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a relativistic approach to large-scale structure

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Wands & Slosar, arXiv:0902.1084

Bruni, Hidalgo, Meures & Wands, arXiv:1307.1478; Bruni, Hidalgo & Wands, arXiv:1405.7006

Bertacca et al, arXiv:1501.03163; Bartolo et al 1506.00915

Fidler, Rampf, Tram, Crittenden, Koyama & Wands arXiv:1505.04756

motivation

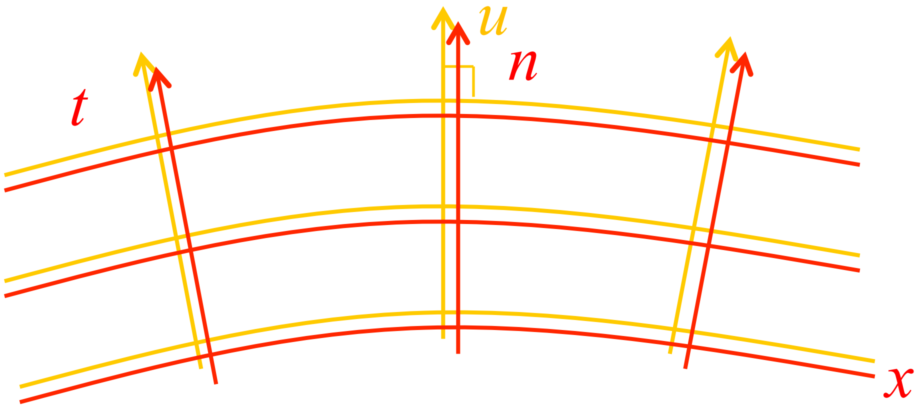
cosmic structure on very largest scales provides window onto the primordial density perturbation and hence models of the very early universe

Can we trust Newtonian models of large-scale structure close to the Hubble scale?

- Coordinate invariance vs absolute time and space
- Curved space vs flat space
- Non-linear constraints vs linear Poisson equation

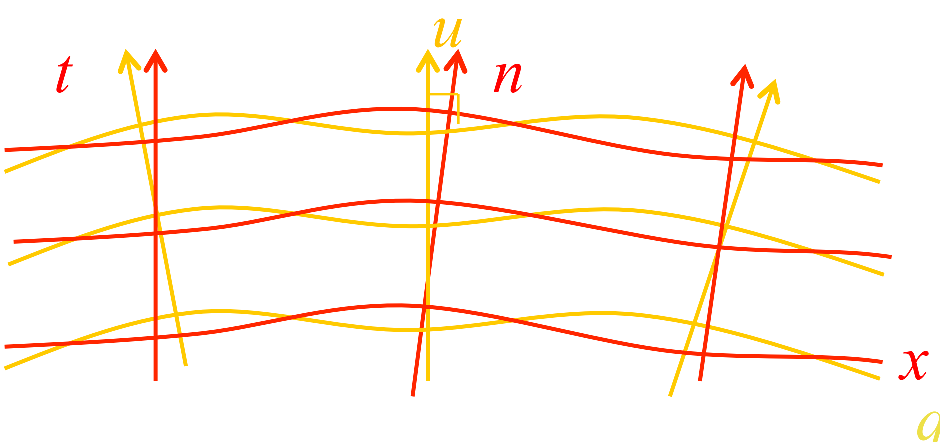
Different approaches:

- Non-linear density field
  - Green & Wald; Adamek et al; (weak-field)
  - Bruni et al (Post-Newtonian (1/c) expansion)
- **Perturbative GR** (early times and large scales)
  - Matarrese et al; Hwang et al; Yoo et al; … ***this talk***



FRW cosmology
“Newtonian”
unique comoving time and coordinates on maximally symmetric space

but no unique choice of time (slicing) and space coordinates (threading) in an inhomogeneous universe



FRW cosmology
+ perturbations

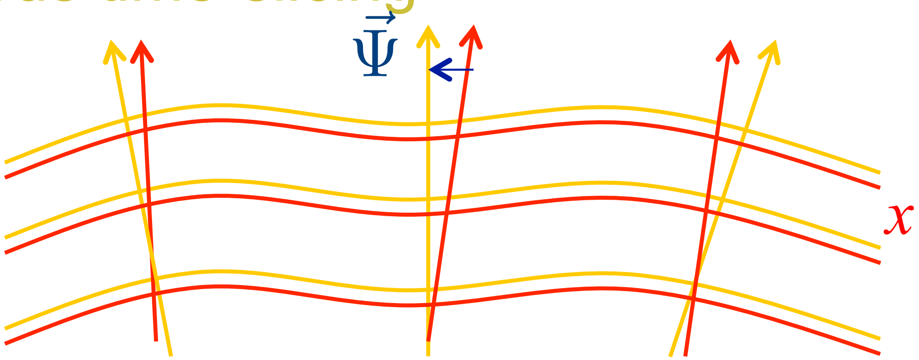
comoving-orthogonal coordinates (τ, q)

Newtonian frames
vs Relativistic comoving gauges

- Newtonian (absolute) time, t

  = comoving-synchronous time-slicing

$$\vec{x} = \vec{q} + \vec{\Psi}(\vec{q}, t)$$



- Lagrangian coordinates, q

  = comoving spatial coordinates – *unique!*

- Eulerian coordinates, x (note: comoving only in background)

  total-matter gauge (same spatial coords as conformal Newtonian)

  *or N-body gaug*e (see Fidler, et al, arXiv:1505.04756)

First-order matter perturbations in GR

- Energy and momentum conservation
  - comoving density contrast:

$$\dot{\delta} = -\vec{\nabla}.\vec{v} - 3\cancel{\dot{\mathcal{R}}}$$

  - total-matter velocity:

$$\dot{\vec{v}} + H\vec{v} = \vec{\nabla}\Phi$$

- Energy constraint:
  - Conformal Newtonian potential

$$\nabla^2\Phi = -4\pi G\bar{\rho}a^2\delta$$

- Momentum constraint:
  - Comoving curvature perturbation (for zero pressure):

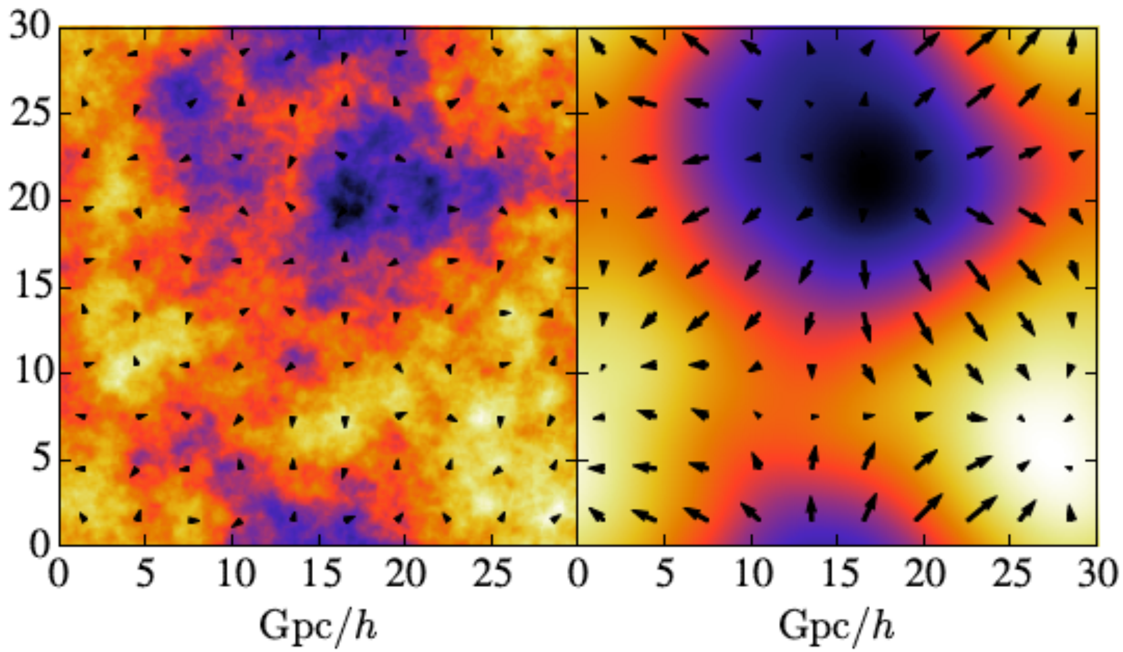
$$\dot{\mathcal{R}} = 0$$

First-order displacement in total-matter gauge

Fidler, Rampf, Tram, Crittenden, Koyama & Wands, arXiv:1505.04756

$$\dot{\delta} = -\vec{\nabla}.\vec{v} - 3\dot{\mathcal{R}} \quad \Rightarrow \quad \vec{\nabla}.\vec{\Psi} = -\delta - 3\mathcal{R}$$

Newtonian displacement   ***Relativistic correction in total matter***



GR volume distortion, $\mathcal{R}$, absent in Newtonian N-body density

$$\rho_{\mathrm{N-body}} = a^{-3} \sum_{\mathrm{particles}} m_p \delta^{(3)}(\vec{x} - \vec{x}_p) = \rho(1 + 3\mathcal{R})$$

First-order matter pertbns in ***N-body gauge***

Fidler, Rampf, Tram, Crittenden, Koyama & Wands, arXiv:1505.04756

- N-body density = physical comoving density

$$\rho \equiv \rho_{\mathrm{N-body}} = a^{-3} \sum m_p \delta^{(3)}(\vec{x} - \vec{x}_p)$$

- N-body displacement = Newtonian displacement

$$\vec{\nabla}.\vec{\Psi} = -\delta$$

- Energy and momentum conservation
  - comoving density contrast: $$\dot{\delta} = -\vec{\nabla}.\vec{v}$$
  - N-body velocity: $$\dot{\vec{v}} + H\vec{v} = \vec{\nabla}\Phi + \cancel{\vec{\nabla}\gamma}$$
- Energy constraint: $$\nabla^2\Phi = -4\pi G\bar{\rho}a^2\delta$$
  - Conformal Newtonian potential
- Momentum constraint: $$3\nabla^2\gamma \equiv \ddot{\mathcal{R}} + H\dot{\mathcal{R}} = 0$$
  - Comoving curvature perturbation constant (for zero pressure)

So does Newtonian = GR for ΛCDM?

- “even to the second order perturbation equations for the relativistic irrotational flow… coincide exactly with the previously known Newtonian equations”

  Hwang & Noh gr-qc/0412128

  - fluid flow evolution equations are the same in comoving-orthogonal time-slicing
  - ***but*** there are **non-linear constraints in GR**
    - GR corrections to non-linear density growing mode at second- and higher-order from a given primordial perturbation, e.g., ζ from inflation

What are the non-linear initial conditions in GR?

Bruni, Hidalgo & Wands (2014)

Matter density, ρ, and expansion, Θ, in ΛCDM

- *GR constraint relates density and expansion to spatial curvature*

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + {}^{(3)}R = 16\pi G\rho + 2\Lambda$$

Non-linear perturbations about FRW:
$$\begin{aligned} \Theta(t,x^i) &= 3H(t) + \theta(t,x^i)\,, \\ \rho(t,x^i) &= \bar{\rho}(t)\left[1+\delta(t,x^i)\right] \end{aligned}$$

At early times use large-scale limit – *gradient expansion*

$$\delta \sim \theta \sim \sigma \sim {}^{(3)}R \sim \nabla^2$$

- GR constraint becomes

$$\frac{{}^{(3)}R}{4} + H\theta = 4\pi G\bar{\rho}\delta + \mathcal{O}(\nabla^4)$$

spatial curvature is a non-linear function of the metric perturbation

Non-Gaussian density field from Gaussian $\zeta(x)$

Peak-background split: long and short wavelength modes

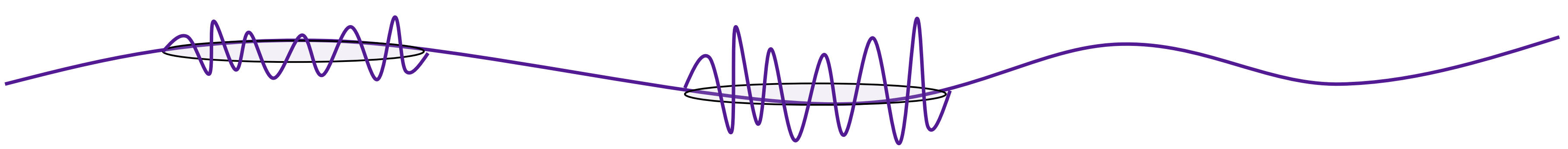
$$\zeta = \zeta_s + \zeta_\ell$$

Long-wavelength mode modulates spatial curvature (rescales background)

$$R = \exp(-2\zeta_\ell)R_s + \mathcal{O}(\nabla\zeta_\ell)$$

and hence growing mode / large-scale density perturbation:

$$\delta_m = \exp(-2\zeta_\ell)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$



large scale ζ_l modulates smaller scale δ_s
on fixed comoving (coordinate) scale

Compare GR curvature R(x) and non-Gaussian $\Phi(x)$

GR density perturbations from Gaussian $\zeta(x)$

$$\delta_m = \exp(-2\zeta_\ell)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$
$$= (1 - 2\zeta_\ell + 2\zeta_\ell^2 - \frac{4}{3}\zeta_\ell^3 + \ldots)\delta_s + \mathcal{O}(\nabla\zeta_\ell)$$

Newtonian density perturbations from non-Gaussian $\Phi(x)$

$$\delta_m = (1 + 2f_{NL}\phi_\ell + 3g_{NL}\phi_\ell^2 + 4h_{NL}\phi_\ell^3 + \ldots)\delta_s + \mathcal{O}(\nabla\phi_\ell)$$

compare term by term using linear relation: $\phi_\ell = (3/5)\zeta_\ell$

Einstein's signature in large-scale structure:

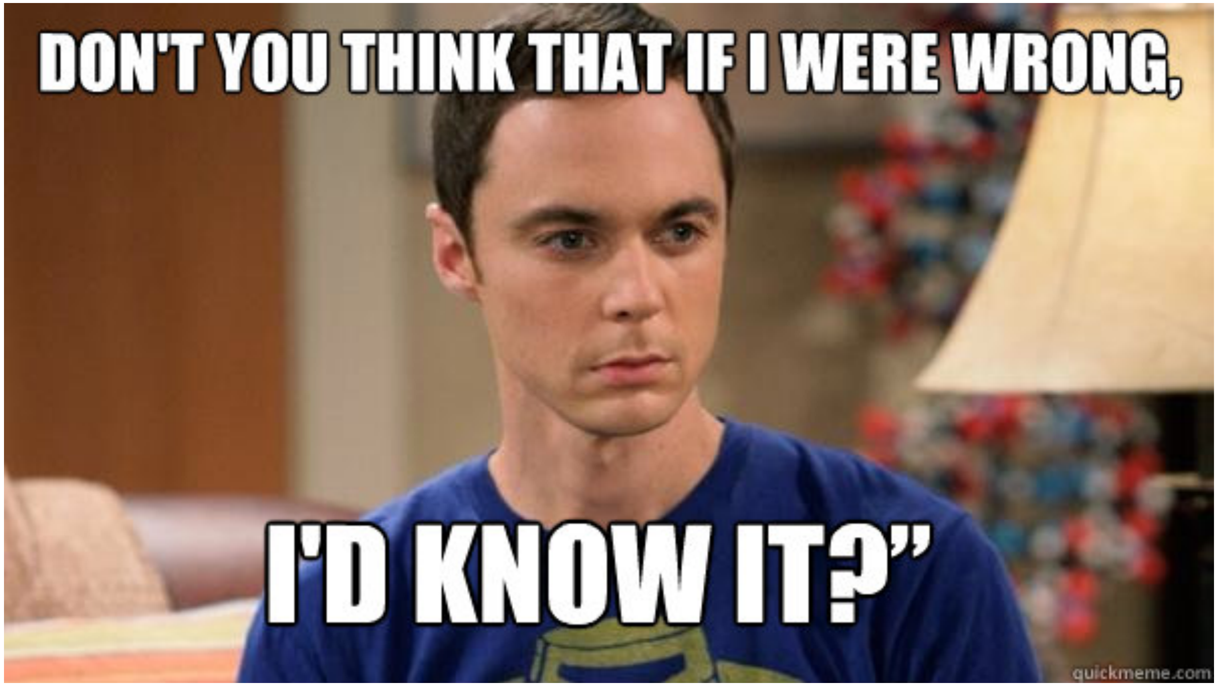
$$\boxed{f_{NL}^{GR} = -\frac{5}{3} \quad , \quad g_{NL}^{GR} = \frac{50}{27} \quad , \quad h_{NL}^{GR} = -\frac{125}{81} \quad \ldots}$$

Bartolo et al; Verde & Matarrese;
Bruni, Hidalgo & Wands (2014)

Newtonian local-type nG ➔ scale-dependent bias

$f_{NL}^{GR} = -5/3$ ➔ *scale-dependent galaxy bias in GR ???*

Bruni, Hidalgo & Wands, arXiv:1405.07006
Dai, Pajer & Schmidt, arXiv:1504.00351
de Putter, Dore & Green, arXiv:1504.05935
Bartolo et al arXiv:1506.00915



No scale-dependent bias* in GR from Gaussian $\zeta(x)$

*at least in simplest bias model

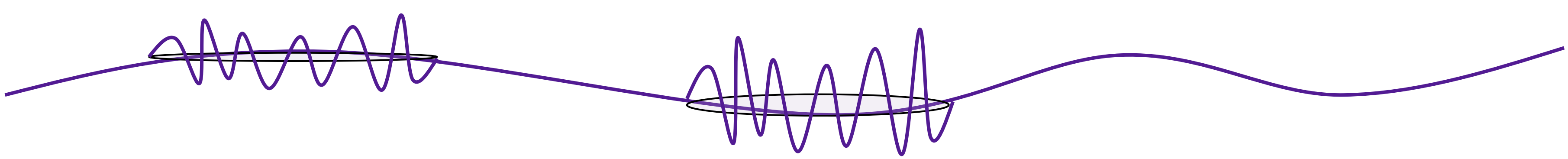
Long-wavelength mode also rescales local physical volume and mass

$$M = \exp(3\zeta_\ell)\bar{M} + \mathcal{O}(\nabla\zeta_\ell)$$

Hence to compare abundance of halos of same physical mass need to compare local variance on different local comoving coordinate scale

Scale-dependence of primordial spectrum exactly cancels the change in variance on fixed coordinate scale

$$\sigma^2_{\bar{M}}|_{\zeta_\ell} = \exp(4\zeta_\ell)\sigma^2_M|_{\zeta_\ell} = \bar{\sigma}^2_{\bar{M}}$$



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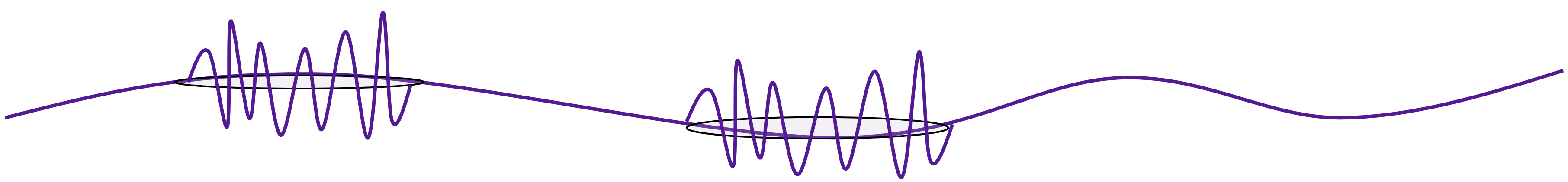

*large scale ζ_l does **not** modulate smaller scale δ_s on fixed mass scale*

Conclusions

- **Newtonian models work to first-order**
  - but need consistent interpretation within GR
  - local Lagrangian bias defined in synchronous-comoving gauge
  - N-body numerical simulations describe particle displacements in novel N-body gauge

- **Einstein gravity** imprint **in nonlinear initial conditions**
  - **Gaussian metric perturbations** from inflation, $\zeta(x)$, generate **non-Gaussian density field**
  - **not equivalent to Newtonian non-Gaussianity**
  - **no scale-dependent galaxy bias** in simplest bias models
  - but there will be a **non-zero galaxy bispectrum**
  - observations also introduce important non-linearities