How to use geodetic VLBI to measure relativistic light deflection from extragalactic objects

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Gravitational delay (difference of two Shapiro delays) is linked to deflection angle

$$\tau_{GR} = \tau_{grav} + \tau_{coord} = \frac{2GM}{c^3} \ln \left| \frac{r_1}{r_2} \right| + \left( \frac{r_1 \cdot s}{r_2} \right) + \frac{(b \cdot s)}{c} \frac{2GM}{c^2 r} = \alpha \frac{b}{c} \sin \varphi \cos A$$
General relativity – light bending

Special expeditions to observe total Solar eclipses since 1919

Now VLBI is doing it every session!
Light deflection in VLBI

\[ \alpha = \frac{4GM}{c^2 R} \]  
\[ \alpha = \frac{2GM}{c^2 R} \frac{\sin \theta}{1 - \cos \theta} \]

at the small angle approximation, Einstein (1915)


\[ V(r) \sim \frac{1}{r} \quad \alpha \sim \text{ctg} \frac{\theta}{2} \]

For a radio source within 1° from Sun
Light deflection angle 0552+398

\[ \alpha = \frac{2GM}{c^2 R} \frac{\sin \theta}{1 - \cos \theta} \]

Real data, 1991-2001, each session, each radio source
Deflection residuals \(0119+115\) 2012-2014

Residuals are positive at large elongation angles!
Deflection residuals $0119+115$ 2011-2014

\[ \Delta\gamma = \Delta\alpha \frac{c^2 r}{GM} \frac{1 - \cos \theta}{\sin \theta} \]

Two estimates $\Delta\gamma = -27 +/\ -8$ and $-28 +/\ -5$ within 4 degrees from the anti Solar point.
Brane world gravity

Randall and Sundrum (hep-th 9906064, 1999); Rubakov, UFN, (2001)

\[ V(r) = -\frac{G(4)}{r} \left(1 + \frac{\text{const}}{k^2 r^2}\right). \]

\[ V(r) \sim \frac{1}{r^3} \]

\[ \alpha \sim \text{ctg} \theta \]

For a radio source within 1° from Sun, magnitude is conditional
Expected change in coordinates

\[ V(r) = -\frac{G(4)}{r} \left( 1 + \frac{\text{const}}{k^2 r^2} \right). \]

\[ \alpha \sim \cot \theta \]

0119+115, 2012-2014 (residuals)

Correction to right ascension
List of quasars within 0°.1 from ecliptic

<table>
<thead>
<tr>
<th>Quasar</th>
<th>Ecliptic Angle</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0055+060</td>
<td>0°.075</td>
<td></td>
</tr>
<tr>
<td>0547+234</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>0558+234</td>
<td>-0.023</td>
<td></td>
</tr>
<tr>
<td>0603+234</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>0723+219</td>
<td>-0.070</td>
<td></td>
</tr>
<tr>
<td><strong>0725+219</strong></td>
<td>-0.00187</td>
<td>7”</td>
</tr>
<tr>
<td>0741+214</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>0749+211</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>0956+124</td>
<td>-0.095</td>
<td></td>
</tr>
<tr>
<td>1226-028</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>1346-109</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>1437-153</td>
<td>0.036</td>
<td></td>
</tr>
<tr>
<td>1907-224</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>2243-081</td>
<td>-0.065</td>
<td></td>
</tr>
<tr>
<td>2322-040</td>
<td>0°.008</td>
<td>~25”</td>
</tr>
</tbody>
</table>

11 Jan 2016
Secondary deflection angle (two telescopes)

\[ \Delta \theta = \theta_2 - \theta_1 \sim b / r \sim 8'' \]

\[ \alpha_1 = \frac{4GM}{c^2 R_1} \quad \alpha_2 = \frac{4GM}{c^2 R_2} \]

\[ \alpha'' = \alpha_2 - \alpha_1 \sim \frac{4GMb}{c^2 R^2} \]

<table>
<thead>
<tr>
<th>R (km)</th>
<th>( \alpha )</th>
<th>( \alpha'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>700,000 (grazing light, 0°.25)</td>
<td>1°.75</td>
<td>0°.00125</td>
</tr>
<tr>
<td>2.8\cdot10^6 (1°)</td>
<td>0°.45</td>
<td>0°.00078</td>
</tr>
<tr>
<td>5.6\cdot10^6 (2°)</td>
<td>0°.22</td>
<td>0°.00019</td>
</tr>
<tr>
<td>15\cdot10^6 (5°)</td>
<td>0°.09</td>
<td>0°.00003</td>
</tr>
</tbody>
</table>

b=10000 km

Depends on baseline length!
Light deflection at two-telescope observations

\[ \Delta \theta = \theta_2 - \theta_1 \sim b / r \sim 8'' \]

\[ \alpha_1 = \frac{4GM}{c^2 R_1} \quad \alpha_2 = \frac{4GM}{c^2 R_2} \]

\[ \alpha''' = \alpha_2 - \alpha_1 \sim \frac{2GMb}{c^2 R^2} \]

\[ \alpha''' = \frac{2GMb}{c^2 R^2} \frac{\sin \varphi \cos 2A}{\cos A} \]

Each baseline produces own deflected image
Secondary deflection angle with space VLBI

\[ \alpha = \frac{4GM}{c^2 R} \]

\[ \alpha'' = \frac{2GMb \sin \varphi \cos 2A}{c^2 R^2 \cos A} \]

\( b, A, \varphi \) - variable

\( M = 10^{12} \) Solar Mass, \( b \sim 10^+ \) au, \( r \sim 1 \) Gpc
Light deflection at two-telescope observations

Distant galaxy with unknown mass is observed with several very long baselines

Remote mass could be estimated instantly

\[ \alpha'' = \frac{2GMb \sin \varphi \cos 2A}{c^2 R^2 \cos A} \]
Any Questions?

Thank you for your attention!