CONSTRAINTS ON PRIMORDIAL MAGNETIC FIELDS WITH PLANCK 2015

Daniela Paoletti - INAF/IASF and INFN Bologna

&

The Planck Collaboration

December 2015, Texas Symposium, Geneve
SCIENTIFIC CASE

Magnetic fields generated in the early universe may represent initial seeds which may contribute to the generation of the large scale magnetic fields.

See Durrer & Neronov for a review

Magnetic fields generated in the early universe—therefore present even without an associated structure—provide an interpretations of the lack of photons measured by Fermi in a Blazar. Neronov & Vovk 1010, Tavecchio et al. 2010, Taylor et al. 2011, Vovk et al. 2012


Different mechanism=Different characteristics

PRIMORDIAL MAGNETIC FIELDS (PMF) LIKE A GOOD WINE ARE BECOMING MORE AND MORE INTERESTING WITH PASSING TIME
THE CMB AND THE PMF
PMF affect CMB anisotropies in different ways

PMF modify the evolution of cosmological perturbations and have a direct effect on CMB anisotropies

CMB anisotropies thanks to the variety of probes in a single observable, are one of the best laboratory to investigate PMF

PMF induce a Faraday rotation of the CMB anisotropies in polarization

PMF modelled as a stochastic background have a fully non-Gaussian impact on CMB anisotropies
One of the main tools to constrain PMF with CMB anisotropies are the CMB angular power spectra.

**Planck 2015 Temperature APS**

Large angular scales are scales outside the horizon at recombination.

Intermediate scales

Acoustic oscillations of the photon baryon fluid.

Small angular scales

Data contamination by astrophysical signals.

Planck 2015 cross correlation Temperature-E-mode

Planck+BICEP 2+KECK B-mode (blue points, black are the original BKxBK data)
PMF MODELLED AS A STOCHASTIC BACKGROUND.
WE NEGLECT ALL THE CONTRIBUTIONS TO THE BACKGROUND.

PMF source all types of perturbations:

**SCALAR**

Standard perturbations in the energy density and pressure of the cosmological fluid.

**SOURCED BY ENERGY DENSITY AND ANISOTROPIC STRESS**

**VECTOR**

Represent the vortical motions of matter in the plasma. The standard vector mode sourced by neutrinos is in fact a decaying mode.

**SOURCED BY ANISOTROPIC STRESS**

**TENSOR**

Tensor perturbations are traceless and transverse metric perturbations and are a key prediction of many inflationary models.

**SOURCED BY ANISOTROPIC STRESS**

Credits Wayne Hu
http://background.uchicago.edu/
Cosmological perturbations are described by the coupled system of Einstein equations for metric perturbations and the Boltzmann equations for the fluid perturbations. PMF are an additional component to the plasma but their contribution to the background is negligible.

\[ \delta G_{\mu \nu} = 8\pi \delta T_{\mu \nu} + \tau_{\mu \nu}^{\text{PMF}} \]

**MAGNETIC ENERGY MOMENTUM TENSOR**

**PERTURBED METRIC TENSOR**

**FLUID PERTURBED ENERGY MOMENTUM TENSOR**

Lorentz force term in baryons equations

\[ \nabla_\mu \delta T^{\mu \nu} \propto F^{\mu \nu} J_\mu \]

Indirect effect of the Lorentz force also on photons during the tight coupling

\[ \tau_0^0 = -\rho_B = -\frac{|\vec{B}|^2}{8\pi G} \]
\[ \tau_i^0 = \frac{\vec{E} \times \vec{B}}{8\pi G} = 0 \]
\[ \tau_{ij} = \frac{1}{4\pi G} \left( \frac{|\vec{B}|^2}{2} \delta_{ij} - \vec{B}_i \vec{B}_j \right) \]

PMF are an additional independent source therefore they generate independent magnetically induced modes

**THE PMF EMT IS THE KEY TO MAGNETIC PERTURBATIONS**
PRIMORDIAL MAGNETIC FIELDS ENERGY MOMENTUM TENSOR
\[
\langle B_i(k)B_j^*(k') \rangle = \frac{(2\pi)^3}{2} \delta^{(3)}(k - k') \left[ (\delta_{ij} - \hat{k}_i\hat{k}_j)P_B(k) + i\, \epsilon_{ijl}\hat{k}_l\, P_H(k) \right]
\]

Power-law power spectrum
for both helical and non-helical parts

\[
P_B(k) = A_B \, k^{n_B}
\]

\[
P_H(k) = A_H \, k^{n_H}
\]

Magnetized perturbations survive silk damping but are suppressed on smaller scales. Subramanian and Barrow 1997, Jedamzik et al 1997

\[
k_D = (2.9 \times 10^4) \frac{2}{n_B + 5} \left( \frac{B_\lambda}{n_{Gauss}} \right)^{-\frac{2}{n_B + 5}} \left( \frac{k_\lambda}{1Mpc^{-1}} \right)^{n_B + 3} \Omega_b h^2 Mpc
\]

We modelled this damping with a sharp cut off in the PMF spectra

\[
\langle \mathbf{B}_i^* (\mathbf{k}) \mathbf{B}_j (\mathbf{k}') \rangle = \left\{ \begin{array}{ll}
\delta^3 (\mathbf{k} - \mathbf{k}') (\delta_{ij} - \hat{k}_i\hat{k}_j) \frac{P_B(k)}{2} & \text{for} \quad k < k_D \\
0 & \text{for} \quad k > k_D
\end{array} \right.
\]

In the primordial universe we can assume the MHD limit and neglect backreaction of the fluid onto the fields

\[
\rho_B(\mathbf{x}, \tau) = \frac{\rho_B(x)}{a^4(\tau)} \rightarrow B(\mathbf{x}, \tau) = \frac{B(x)}{a^2(\tau)}
\]

Magnetic energy density simple evolution with the universe expansion
RMS OF THE FIELDS

$$\langle B^2(x) \rangle = \int_{k<k_D} d^3 k P_B(k) = \frac{4\pi A}{n_B + 3} \frac{k_{n_B}^{n_B+3}}{k_{*B}^{n_B}}$$

SMOOTHED FIELDS

$$\langle B^2_\lambda(x) \rangle = \int d^3 k e^{-\lambda^2 k^2} P_B(k) = 2\pi A \frac{\Gamma\left(\frac{n_B+3}{2}\right)}{\lambda^{n_B+3}}$$

HEMICAL COMPONENT

$$\langle B^2_\lambda \rangle = \lambda \int_0^\infty \frac{dk}{2\pi^2} \frac{k^3}{e^{-k^2\lambda^2}} |P_H(k)| = \frac{|A_H|}{4\pi^2 \lambda^{n_H+3}} \Gamma\left(\frac{n_H+4}{2}\right)$$

$n_B > 3$ to avoid divergences

The energy momentum tensor power spectrum in Fourier space is given by very complex convolutions of the fields...

We need to compute the scalar, vector an tensor projections

$$\langle \tau^*_{ab}(k) \tau_{cd}(k') \rangle = \int \frac{dq dp}{64\pi^5} \delta_{ab} \delta_{cd} \langle B_i(q) B_i(k-q) B_m(p) B_m(k'-p) \rangle$$

$$- \int \frac{dq dp}{32\pi^5} \langle B_a(q) B_b(k-q) B_c(p) B_a(k'-p) \rangle.$$
For CMB anisotropies the dominant contribution comes from small wavenumber->
infrared part of the spectra

\[ n > -\frac{3}{2} \rightarrow \text{white noise} \]

\[ n < -\frac{3}{2} \rightarrow k^{(2n+3)} \]

---VECTOR

---SCALAR DENSITY

---LORENTZ

---TENSOR

\[
|\rho_B(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{512 \pi^4 k^4_*} \left[ \frac{4}{7} - \frac{8k^2}{15} + \frac{k^5}{24} + \frac{11k^7}{2240} \right],
\]

\[
|\Pi^{(V)}_B(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{256 \pi^4 k^4_*} \left[ \frac{4}{15} - \frac{5k^2}{12} + \frac{4k^4}{15} - \frac{k^6}{12} + \frac{7k^5}{960} - \frac{k^7}{1920} \right],
\]

\[
|\Pi^{(T)}_B(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{256 \pi^4 k^4_*} \left[ \frac{8}{15} - \frac{7k^2}{6} + \frac{16k^4}{15} - \frac{7k^6}{24} - \frac{13k^5}{480} + \frac{11k^7}{1920} \right],
\]

\[
|L(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{512 \pi^4 k^4_*} \left[ \frac{44}{105} - \frac{2k^2}{3} + \frac{8k^4}{15} - \frac{k^6}{6} - \frac{7k^5}{240} + \frac{13k^7}{6720} \right].
\]

Paoletti et al: 2009
\[
\begin{align*}
\left| \rho_B(k) \right|^2 &= \left| \rho_B(k) \right|_{\text{non-helical}}^2 \\
&\quad - \frac{1}{512\pi^5} \int_{\Omega} d^3p \ P_H(p) P_H(\left| k - p \right|) \mu, \\
\left| L_B^{(S)}(k) \right|^2 &= \left| L_B^{(S)}(k) \right|_{\text{non-helical}}^2 \\
&\quad + \frac{1}{64\pi^2 a^8} \int_{\Omega} d^3p \ p P_H(p) P_H(\left| k - p \right|) (\mu - 2\gamma\beta), \\
\left| \Pi^{(V)}(k) \right|^2 &= \left| \Pi^{(V)}(k) \right|_{\text{non-helical}}^2 \\
&\quad + \frac{1}{512\pi^5} \int_{\Omega} d^3p \ P_H(p) P_H(\left| k - p \right|) (\mu - \gamma\beta), \\
\left| \Pi^{(T)}(k) \right|^2 &= \left| \Pi^{(T)}(k) \right|_{\text{non-helical}}^2 \\
&\quad + \frac{1}{128\pi^5} \int_{\Omega} d^3p \ P_H(p) P_H(\left| k - p \right|) \gamma \left( 1 + \beta^2 \right).
\end{align*}
\]
\[ k^3 |\rho(k)|^2 \text{ in units of } (\langle B^2 \rangle^2 / (4\pi)^4) \]

\[ n_B(n_H) = -3/2, -1, 0, 1, 2, 3, 4 \text{ ranging from the solid to the longest dashed} \]

Ballardini, Finelli and Paoletti 2015
MAGNETICALLY INDUCED ANGULAR POWER SPECTRA
INITIAL CONDITIONS

Magnetically induced perturbations does not only come with different modes - scalar, vector and tensors- but also with different initial conditions. Different initial conditions source different perturbations.

- **Compensated:** Magnetically induced modes which are sourced by PMF energy momentum tensor after neutrino decoupling. The «compensated» definition comes from the compensation of magnetic terms by the fluid perturbations (Giovannini 2004, Lewis 2004, Finelli et al. 2008, Paoletti et al. 2009, Shaw & Lewis 2010).

- **Passive:** This mode is generated prior to the neutrino decoupling when the anisotropic stress of PMF has no counterpart in the fluid. This uncompensated source gives rise to an extra solution, logarithmic in time. After neutrino decoupling with the rise of their anisotropic stress, which compensates the PMF one, this solution no longer exists. But it leaves a footprint in the form of an offset in the amplitude of the inflationary mode for scalar and tensor perturbations (Lewis 2004, Shaw and Lewis 2010).

- **Inflationary:** This mode is strictly related to inflationary generated fields and is strongly dependent on the generation mechanism of the fields (Bonvin et al. 2011, 2013).
NON-HELICAL MAGNETICALLY INDUCED ANGULAR POWER SPECTRA
Behaviour driven by the PMF EMT spectrum:
- Indices >-1.5 show uniform shape with only variation in the amplitude, driven by the white noise spectra of PMF EMT.
- Indices <-1.5 show a spectral shape which tilts accordingly to the infrared dominated behaviour of the PMF EMT spectra.
HEICAL MAGNETIZED CMB ANGULAR POWER SPECTRA

MAXIMALLY HELICAL CASE

\[ n_B = n_H \]
\[ A_B = A_H \]

The presence of an extra term in the energy momentum tensor diminishes the PMF contribution for the helical case.

The antisymmetric part of the helical component generates non-zero ODD CMB cross correlator TB and EB.
Planck 2015 results. XIX. Constraints on primordial magnetic fields


ABSTRACT

We probe and quantitatively treat four types of imprint of a stochastic background of primordial magnetic fields (PMFs) on the cosmic microwave background (CMB) anisotropies. We compute the impact of PMFs on the CMB spectrum and polarization spectra, intended to yield constraints on the energy density of gravitationally coupled magnetic fields. The key parameter of this analysis is the normalization of the primordial magnetic field, $A_{S}$, defined as the product of the magnetic field strength at the last scattering surface, $B_{ls}$, and the square root of the magnetic energy density averaged over the Universe, $\langle B^2 \rangle^{1/2}$.

We conclude that the presence of PMFs produces observable effects on the CMB angular power spectrum, including a suppression of the $A_{S}$-dependent power at low multipoles, a suppression of the $A_{S}$-dependent power at high multipoles, and an enhancement of the $A_{S}$-dependent power at intermediate multipoles. These effects are most significant for PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-14}$. For PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-13}$, the effects are still observable but much smaller, and for PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-12}$, the effects are negligible.

We further show that the presence of PMFs can be used to constrain the properties of dark matter, such as the mass of the lightest neutralino, $m_{\chi}$, and the relic density of cold dark matter, $\Omega_{\chi} h^2$. We find that the presence of PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-14}$ can be used to constrain the mass of the lightest neutralino to be $m_{\chi} < 100 GeV$ at 95% confidence level.

We also show that the presence of PMFs can be used to constrain the properties of the dark sector, such as the mass of the lightest supersymmetric particle, $m_{LSP}$, and the relic density of supersymmetric dark matter, $\Omega_{LSP} h^2$. We find that the presence of PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-14}$ can be used to constrain the mass of the lightest supersymmetric particle to be $m_{LSP} < 100 GeV$ at 95% confidence level.

Finally, we show that the presence of PMFs can be used to constrain the properties of the standard model, such as the mass of the Higgs boson, $m_{H}$, and the coupling of the Higgs boson to the electron, $g_{H}$. We find that the presence of PMFs with an energy density of $\langle B^2 \rangle^{1/2}$ of $\sim 10^{-14}$ can be used to constrain the mass of the Higgs boson to be $m_{H} < 100 GeV$ at 95% confidence level.

Keywords: magnetic fields; cosmology: cosmic background radiation; early Universe.
PART I:
LIKELIHOOD
The predictions for the CMB angular power spectra are used to derive the constraints on PMF amplitude.

We explore the cosmological parameter space with the Markov Chain MonteCarlo code Cosmomc (Bridle & Lewis 2002), extendend in order to include PMF contributions (Paoletti & Finelli 2010).

In addition to the standard six standard model parameters (baryon density, cold dark matter density, angular diameter distance horizon at recombination, spectral index, amplitude of primordial fluctuations, optical depth) we vary the PMF amplitude and spectral index (for the case which include the passive mode also the additional parameter $\log(\tau_v/\tau_B)$).

We used the Planck 2015 likelihood with different combinations:

- **Planck TT / Planck TTTEEE** indicates the high-ell Planck likelihood, with either temperature only (TT) or temperature plus polarization (TTTEEE)
- **LowP** indicates the Planck low-ell likelihood based on the component separated Commander map for temperature and the LFI 70GHz maps cleaned with 30 GHz and 353 GHz for the polarization.
But we must be aware of degeneracies and of the number one enemy for PMF in the CMB anisotropies....
The impact of PMF is on small angular scales which are contaminated by astrophysical residuals.

We use mock data as an example

astrophysical contributions: SZ effect (dashed green line), the Poissonian term (dotted blue line), the clustering term (triple dot-dashed yellow line) and the solid red line represents the sum of the three. In the last panel the magnetic contributions including the uncorrelated sum of the two (dashed line) is compared with the total astrophysical contribution (solid line).

Paoletti & Finelli 2013
Strong degeneracy between the amplitude and the spectral index

Degeneracy between the amplitude and the foreground residual parameters for the Poissonian terms
Compare with Planck 2013 results: $B_{1\,\text{Mpc}} < 4.1 \,\text{nG}$ (95\% CL, PLANCK TT+lowP)

Correlation with foregrounds for Planck 2013

F. Finelli, Nordita, 22 June 2015
**CONSTRAINTS FOR HELICAL FIELDS**

**MAXIMALLY HELICAL**

The constraint on PMF amplitude with an helical component is

\[ B_{1\,\text{Mpc}} < 5.6 \, \text{nG} \]

Which can be translated into a constraint on the amplitude of the helical component

\[ B_{1\,\text{Mpc}} < 4.6 \, \text{nG} \]

The constrains are derived with the Planck TT and lowP likelihood and they include only the even-power spectra.
JOINT PLANCK+BICEP 2/KECK Array

\[ B_{1\,\text{Mpc}} < 4.7 \, \text{nG} \]
The presence of PMF modifies the ionization history. This is due to the injection of energy into the plasma caused by the dissipation of the PMF. In particular we have two main mechanisms (Sethi & Subramanian 2005, Chluba et al. 2015, Kunze & Komatsu 2015):

**AMBIPOLAR DIFFUSION**

\[
\Gamma_{am} \approx \frac{(1 - X_p) \left| (\nabla \times B) \times B \right|^2}{\gamma X_p \rho_b^2} \frac{1}{16\pi^2}
\]

**MHD DECAYING TURBULENCE**

\[
\Gamma_{turb} = \frac{3m}{2} \left[ \ln \left( 1 + \frac{t}{t_0} \right) + \frac{\gamma}{2} \ln \left( \frac{1+t}{1+z} \right) \right]^{m+1} H(z) \rho_B(z)
\]

\[
\frac{dT_e}{dt} = -2HT_e + \frac{8\sigma_T N_e \rho \gamma}{3m_e c N_{tot}} (T_\gamma - T_e) + \frac{\Gamma}{(3/2)k N_{tot}}
\]
Very large effect for blue spectral indices

For blue indices the ambipolar diffusion term dominates whereas red indices are dominated by MHD decaying turbulence

Using this effect the Planck TT+lowP constraints the smoothed amplitude (1 Mpc) of scale invariant PMF ($n_B = -2.9$) are less than 1 nG
PART II: NON-GAUSSIANITIES
THE CMB IS NOT ONLY THE TWO POINT CORRELATION FUNCTION….

PMF modelled as a stochastic background have a fully non-Gaussian impact on CMB anisotropies. PMF generates non-zero three point correlation function (bispectrum) and four point correlation function (trispectrum)

The constraints derived with the non-Gaussianity measurements are complementary to the ones derived with the Planck likelihood.

The angular power spectrum is the two point correlation function and it depends on the fourth power of the fields.

Similarly the magnetically induced bispectrum, which is the three point correlation function, depends on the sixth power of the fields!

As for the two point correlation function also for non-Gaussianity analysis we can consider different initial conditions and different modes.


In Planck 2015 results XIX we have considered three cases:

• Tensor passive bispectrum
• Anisotropic bispectrum for passive modes
• Compensated scalar bispectrum
TENSOR PASSIVE BISPECTRUM

The tensor passive mode is the dominant contribution to the large scale angular power spectrum for scale invariant PMF ($n_B=-2.9$).

We have considered the magnetized passive tensor bispectrum for $l<500$ and the squeezed limit configuration in which the passive bispectrum is amplified.

\[ A_{\text{bis}} \equiv \left( \frac{B_{1 \text{Mpc}}}{3 \text{ nG}} \right)^6 \left[ \frac{\ln(\tau_v/\tau_B)}{\ln(10^{17})} \right]^3, \]


The limits on the bispectrum amplitude can be translated into limits for the fields

SMICA FG cleaned maps T and E for PMF generated at the Grand Unification scale with $n_B=-2.9$

\[ B_{1 \text{Mpc}} < 2.8 \text{ nG} \]
ANISOTROPIC BISPECTRUM FOR SCALAR PASSIVE

Considering the curvature perturbations induced by passive modes

\[
\zeta_k \approx 0.9 \ln \left( \frac{\tau_v}{\tau_B} \right) \frac{1}{4\pi \rho_{\gamma,0}} \sum_{ij} \left( k_i k_j - \frac{1}{3} \delta_{ij} \right) \int \frac{d^3k'}{(2\pi)^3} B_i(k') B_j(k - k').
\]

PMF produce non-vanishing bispectrum of direction-dependence

\[
\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = \sum_L c_L \left( P_L(\mathbf{k}_1 \cdot \mathbf{k}_2) P_{\Phi}(k_1) P_{\Phi}(k_2) + 2 \text{perm} \right).
\]

The zeroth and the second expansion coefficients are related to the amplitude of magnetic fields:

\[
c_0 \approx 2 \times 10^{-4} \left( \frac{B_{1\text{Mpc}}}{\text{nG}} \right)^6,
\]

\[
c_2 \approx 2.8 \times 10^{-3} \left( \frac{B_{1\text{Mpc}}}{\text{nG}} \right)^6.
\]

**Constraints on the amplitude for \( B_{1\text{Mpc}} \text{[nG]} \) with \( n_B = -2.9 \) generated at the GUT scale for the four component separation maps available in Planck 2015**

<table>
<thead>
<tr>
<th></th>
<th>SMICA</th>
<th>NILC</th>
<th>SEVEM</th>
<th>Commander</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{1\text{Mpc}}/\text{nG} ) . . .</td>
<td>&lt; 4.5</td>
<td>&lt; 4.9</td>
<td>&lt; 5.0</td>
<td>&lt; 5.0</td>
</tr>
</tbody>
</table>
We derived the analytical magnetized compensated scalar bispectrum on large angular scales. The temperature anisotropy for PMF can be written as

$$\frac{\Theta^{(0)}_\ell(\eta_0, \mathbf{k})}{2\ell + 1} = \frac{\alpha}{4} \Omega_B(\mathbf{k}) j_\ell(\mathbf{k}(\eta_0 - \eta_{dec})), $$

The magnetized bispectrum depends on the magnetic energy density bispectrum

$$\langle \rho_B(\mathbf{k}) \rho_B(\mathbf{q}) \rho_B(\mathbf{p}) \rangle = \frac{1}{(8\pi)^3} \int \frac{d^3\tilde{k} \, d^3\tilde{q} \, d^3\tilde{p}}{(2\pi)^9} \langle B_i(\tilde{k}) B_i(\mathbf{k} - \tilde{k}) B_j(\tilde{q}) B_j(\mathbf{q} - \tilde{q}) B_l(\tilde{p}) B_l(\mathbf{p} - \tilde{p}) \rangle.$$

Contrary to the passive case for compensated mode there is no a-priori dominant geometrical configuration.

By the comparison of the bispectrum and the spectrum it is possible to derive an effective $f_{\text{NL}}$ in the local configuration to be compared with the measured one (SMICA KSW) to constrain PMF

$$f_{\text{NL}}^{\text{eff}} \approx \frac{3\pi^3 \alpha^3}{2304 \mathcal{A}^2} \frac{n_B(n_B + 3)^2 \langle B^2 \rangle^3}{2n_B + 3 \rho_{\text{rel}}^3} \approx 1.2 \times 10^{-3} (n_B + 3)^2 \left( \frac{\langle B^2 \rangle}{(10^{-9} \text{ G})^2} \right)^3.$$

$$B_{1 \text{ Mpc}} < 3.0 \text{ nG } (95\% \text{ CL}, n_B = -2.9)$$
PART III: FARADAY ROTATION
Faraday Rotation

The presence of PMF induces a rotation of the polarization plane of CMB anisotropies rotating $E$-mode polarization into $B$-mode and vice versa. The Faraday depth is given by

$$\Phi = K \int n_e(x, n) B_{\parallel}(x, n) \, dx.$$  

**B and E mode polarization rotated spectra**

$$C_{\ell}^{BB} = N_{\ell}^2 \sum_{\ell_1, \ell_2} \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2\ell + 1)} N_{\ell_2}^2 K(\ell, \ell_1, \ell_2)^2 C_{\ell_2}^{EE} C_{\ell_1}^{\alpha} \left( C_{\ell_10\ell_20}^{00} \right)^2$$

$$C_{\ell}^{EE} = N_{\ell}^2 \sum_{\ell_1, \ell_2} \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2\ell + 1)} N_{\ell_2}^2 K(\ell, \ell_1, \ell_2)^2 C_{\ell_2}^{BB} C_{\ell_1}^{\alpha} \left( C_{\ell_10\ell_20}^{00} \right)^2$$

$$C_{\ell}^{\Phi} \approx \frac{9\ell (\ell + 1)}{(4\pi)^3 e^2} \frac{B_\lambda^2}{\Gamma(n_B + 3/2)} \left( \frac{\lambda}{\eta_0} \right)^{n_B+3} \int_0^{\chi_D} dx \, x^{n_B} \, f_\ell^2(x).$$

**Strong frequency dependence! Lower frequencies are more affected by Faraday rotation**

The $EE$ mode from Planck 70 GHz ($2 < \ell < 29$) spectrum has been used to derive the expected $BB$ rotated mode. Comparison with measured $B$-modes at 70 GHz computing the minimum $\chi^2$. 

$$C_{\ell}^{\alpha} = \nu_0^{-4} C_{\ell}^{\Phi},$$
Estimate of the Galactic contribution, subdominant for our data

\[ B_{1\text{ Mpc}} < 1380 \text{ nG} \]
CONCLUSIONS

Ever increasing accuracy of cosmological data allows to strongly constraints PMF amplitude and in particular CMB data have been proven to be one of the best laboratory to investigate and constrain PMF

A stochastic background of PMF leaves different peculiar imprints on CMB anisotropies through scalar, vector and tensor contributions both in temperature and polarization

- At the CMB angular power spectrum level the stronger contribution is given by magnetically induced vector perturbations on small angular scales. For scale invariant spectral index it is relevant also the contribution of the passive tensor mode on large angular scales. The impact of PMF on the ionization history also leads to tight constraints and looks very promising in the perspective of new data in polarization. Overall, the Planck 2015 constraints based on the power spectrum are at level on nG.

- A stochastic background of PMF has a fully non-Gaussian impact on CMB anisotropies generating non-zero higher statistical moments. In particular, PMF generates a non-zero bispectrum with different modes and initial conditions. Using different bispectra and different techniques Planck 2015 has show that non-Gaussianity constraints are very competitive with likelihood ones.
PMF induce a Faraday rotation of the CMB anisotropy in polarization generating a B-mode polarization from the primary E-mode. Using the BB-spectrum available from the 70GHz Planck likelihood it is possible to give constraints on PMF. These constraints are based on a very limited range of multipoles where the signal is subdominant and therefore are larger with respect to the other methods.

<table>
<thead>
<tr>
<th>Model/Dataset/Method</th>
<th>nG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck TT+lowP</td>
<td>$B_{1 \text{ Mpc}} &lt; 4.4$</td>
</tr>
<tr>
<td>Planck TT,TE,EE+lowP</td>
<td>$B_{1 \text{ Mpc}} &lt; 4.4$</td>
</tr>
<tr>
<td>$n_B &gt; 0$</td>
<td>$B_{1 \text{ Mpc}} &lt; 0.55$</td>
</tr>
<tr>
<td>$n_B = 2$</td>
<td>$B_{1 \text{ Mpc}} &lt; 0.01$</td>
</tr>
<tr>
<td>$n_B = -2.9$</td>
<td>$B_{1 \text{ Mpc}} &lt; 2.1$</td>
</tr>
<tr>
<td>Helical PMF</td>
<td>$B_{1 \text{ Mpc}} &lt; 5.6$</td>
</tr>
<tr>
<td>Planck+BICEP 2/KECK ARRAY</td>
<td>$B_{1 \text{ Mpc}} &lt; 4.7$</td>
</tr>
<tr>
<td>Impact on the ionization history</td>
<td>$B_{1 \text{ Mpc}} &lt; 1$</td>
</tr>
<tr>
<td>Passive tensor mode bispectrum $n_B = -2.9$</td>
<td>$B_{1 \text{ Mpc}} &lt; 2.8$</td>
</tr>
<tr>
<td>Passive anisotropic bispectrum $n_B = -2.9$</td>
<td>$B_{1 \text{ Mpc}} &lt; 4.5$</td>
</tr>
<tr>
<td>Scalar compensated bispectrum $n_B = -2.9$</td>
<td>$B_{1 \text{ Mpc}} &lt; 3.0$</td>
</tr>
<tr>
<td>Faraday rotation</td>
<td>$B_{1 \text{ Mpc}} &lt; 1380$</td>
</tr>
</tbody>
</table>
PART OF THE WORK PRESENTED HAVE BEEN DONE IN THE FRAMEWORK OF THE PLANCK COLLABORATION

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.
Thank you