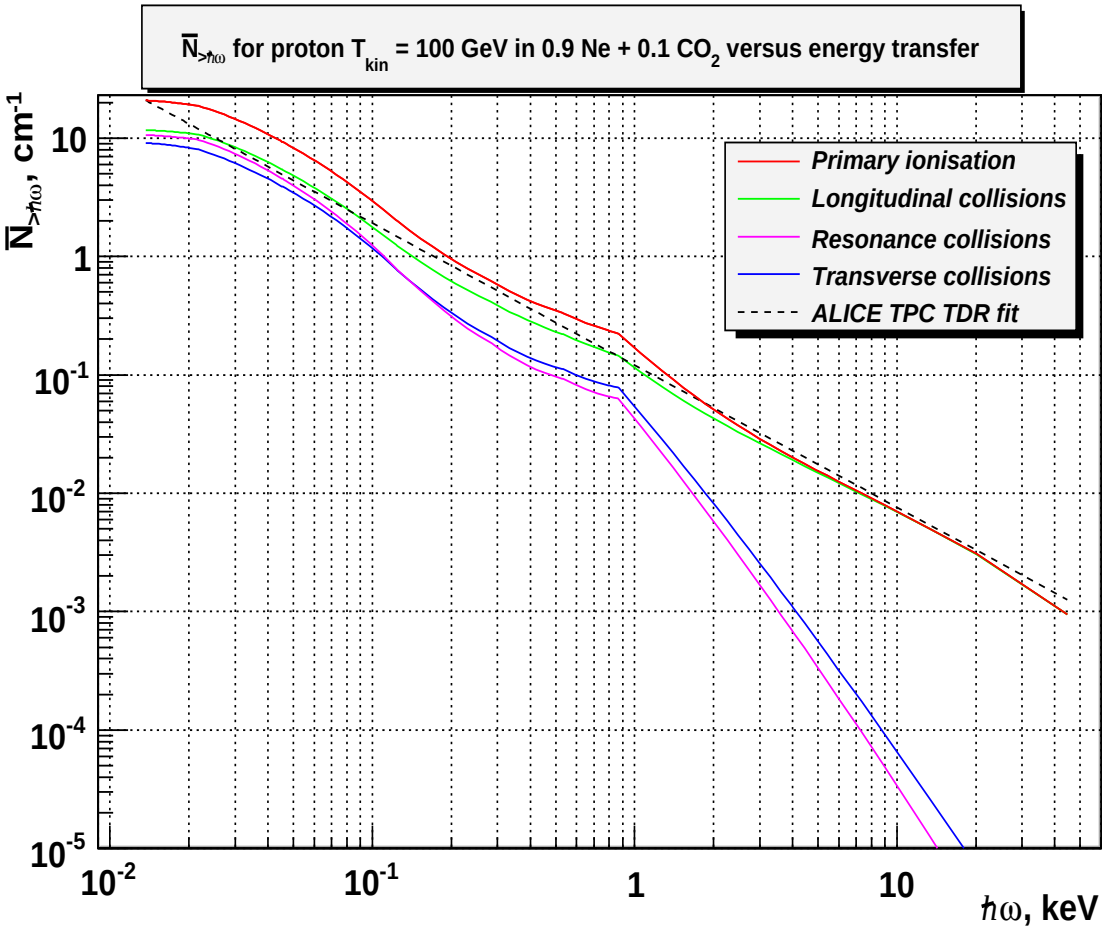
The majority ($\sim 90-95\%$) of longitudinal collisions are resonance ones.



Resonance collisions (direct exitation of shell electron) are about 90% of longitudinal collisions.



Resonance collisions contribute mainly for soft energy transfers while for $\gamma \sim 4$ Cherenkov energy loss is small. ALICE fit is $14.35/(\hbar\omega)^{1.2}$.



Resonance collisions contribute mainly for soft energy transfers while for $\gamma \sim 100$ Cherenkov energy loss is about resonance one.

Cerenkov photons are absorbed in the vicinity of the particle track and produce photo-electrons. Thus the total ionisation is defined by photo-, resonance and Rutherford secondary electrons. The practical electron ranges, R_e can be estimated using empirical formula [7]:

$$R_e(E) = \frac{A\,T_{kin}}{\rho}\left[1 - \frac{B}{1 + C\,T_{kin}}\right],$$

where T_{kin} $(=\hbar\omega)$ is the electron kinetic energy, and A, B, C are fitting parameters. In the case of Cerenkov photon absorption, its free path λ_γ should be added:

$$\lambda_\gamma = \frac{1}{\sigma_\gamma \rho},$$

where σ_γ is the photo-absorption cross-section and ρ is the material density. The average practical ionisation ranges perpendicular to the track read:

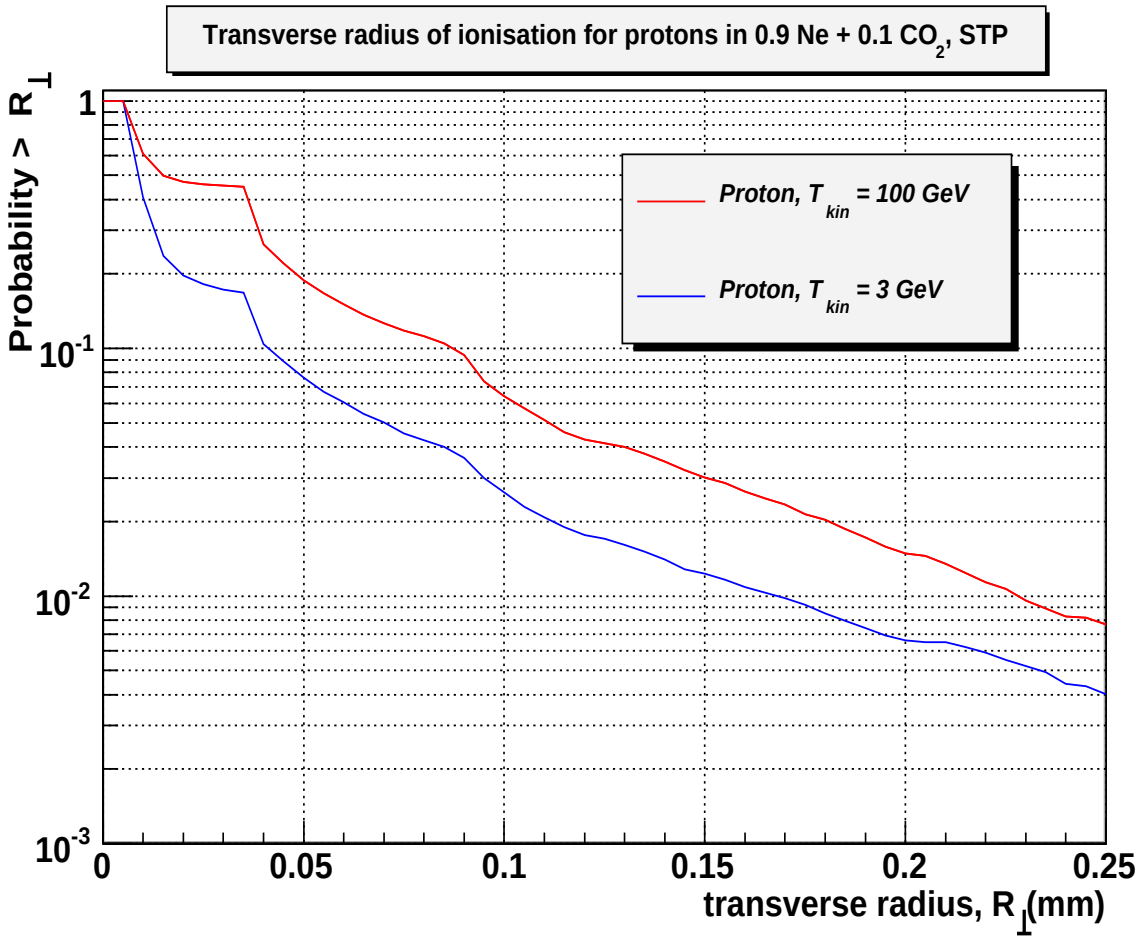
$$< R^2_{cer} >_\Omega = \frac{2}{3}(\lambda_\gamma + R_e)^2, \quad < R^2_{res} >_\Omega = \frac{2}{3}R^2_e, \quad < R^2_{Ruth} >_\Omega = R^2_e,$$

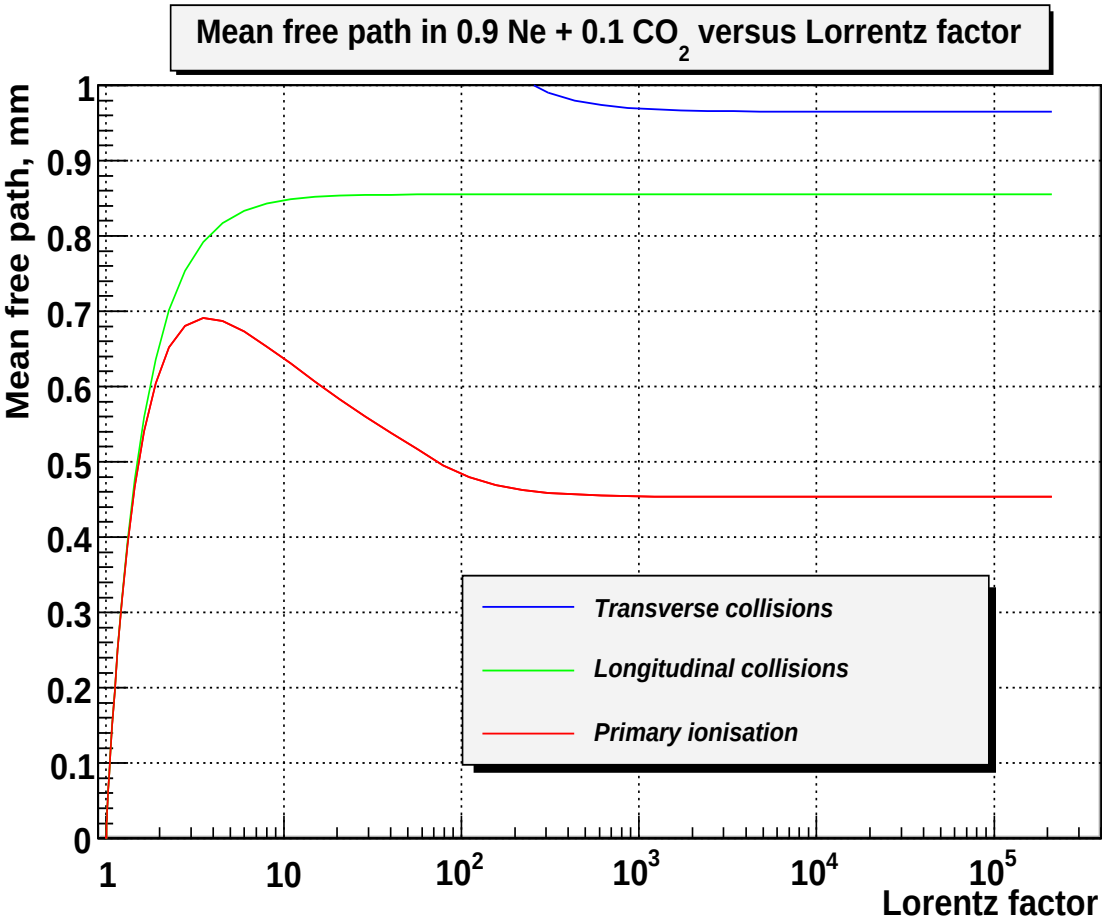
since Cerenkov photons and resonance electrons are widely distributed in Ω-forward semi-sphere, while Rutherford electrons are approximately perpendicular to the particle track.

MRS ranges of secondary electrons produced by protons
in 90% Ne + 10% CO_2 gas mixture, STP (1 atm, 0 °C)
for 1000 events, (range, μm/% events)

proton T_{kin}, GeV	Cerenkov	Resonance	Rutherford	Total, μm
1	248/5%	13/86%	66/9%	60
3	218/14%	13/78%	65/8%	83
100	181/43%	15/51%	45/6%	119
1000	252/47%	15/47%	46/6%	173

Ionisation clusters are not point-like. The RMS ranges are however small ($\sim 10\%$) compared to ALICE TPC diffusion spread, $\sigma_{\perp,\parallel} = D_{\perp,\parallel}\sqrt{L_{drift}}$ ($D_\perp \sim D_\parallel \sim 200\mu\text{m}/\sqrt{\text{cm}}$).


Transverse radius of ionisation for protons in 0.9 Ne + 0.1 CO2, STP
Probability > R⊥
1
10-1
10-2
10-3
0
0.05
0.1
0.15
0.2
0.25
transverse radius, R⊥(mm)
Proton, Tkin = 100 GeV
Proton, Tkin = 3 GeV



Track is sequence of ionisation clusters with $R < \lambda$. Ionisation takes about 30 % of relativistic particle trajectory length.

4 Ionisation vs. Energy Transfer ($\hbar\omega$)

Mean ionisation, $\bar{n}_i$ (mean number of ion (electron-ion in gas) pairs) produced by electron with the kinetic energy, T_{kin}, fully absorbed in medium, reads by empirical definition:

$$\bar{n}_i = \frac{T_{kin}}{W},$$

where W is the mean energy required for creation of one ion pair (working function). Due to shell binding the ionisation process is not continuous ($n_i^{max} \sim T_{kin}/I_1$), and its fluctuation, σ_i, less than poissonian - $\sigma_i^2 = F\,\bar{n}_i$, where F (~ 0.2) is Fano factor. Gaussian($\bar{n}_i, \sigma_i$) is often used to estimate the ionisation distribution. For $T_{kin} \gg I_K$ the working function $\sim$ constant - W_o. For smaller energies like typical $\hbar\omega$ it depends (due to shell effects) on energy absorbed - $W(\hbar\omega)$, so $\bar{n}_i = \hbar\omega/W(\hbar\omega)$. Empirically all shell effects are included in this relation ($T_{kin} = \hbar\omega$). The HEED program [8] proposed (among others) the empirical relation for $W(\hbar\omega)$:

$$W(\hbar\omega) = \frac{W_o}{1 - \left(\frac{V}{\hbar\omega}\right)^2}, \quad V = \frac{W_o}{2}.$$

5 Empirical or Modeled Ionisation (?)

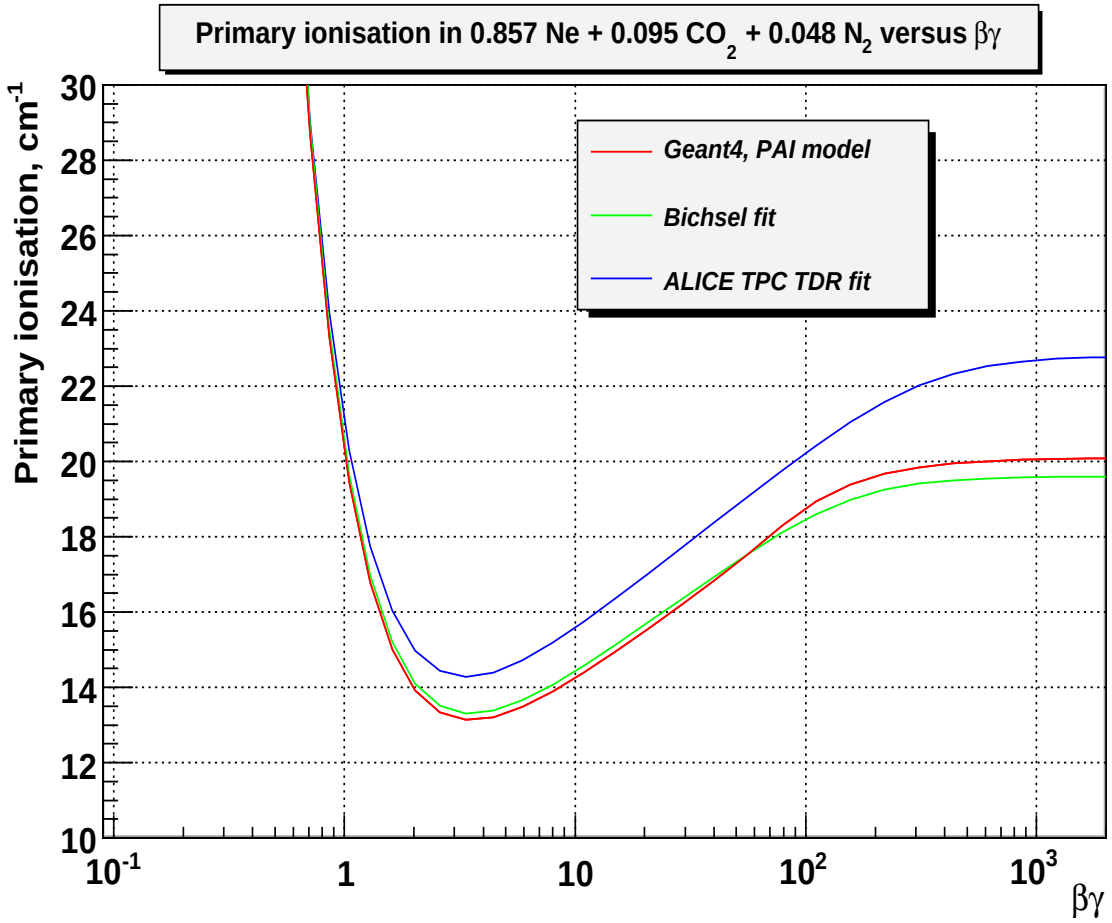
Empirical relations shown above describe in average all shell effects. Therefore, the description of ionisation should be either fully empirical or recursively (atom excitation-relaxation) modeled. There are issues in model approach [8]:

1. According to PAI model the resonance energy transfers are shell related only. Both Cerenkov and Rutherford energy transfers are collective. Cerenkov photons do not produce direct ionisation.

2. Resonance collisions dominate at very low particle energies where PAI model based on Born approximation is not valid.

3. Recursive process of atom excitation-relaxation (down to $T^e_{kin} < I_1$) is time consuming. We do not have efficient precise models.

4. Ionisation is measured but always in energy calibration. We see ionisation but do need energy loss! Ionisation itself can be excluded from simulation based on space distribution of charged energy loss - $d(\hbar\omega)/d(\text{gas volume})$.



In 2004 test beam of ALICE TPC not TDR gas mixture was used (other contents, temperature and pressure). Gas mixture should be fixed before preparation of theory driven paramerisation. Fit according to H. Bichsel model for Ne at T = 19 °C, P = 696 mbar was provided by Peter Christiansen, Lund.

6 Summary

1. PAI model provides information for the preparation of parametrized generator of ionization clusters in sensitive detectors.
2. The efficient volume of ionisation deposition depends on particle energy. For relativistic particles Cerenkov photons are responsible for approximately half of events. This is not direct ionisation on the particle track!
3. PAI driven parametrisation can be implemented. It depends however on gas mixture and its density (temperature, pressure).

References

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