Gravitational waveform from binary neutron star mergers: Numerical Relativity & Effective one body

Kenta Hotokezaka
(Hebrew University of Jerusalem)

Maseru Shibata (Kyoto University), Kotarou Kyutoku (RIKEN), Yuichiro Sekiguchi (Toho University)
Introduction: Neutron Star

Mass ~ 1.4 Msun, Radius ~ 10km

- Crust
- Nuclear density
- Core
  - Composition? (neutron, hyperon, quark)
  - Interaction?
  - etc

Many mysteries in deep inside of neutron stars.
Introduction: Neutron Star EoS & M-R

1 to 1 correspondence between EOS and M-R relation. Measurements of Mass & Radius => High-dense material.
GWs as a probe of NSs

- The masses can be measured using the chirp signal
- Neutron Star’s size affects gravitational-waves of mergers

Tidal effect in pre-mergers:
- Flanagan & Hinderer 2008
- Hinderer et al 2010
- Damour, Nagar, & Villain 2012
- Agathos et al 2015

Post-merger:
- Bauswein & Janka 2012
- Baustein et al 2014
- Hotokezaka et al 2013

Inspiral-merger-post merger:
- Read et al 2010, 2013

Cut off at the merger:
- Kiuchi et al 2010, Lackey et al 2014
Tidal interaction in binaries

The orbital motion is affected by the tidal effect. The leading of the tidal effect is 5PN order.

\[ E_{\text{int}} \approx -\frac{M_1}{r} \left( M_2 + \frac{3Q_2}{2r^2} \right) \]

\[ Q_2 \sim \Lambda_2 \frac{GM_1}{r^3} \]

Tidal deformability parameter

\[ \Lambda_2 = \frac{2}{3} \frac{R_2^5}{G} k_2 \]

Dimensionless tidal Love number
Tidal deformability of neutron stars

Hinderer et al 2010, see also Damour and Nagar 2009

$$\Lambda_2 = \frac{2}{3} \frac{R_2^5}{G} k_2$$

Questions:
What is the measurability of tidal deformability parameters?
Can we distinguish a BNS merger from a BBH with the same mass?
The tidal effect is stronger in later times of inspiral. => We want to use the information up to the merger.

In fact, Read et al 2014 show that hybrid waveforms of analytic and NR waveforms improve the measurability of tidal deformability parameters. (Favata 2014, Yagi & Yunes 2014, Wade et al 2014)

But, the truncation error of post-Newtonian causes parameter estimation biases.

Motivated by this, we compute late-inspiral waveforms using numerical relativity
Long-term : ~15 orbits
Low eccentricity : e < 0.001  

see Kyutoku et al 2014 for a method
The finite resolution effects systematically cause overestimates of the tidal effects.

=> A extrapolation procedure is needed.
Resolution Extrapolation

\[ h^{2.2}(t_{\text{ret}}) = A^{2.2}(t_{\text{ret}}) \exp[i\Phi(t_{\text{ret}})]. \]

\[ t_{\text{ret}} \rightarrow \eta t_{\text{ret}} \text{ and } \Phi \rightarrow \eta \Phi. \quad \text{Uniformly stretching} \]

\[ I = \min_{\eta', \phi} \int_{t_i}^{t_f} dt_{\text{ret}} |A^{2.2}_2(\eta' t_{\text{ret}}) \exp[i\eta' \Phi(\eta' t_{\text{ret}}) + i\phi]|^2 - A^{2.2}_1(t_{\text{ret}}) \exp[i\Phi_1(t_{\text{ret}})]|^2 \]

taking a limit \( \Delta_x \rightarrow 0 \).
Resolution Extrapolation
stretching

Difference between the extrapolated and highest
Merger time: \(~ 0.3\) ms
Total phase : \(~ 1\) radian

the two extrapolated
Merger time: \(~ 0.1\) ms
Total phase : \(~ 0.5\) radian
Comparison with Effective-one body

Effective one body waveforms with tidal (Bernuzzi et al 2015) agree very well with NR ones up to a last few cycles.

In a last few cycle,
Phase difference: 0.5 ~ 1 rad
(Better for Smaller NS)

Hybridize these waveforms
Hybrid waveform
Hybrid waveform

![Graph showing hybrid waveforms with labels for different waveforms and markers indicating Tidally dominated, Merger, and Hypermassive Neutron Star. The x-axis represents frequency (kHz) ranging from 0.2 to 4 kHz, and the y-axis represents $2f^{1/2}|h(f)| (D_{eff} = 100 \text{ Mpc})$ ranging from $10^{-23}$ to $10^{-22}$. Various waveforms are plotted including APR4, SFHo, DD2, TMA, TM1, BBH, and aLIGO.]
Measurability (Distinguishability)

\[ ||h_1 - h_2||^2 := \min_{\Delta t, \Delta \phi} \left[ 4 \int_{f_i}^{f_f} \left| \tilde{h}_1(f) - \tilde{h}_2(f)e^{i(2\pi f \Delta t + \Delta \phi)} \right|^2 \right] df \]

\[ ||h_1 - h_2|| > 1 \quad \text{Marginally distinguishable} \]

\[ \delta \Lambda_{\text{rand}} = \frac{|\Lambda_1 - \Lambda_2|}{||h_1 - h_2||} \]

see Lindblom et al 2009
Read et al 2013
Measurability of tidal deformability

The plot shows the comparison between different equations of state and waveforms. The y-axis represents the measureability of tidal deformability, defined as \( \text{Def} = 200 \text{ Mpc} \|h_1-h_2\| \). The x-axis represents the parameter \( \delta \Lambda \), ranging from 0 to 1600. Different line segments correspond to different equations of state and waveforms, such as APR4-SFH0, TMA-TM1, DD2-TM1, SFho-DD2, APR4-DP2, and SFHo-TM1. The crosses denote the results of the comparison between binary black hole (BBH) and binary neutron star (BNS) systems.

The dashed curve in each plot is a fitting formula in the form \( f(\delta \Lambda) \). The crosses denote the results of the comparison between different systems for (50 Hz, 1 kHz) and (50 Hz, 2 kHz), respectively. The values of \( \delta \Lambda \) range from 0.561 to 3.07, as shown for each value of \( f \).

The plot also includes the marginally distinguishable regions for different frequencies, with lines for 50-2000 Hz and 50-1000 Hz.
EOB vs NR

Deff = 200 Mpc

<table>
<thead>
<tr>
<th>0.05–2 kHz</th>
<th>APR4</th>
<th>SFHo</th>
<th>DD2</th>
<th>TMA</th>
<th>TM1</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOB:APR4</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOB:SFHo</td>
<td></td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOB:DD2</td>
<td></td>
<td></td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOB:TMA</td>
<td></td>
<td></td>
<td></td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>EOB:TM1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.1</td>
</tr>
</tbody>
</table>

EOB is working quite well for Deff > 200 Mpc
Conclusion

We compute long-term (15 orbit) gravitational waveforms with errors of \(\sim 0.5\) rad.

Current tidal Effective one body formalism is good to describe NS-NS inspirals up to the merger. (indistinguishable to NR waveforms for SNR<20)

For a NSNS merger event Deff~200Mpc (SNR~17), we can distinguish between BBH and BNS NSs with R<12km and R>13 km