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Gravitational waves in a bigravity model: from inflation to present

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based on works by Giulia Cusin, Ruth Durrer, PG and Mariele Motta
(arXiv:1412.5979 and arXiv:1505.01091)

Outline

- Review of the cosmological background in bigravity
- Focus on the "$\beta_1\beta_4$ model"
- Cosmological evolution of tensor perturbations
- Generation of tensor perturbations during inflation

- We consider cosmological (i.e. spatially homogeneous and isotropic) background solutions (in conformal time) of Hassan & Rosen bigravity [1109.3515]:

$$g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau)\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right)$$

$$f_{\mu\nu}dx^\mu dx^\nu = b^2(\tau)\left(-c^2(\tau)d\tau^2 + \delta_{ij}dx^i dx^j\right)$$

- Physical and conformal Hubble parameters, ratio of the scale factors:

$$H = \frac{\mathcal{H}}{a} = \frac{a'}{a^2}\,, \quad H_f = \frac{\mathcal{H}_f}{b} = \frac{b'}{b^2\,c}\,, \quad r = \frac{b}{a}\,, \quad {}' \equiv \frac{d}{d\tau}$$

- The energy-momentum tensor has the form of a perfect fluid:

$$T_{\mu\nu} = (p+\rho)\; u_\mu u_\nu + p\, g_{\mu\nu}\,, \quad \rho' = -3(\rho + p)\,\mathcal{H}\,, \quad p = w\rho\,.$$

- The two Friedmann equations are:

$$H^2 = \frac{8\pi G}{3}\;(\rho + \rho_g)\;, \quad H_f^2 = \frac{m^2}{3}\left(\frac{\beta_1}{r^3} + \frac{3\beta_2}{r^2} + \frac{3\beta_3}{r} + \beta_4\right)\,,$$

$$\rho_g = \frac{m^2}{8\pi G}\left(\beta_3\, r^3 + 3\beta_2\, r^2 + 3\beta_1\, r + \beta_0\right)$$

- Bianchi constraint: $$(\beta_3\, r^2 + 2\beta_2\, r + \beta_1)(\mathcal{H} - \mathcal{H}_f) = 0$$

There are **two branches of solutions:** Koennig & al. [1312.3208], Lagos & Ferreira [1410.0207]

1) BRANCH I: $\beta_3 r^2 + 2\beta_2 r + \beta_1 = 0 \;\longrightarrow\; r = \bar{r} = const.$

This branch is equivalent, at background level, to GR with an effective cosmological constant

2) BRANCH II: $\mathcal{H}_f = \mathcal{H}$ $\longrightarrow$ late-time de Sitter phase: promising!

Several possible sub-branches of solutions are possible, depending on the initial value for *r*, for the following set of coupled background equations:

$$c = \frac{\mathcal{H}r + r'}{\mathcal{H}r}\,, \qquad \frac{r'}{r} = \frac{-9\beta_1 r^2 + 3\beta_1 + 3\beta_4 r^3 + r M_p^{-2} m^{-2} \rho_r}{3\beta_1 r^2 + \beta_1 - 2\beta_4 r^3}\mathcal{H}\,,$$

$$\rho_m = M_p^2 m^2 \left(\frac{\beta_1}{r} - 3\beta_1 r + \beta_4 r^2\right) - \rho_r$$

$$\mathcal{H}^2 = a^2 m^2 \frac{\beta_1 + \beta_4 r^3}{3r} \xrightarrow{(\text{at } \tau = \tau_0)} \text{extract initial value } r_0 = r(\tau_0)$$

such that we are in the **infinte branch**

1) **Finite branch**: at early times $r \ll 1$ and r evolves from 0 to a finite value

→ all of the solutions on this branch have either an unviable background evolution, or (exponentially) unstable scalar perturbations in the past

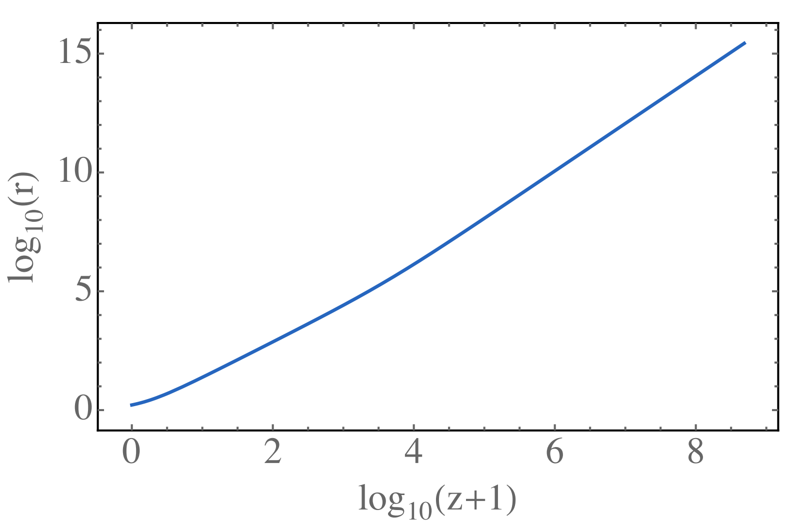
2) **Infinite branch**: at early times $r \gg 1$ and r evolves from infinity to a finite value

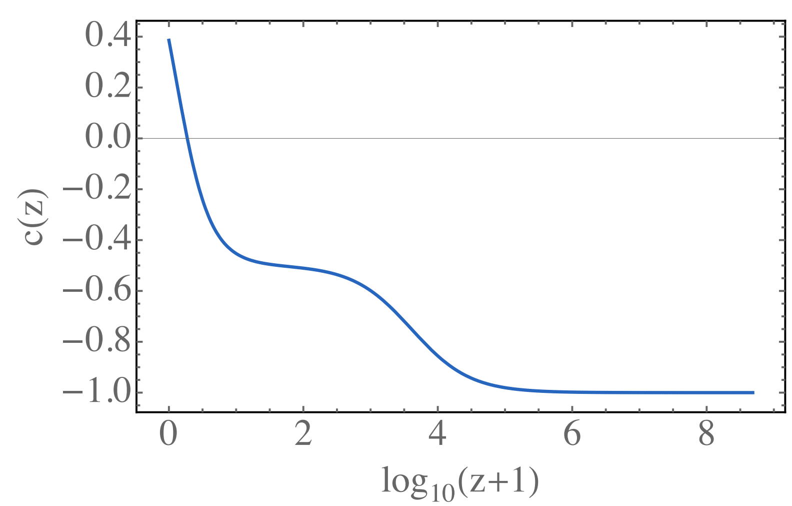
→ there exists a self-accelerating model which seems to have a stable cosmological evolution: the **'$\beta_1\beta_4$ model'** ($\beta_0=\beta_2=\beta_3=0$)

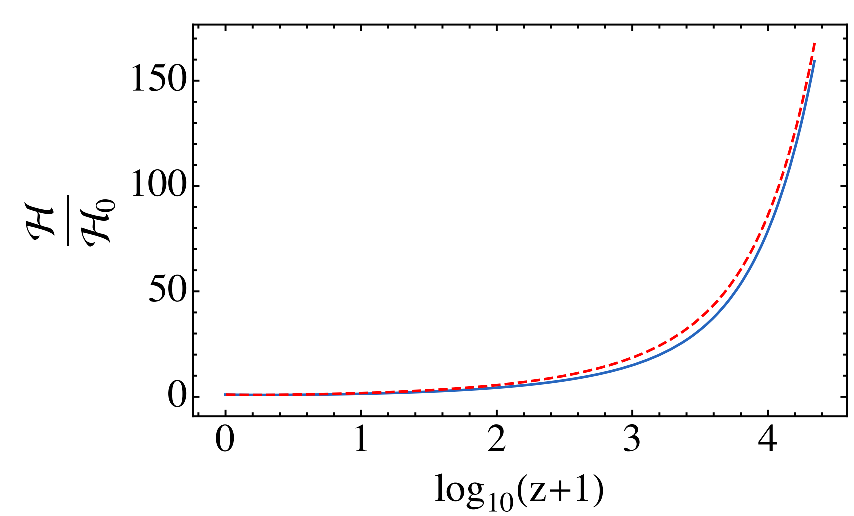
To solve the system of dynamical equations numerically, we use the best-fit values <u>$\beta_1=0.48$, $\beta_4=0.94$</u>, obtained by fitting measured growth data and type Ia supernovae.

Koennig & al. [1407.4331]

Cosmological evolution of background quantities:







Tensor metric perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2\, h_{g\mu\nu}$$
$$f_{\mu\nu} = \bar{f}_{\mu\nu} + b^2\, h_{f\mu\nu}$$

$$(h_g)_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h_{ij}^{(g)} \end{pmatrix},$$

$$(h_f)_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h_{ij}^{(f)} \end{pmatrix},$$

$$\partial_i h^{ij}_{(g,f)} = 0\,, \qquad \delta^{ij} h_{(g,f)ij} = 0\,.$$

Tensor perturbations in the $\beta_1\beta_4$ model

Comelli & al. [1202.1986], Lagos & Ferreira [1410.0207]

$$h_g'' + 2\mathcal{H}\, h_g' + k^2 h_g \overset{C_g}{\boxed{+\, m^2 a^2 r\, \beta_1\, (h_g - h_f)}} = 0$$

$$h_f'' + \left[2\left(\mathcal{H} + \frac{r'}{r}\right) - \frac{c'}{c}\right] h_f' + c^2 k^2\, h_f \overset{C_f}{\boxed{-\, m^2 \beta_1 \frac{c\, a^2}{r}\, (h_g - h_f)}} = 0$$

During RD (when we have to put the initial conditions):

1) $r \gg 1\,,\; r \propto a^{-2}\,,\; c \simeq -1 \simeq const. \Rightarrow C_f \ll C_g\,, [\ldots] \to -2\mathcal{H}$

2) $\mathcal{H} = 1/\tau$

The e.o.m. then become:

$$h_g'' + \frac{2}{\tau}\, h_g' + k^2 h_g + m^2 a^2 r\, \beta_1\, (h_g - h_f) = 0$$

$$h_f'' - \frac{2}{\tau}\, h_f' + k^2\, h_f = 0$$

Solution for the h_f e.o.m. : $\quad h_f = c_3 (k\tau)^2 y_1(ck\tau) - 3 c_4 \dfrac{(k\tau)^2}{(k\tau_{\rm in})^3} j_1(ck\tau)$

Moreover, on super-Hubble scales ($k\tau \ll 1$):

1) k^2-terms become negligible

2) $m^2\beta_1 a^2 r \simeq (0.05\mathcal{H}_0)^2 < k^2$

The e.o.m. become:

$$h_g'' + \frac{2}{\tau}\, h_g' = 0\,, \qquad h_f'' - \frac{2}{\tau}\, h_f' = 0$$

and are solved by:

$$h_g = c_1 + c_2\left(\frac{\tau_{\rm in}}{\tau}\right) \simeq A\,,$$

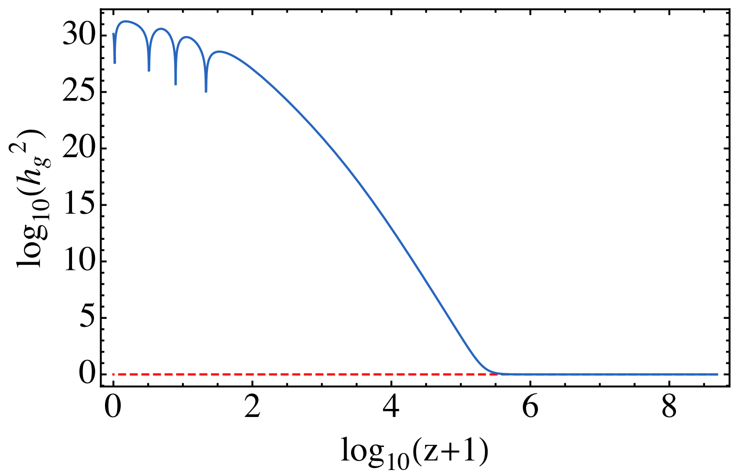
$$h_f = c_3 + c_4\left(\frac{\tau}{\tau_{\rm in}}\right)^3 \simeq B\left(\frac{\tau}{\tau_{\rm in}}\right)^3.$$

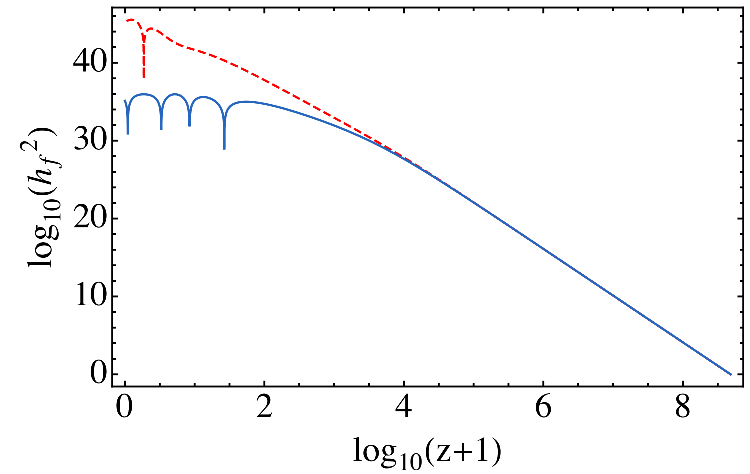
h_f has a growing mode which indicates an instability!

Initial conditions in RD are $h_g(\tau_{\rm in})=A$, $h_f(\tau_{\rm in})=B$

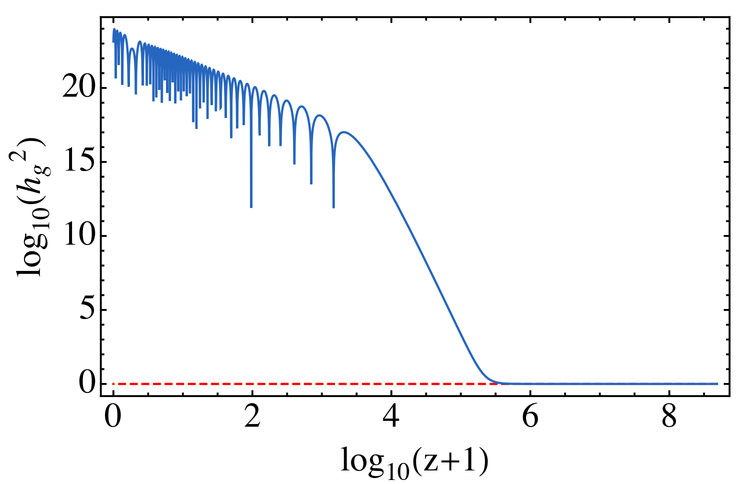
Choosing initial conditions for numerical evolution as A=B=1: h_g tremendously amplified via its coupling to h_f!!

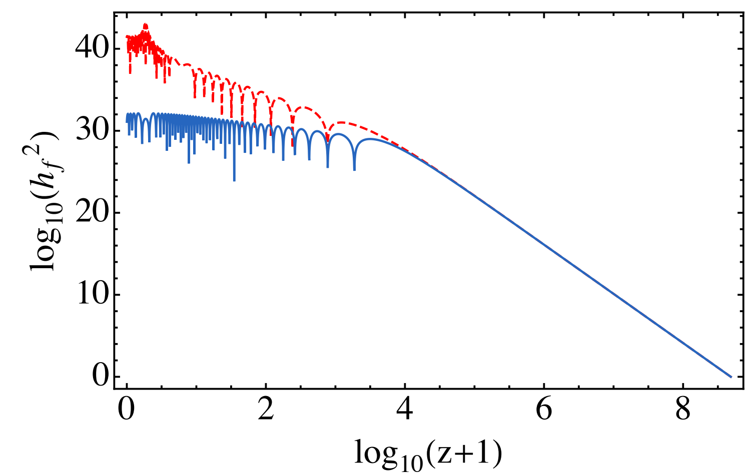
$$k \simeq 10\,\mathcal{H}_0$$



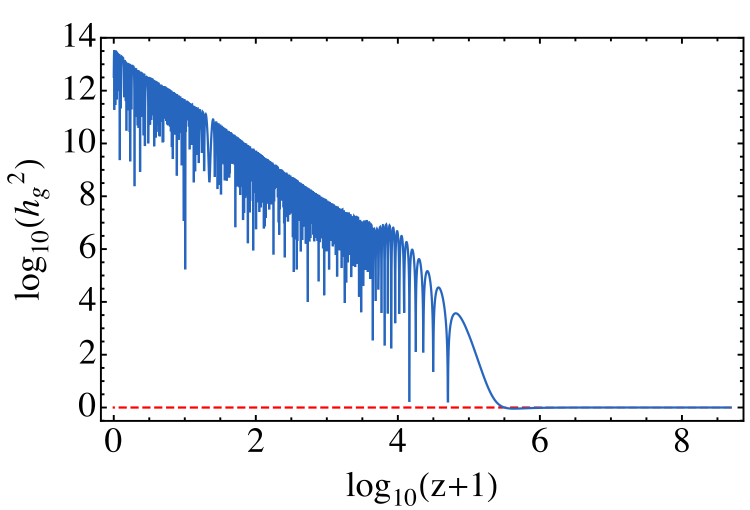


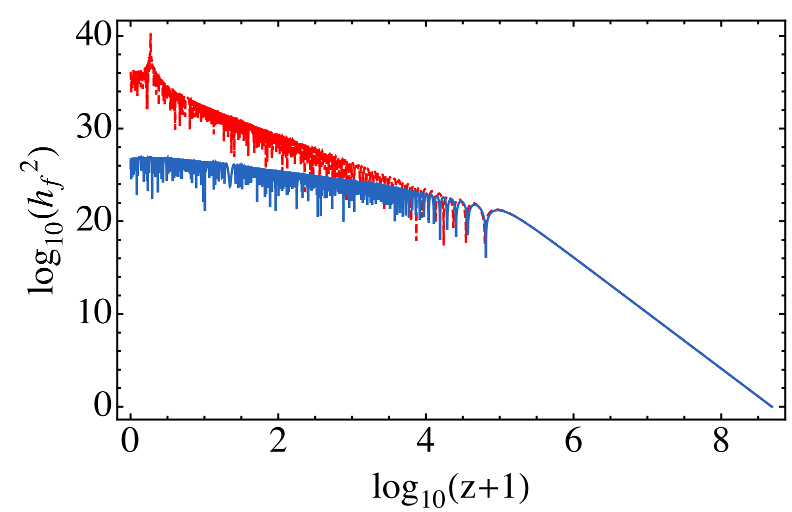
$$k \simeq 100\,\mathcal{H}_0$$





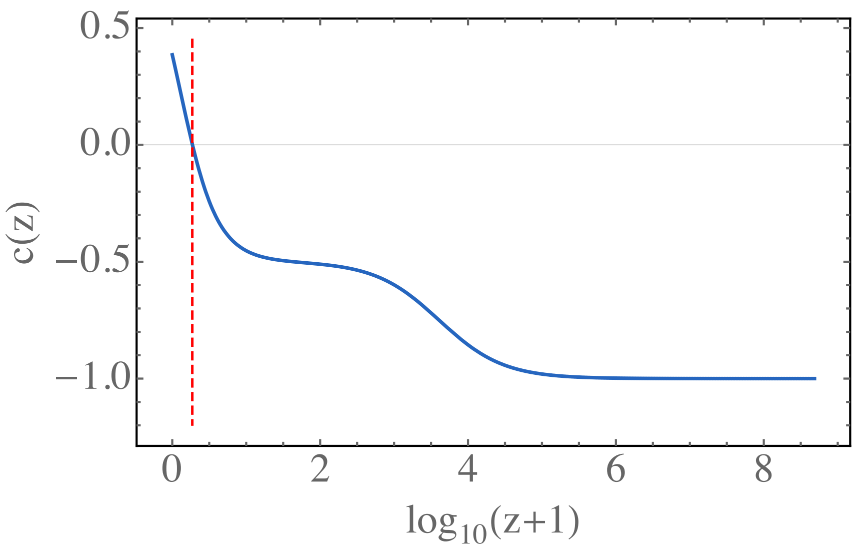
$$k \simeq 2000\,\mathcal{H}_0$$





What is the reason for the h_f instability?

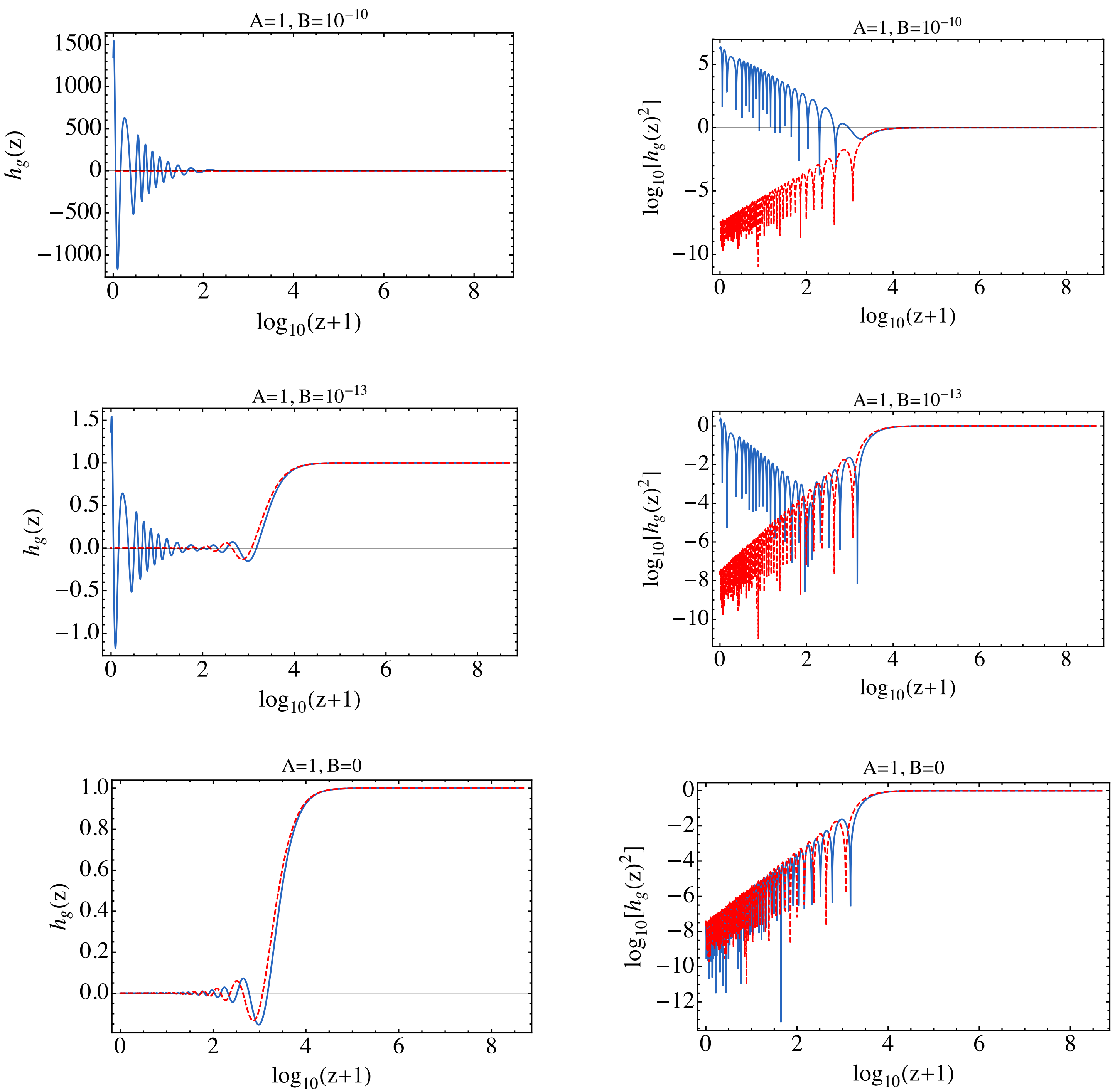
$$S_{\rm kin}^{(\pm 2)} \propto M_g^2 \int d^4x\, a^2 \left((h_g')^2 + r^2\, \frac{1}{c} (h_f')^2 \right)$$



The kinetic term for h_f has the wrong sign until recent time, since c becomes positive only at $z_c \simeq 0.9 \Rightarrow$ <u>the tensor sector is affected by a ghost power-law instability</u>, due to the violation of the *generalised Higuchi bound* on FLRW background.

The initial conditions for this unstable mode h_f need to be significantly suppressed w.r.t. the ones for the physical mode h_g in order to recover the standard (GR) behaviour of GWs!!

Choosing initial conditions for numerical evolution as B≪A: evolution of h_g converges to the GR one as B decreases w.r.t. A



Instability in h_g does not show up until today if B≪A

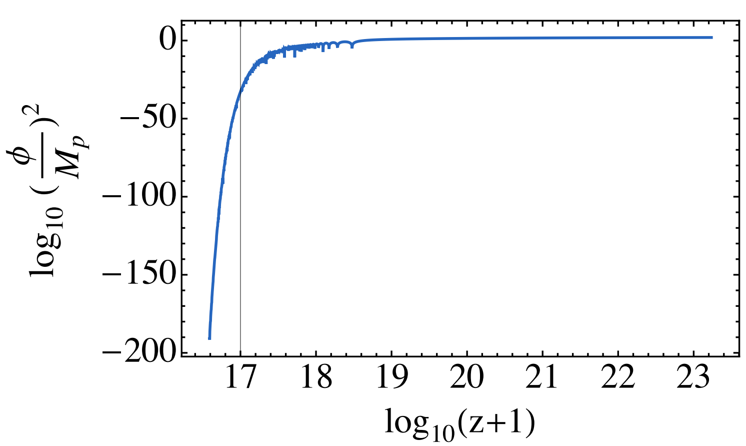
The physical values for B and A are settled at the beginning of RD ➔ we need to study inflation in this model to put constraints!!

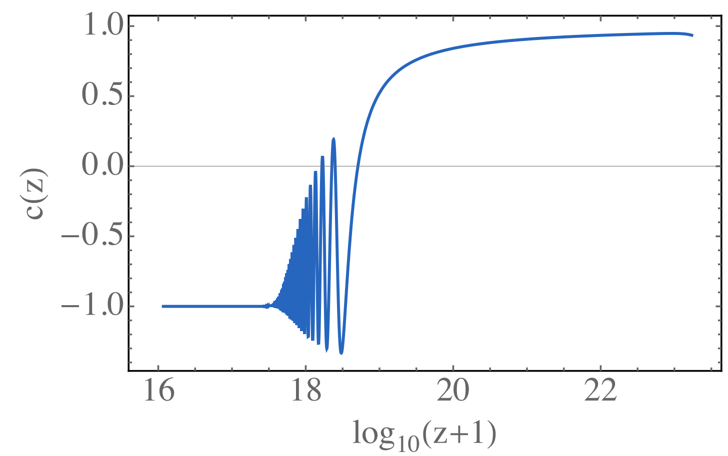
The $\beta_1\beta_4$ model during inflation: the background evolution

The inflationary phase is described by the dynamics of a single (slow-rolling) scalar field ϕ:

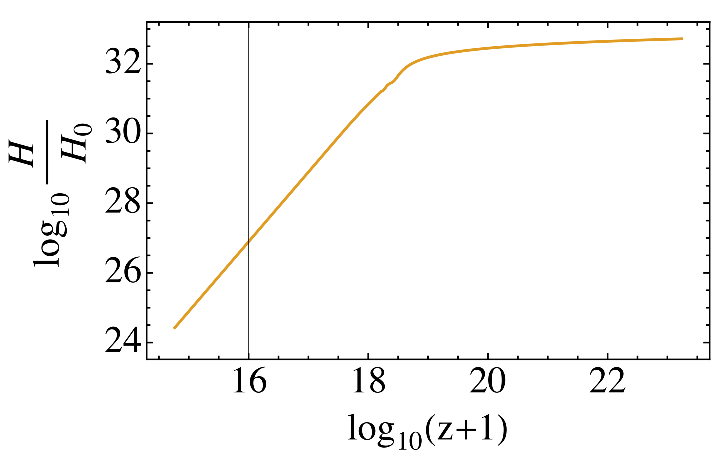
$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\,, \qquad V(\phi) = \frac{1}{2}M_\phi^2\,\phi^2$$

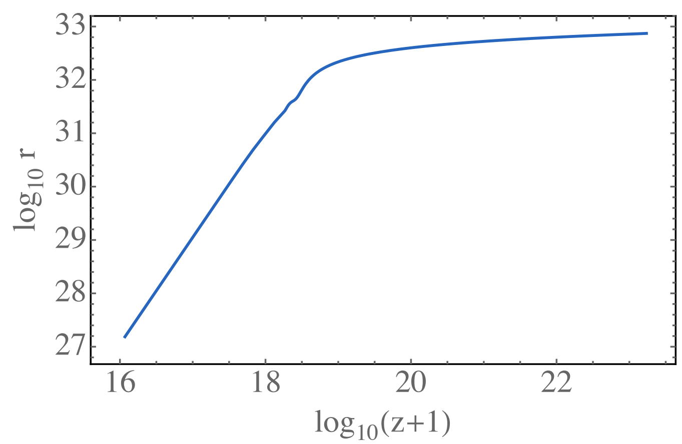
We embed the $\beta_1\beta_4$ bigravity model into this inflationary scenario:





$$c \simeq constant \simeq 1$$





$$r_I \simeq \frac{H_I}{H_0} = constant \gg 1$$

The $\beta_1\beta_4$ model during inflation: tensor perturbations

In a background modelled as pure de Sitter with H=$H_{\rm I}$≃constant, the tensor e.o.m. are:

$$\boxed{\begin{aligned} &h_g'' - \frac{2}{\tau}\, h_g' + k^2 h_g + \left(\frac{H_0}{H_I}\right) \frac{1}{\tau^2}\,(h_g - h_f) = 0 \\ &h_f'' - \frac{2}{\tau}\, h_f' + k^2\, h_f - \left(\frac{H_0}{H_I}\right)^3 \frac{1}{\tau^2}\,(h_g - h_f) = 0 \end{aligned}}$$

$$\Downarrow$$

Power spectra:

$$P_{h_g}(z,k) \simeq \left(\frac{H_I}{M_p}\right)^2 \simeq r_I^2\, P_{h_f}(z,k) \Rightarrow P_{h_g} \gg P_{h_f}\,, \qquad |k\,\tau| \ll 1\,.$$

What about the instability in the tensor sector?

At the end of inflation, for $|k\tau_e| \ll 1$,

$$h_f \sim r_I^{-1} \frac{H_I}{M_p} \left(1 + \left(\frac{k}{H_I} \right)^2 (1 + z_{end})^2 \right) ,$$

In RD, for $|k\tau| \ll 1$,

$$h_f = c_3 + c_4 \frac{1}{(1+z)^3}$$

- Only the decaying mode of h_f turns into the growing mode!
- This decaying mode is severely suppressed:

$$\left(\frac{k}{H_I} \right)^2 (1 + z_{end})^2 = (k\tau_{end})^2 \ll 1$$

- In order for the GWs instability not to show up until today, the coupling term $h_g \Leftrightarrow h_f$ has to be strongly suppressed w.r.t. h_g'' in the e.o.m. for h_g
- Does this yield any bounds on the inflationary energy scale?
- Considering only the decaying mode for h_f at the end of inflation, one finds the condition:

$$\left(\frac{h_f}{h_g} \right) (\tau_0) \lesssim r_I^{-1} \left(\frac{H_0}{H_I} \right)^{-1/2} \simeq \left(\frac{H_0}{H_I} \right)^{1/2} \ll 1$$

No significant bound on the inflationary scale H_I

↓

The $\beta_1\beta_4$ model is not ruled out from the analysis of the tensor perturbations!

(in agreement with Johnson & Terrana [1503.05560])

Conclusions

- In the “$\beta_1\beta_4$ model” of bigravity there is an instability in the f-sector of tensor perturbations during the cosmological evolution which can affect the physical sector through the coupling.
- However, we have found that if the initial value of h_f is severely suppressed w.r.t. the one of h_g, the instability does not show up in the physical sector until today.
- The evolution of GWs is actually ΛCDM-like for every inflation energy scale.

Thank you for your attention

Tensor-to-scalar ratio in bigravity

Standard tensor-to-scalar ratio:

$$r_T = \frac{A^2}{A_s^2} \simeq \frac{H_I^2}{M_p^2 A_s^2} = 16\epsilon \simeq 0.1$$

Tensor-to-scalar ratio in bigravity on very large scales:

$$r_T = 16\epsilon \left(\frac{T_{\rm in}}{T_{\rm eq}}\right)^6 \left(\frac{T_{\rm eq}}{T_0}\right)^3 \simeq \left(\frac{T_{\rm in}}{0.01\text{GeV}}\right)^{10} \left(\frac{B}{A}\right)^2$$

We need to suppress the generation of h_f such that
$B = h_f(\tau_{\rm end}) \ll h_g(\tau_{\rm end}) = A$ at the end of inflation!

Behaviour of h_f during reheating

- We have assumed that the transition between inflation and RD is instantaneous to derive our result.
- What if there is a relatively long reheating phase in between?

$$h'' + \alpha(\tau)h' = 0$$

$$\Downarrow$$

$$h(\tau) = A_1 \int^{\tau} d\tau' \left[\exp\left(-\int^{\tau'} \alpha(\tau'')d\tau''\right)\right] + A_2\,.$$

- There will also be a constant mode during reheating.

What about the inflation energy scale?

- Inflationary energy scale typically above the so-called *strong coupling scale* of the theory, $\Lambda_3=(M_\mathrm{p}m^2)^{1/3} \Longrightarrow$ see Giulia's talk.

- Λ_3 has been derived in a Minkowski background: not completely clear what its actual meaning is on a cosmological background ($\Rightarrow$ also discussion session…)

Vector and scalar perturbations during inflation

- In a de Sitter background, only the gauge-invariant vector mode $\mathcal{V}_i = \mathcal{V}_{g,i} - \mathcal{V}_{f,i}$ propagates.

Comelli & al. [1202.1986], De Felice & al. [1404.0008]

- One finds that, on super-Hubble scales, the vector power spectrum reads

$$P_{\mathcal{V}}(k\,,\tau) \equiv k^3 |k\,\mathcal{V}_i|^2 \simeq \left(\frac{H_I}{M_p}\right)^2 \frac{H_I}{H_0}\,, \qquad\qquad |k\tau| \ll 1$$

- Linear perturbation theory remain viable if at least $P_{\mathcal{V}} < 1 \Rightarrow H_I < 10^{-2}\,\mathrm{GeV}$ which requires a low (but still acceptable) scale of inflation.

- In a de Sitter background, if we neglect the inflaton perturbation, only the scalar mode $\Phi = \Phi_g - 2r_I^2 \Phi_f$ propagates.

Comelli & al. [1202.1986]

- On super-Hubble scales, Φ has a primordial exponential instability.
- This instability is a manifestation that the Higuchi bound is violated in the scalar sector of the $\beta_1\beta_4$ model!

Fasiello & Tolley [1308.1647], Lagos & Ferreira [1410.0207], Koennig [1503.07436]

- One would need a mechanism to modify the scalar sector of the theory in the UV in order to avoid to rule out the model…