DARK ENERGY AS A FIXED POINT OF THE EINSTEIN YANG-MILLS HIGGS EQUATIONS

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Facts

- The current expansion of the Universe is accelerated

- The acceleration started "recently"

- General Relativity mathematically can explain why: cosmological constant

- Λ is way too small to make sense in QFT

Ways out

- Acceleration is just a local effect

- Acceleration is due to "exotic" d.o.f. in the matter sector (quintessence, etc)

- Standard General Relativity does not work at the largest scales (*f(R),* etc*)*

- There are fundamental, global fields to take in account beyond gravity

THE PROPOSAL

"Accelerated expansion corresponds to the only stable, asymptotic fixed point of the Yang-Mills Higgs Einstein equations on a homogeneous and isotropic cosmological spacetime"

"Dark energy as a fixed point of the Einstein Yang-Mills Higgs Equations"
M. Rinaldi, JCAP 1510 (2015) 10, 023
e-Print: arXiv:1508.04576

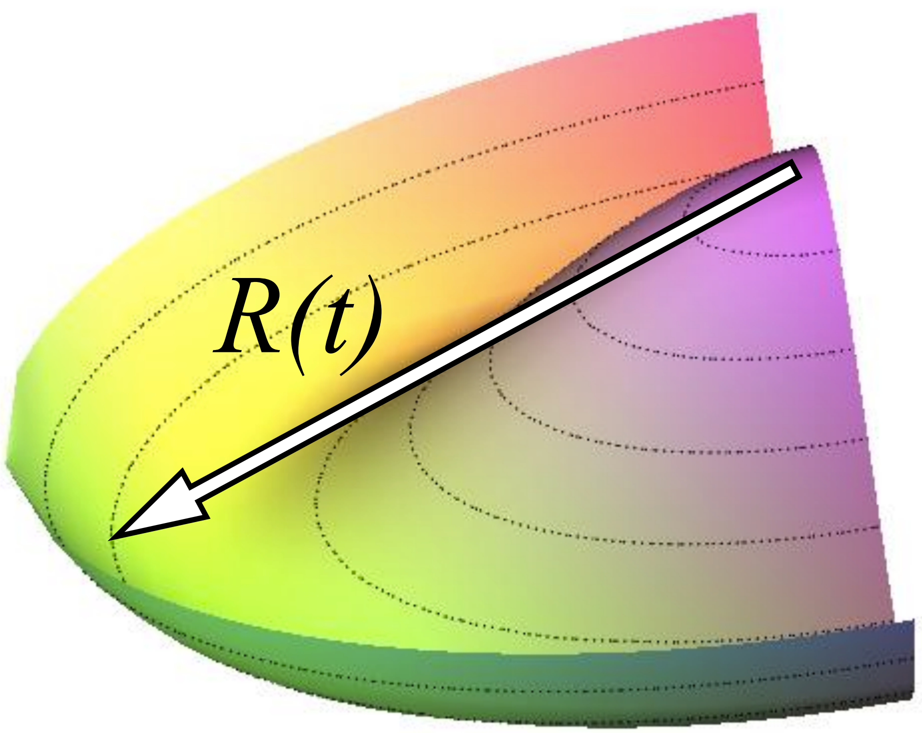
"Higgs Dark Energy"
M. Rinaldi, Class.Quant.Grav. 32 (2015) 045002
e-Print: arXiv:1404.0532

"The dark aftermath of Higgs inflation"
M. Rinaldi, Eur.Phys.J.Plus 129 (2014) 56
e-Print: arXiv:1309.7332

Motivations & History (1)

"Spintessence" (Boyle et al., PLB 2002).

Complex scalar field in *U(1)* potential: the *U(1)* charge, rescaled by cosmological expansion, acts as a time-dependent cosmological constant.



$$\ddot{R} + 3H\dot{R} + V'(R) = \frac{Q^2}{a^6 R^3}$$

Q depends on the phase: slow spinning gives quintessence, large spinning gives spintessence.

The model is very unstable and decays into Q-balls (à la Coleman) except for designer (unnatural) potentials.

It gives an interesting dark matter candidate.

Motivations & History (2)

"Higgs inflation" (Bezrukov et al., PLB 2008).
Jordan frame Lagrangian with non-minimally coupled Higgs doublet

$$L_{\rm tot} = L_{\rm SM} - \frac{M^2}{2}R - \xi H^\dagger H R$$

including Yang-Mills fields!

Einstein frame and unitary gauge: flat potential, good inflationary dynamics.

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\right\} \qquad U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

Unitary gauge has been chosen = Yang-Mills fields neglected
Kaiser et al. studied multifield dynamics for the Higgs in this context.

Ignoring YM is OK during inflation … but what happens at low energy?

Motivations & History (3)

Higgs inflation model rewritten:

$$\frac{\mathcal{L}_{\mathcal{J}}}{\sqrt{g}} = \left(\frac{M_p^2}{2} + \xi \mathcal{H}^\dagger \mathcal{H}\right) R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{1}{4} F^2 - V(\mathcal{H}^\dagger \mathcal{H})$$

SU(N)

At low energy Jordan frame ≃ Einstein frame:

$$\frac{\mathcal{L}_{\mathcal{E}}}{\sqrt{g}} \simeq \frac{M_p^2}{2} R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{1}{4} F^2 - V(\mathcal{H}^\dagger \mathcal{H})$$

The question is whether in this regime the classical dynamics is affected by Gauge Fields + Higgs complex multiplet.

THE FULL MODEL

Full Einstein Yang-Mills + Higgs + matter fluid action:

$$L = \sqrt{|\det g|}\left[\frac{M^2}{2}R - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{2}(D_\mu\Phi^a)(D^\mu\Phi^a) - V(\Phi^a\Phi^a)\right] + L_m$$

where:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu \qquad a = 1, 2, 3$$

$$D_\mu\Phi^a = \partial_\mu\Phi^a + g\epsilon^{abc}A^b_\mu\Phi^c$$

$$V = \frac{\lambda}{4}\left(\Phi^2 - \Phi_0^2\right)^2$$

L_m = perfect fluid Lagrangian (cold dark matter + radiation)

We choose a *SO(3)* representation and we impose isotropy and homogeneity:

$$A^a_0 = 0\,, \qquad A^a_i = f(t)\delta^a_i$$

Equations of motion

$$H^2 = \frac{1}{3M^2}\left[\frac{3\dot{f}^2}{2a^2} + \frac{3g^2 f^4}{2a^4} + \frac{\dot{\Phi}^2}{2} + \frac{g^2 f^2 \Phi^2}{a^2} + V + \rho_{\rm m} + \rho_{\rm r}\right]$$

$$\dot{H} = -\frac{1}{2M^2}\left[\frac{2\dot{f}^2}{a^2} + \frac{2g^2 f^4}{a^4} + \dot{\Phi}^2 + \frac{2g^2 f^2 \Phi^2}{3a^2} + \rho_{\rm m} + \frac{4\rho_{\rm r}}{3}\right]$$

$$\dot{\rho}_{\rm m} = -3H\rho_{\rm m}\,, \quad \dot{\rho}_{\rm r} = -4H\rho_{\rm r} \qquad V = \frac{\lambda}{4}\left(\Phi^2 - \Phi_0^2\right)^2$$

$$\ddot{f} + H\dot{f} + \frac{2g^2 f^3}{a^2} + \frac{2g^2 f \Phi^2}{3} = 0$$

$$\ddot{\Phi}^a + 3H\dot{\Phi}^a + \frac{2g^2 f^2 \Phi^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) = 0\,, \quad a = 1, 2, 3\,.$$

EQUATIONS OF MOTION

$$H^2 = \frac{1}{3M^2}\left[\frac{3\dot{f}^2}{2a^2} + \frac{3g^2 f^4}{2a^4} + \frac{\dot{\Phi}^2}{2} + \frac{g^2 f^2 \Phi^2}{a^2} + V + \rho_{\rm m} + \rho_{\rm r}\right]$$

$$\dot{H} = -\frac{1}{2M^2}\left[\frac{2\dot{f}^2}{a^2} + \frac{2g^2 f^4}{a^4} + \dot{\Phi}^2 + \frac{2g^2 f^2 \Phi^2}{3a^2} + \rho_{\rm m} + \frac{4\rho_{\rm r}}{3}\right]$$

Note that $\dot{H} \leq 0$ can vanish for $a \to \infty$

$$\dot{\rho}_{\rm m} = -3H\rho_{\rm m}\,, \quad \dot{\rho}_{\rm r} = -4H\rho_{\rm r}$$

$$V = \frac{\lambda}{4}\left(\Phi^2 - \Phi_0^2\right)^2$$

$$\ddot{f} + H\dot{f} + \frac{2g^2 f^3}{a^2} + \frac{2g^2 f \Phi^2}{3} = 0$$

Not good for slow-roll:

$$\frac{1}{V}\frac{dV}{d\phi_a} \to \infty$$

$$\ddot{\Phi}^a + 3H\dot{\Phi}^a + \frac{2g^2 f^2 \Phi^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) = 0\,, \quad a = 1, 2, 3\,.$$

Note $\Phi = \Phi_0$ is NOT a solution unless $g = 0$, $f = 0$, or $\Phi_0 = 0$

Late-time acceleration is possible only in the "ultra slow-roll" regime:

$$|\ddot{\Phi}| = |\dot{\Phi}| \ll 1$$

$$\frac{2g^2 f^2 \Phi^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) \simeq 0$$

DYNAMICAL SYSTEM ANALYSIS (1)

Define the new dimensionless variables (' = derivative wrt $N = ln\ a$) :

$$x = \frac{f'}{\sqrt{2}aM}\,, \quad y = \frac{gf^2}{\sqrt{2}MHa^2}\,, \quad v = \frac{1}{MH}\sqrt{\frac{V}{3}}\,, \quad r = \frac{1}{MH}\sqrt{\frac{\rho_{\rm r}}{3}}\,,$$

$$l = \frac{\sqrt{2}Ma}{f}\,, \quad w^i = \frac{gf\Phi^i}{\sqrt{3}MaH}\,, \quad z^i = \frac{(\Phi^i)'}{\sqrt{6}M}\,, \quad i = 1, 2, 3\,,$$

Deceleration parameter and effective equation of state:

$$q = -1 - \frac{H'}{H} = \frac{1}{2}(1 + x^2 + y^2 + r^2 - 3v^2 + 3z^2 - w^2) \qquad \omega_{\rm eff} = \frac{1}{3}(2q - 1)$$

Energy densities:

$$\Omega_{\rm m} \equiv \frac{\rho_{\rm m}}{3M^2H^2} = 1 - (x^2 + y^2 + z^2 + w^2 + v^2 + r^2)$$

$$\Omega_{\rm r} = r^2\,,$$

$$\Omega_{\rm de} = x^2 + y^2 + z^2 + w^2 + v^2$$

$$1 = \Omega_{\rm de} + \Omega_{\rm r} + \Omega_{\rm m}$$

Dynamical System Analysis (2)

System of equations:

$$\alpha = \frac{\sqrt{3\lambda}}{g}$$

(Note that the system depends on the ratio ***α*** only. In SM ***α* = 0.96**)

$$
\begin{aligned}
l' &= l(1 - lx)\,, \\
x' &= (q-1)x - l(w^2 + 2y^2)\,, \\
y' &= y(q - 1 + 2xl)\,, \\
r' &= (q-1)r\,, \\
v' &= v(q+1) + \alpha l(w_1 z_1 + w_2 z_2 + w_3 z_3)\,, \\
w_i' &= w_i(q + lx) + \sqrt{2} l y z_i\,, \quad i = 1,2,3\,, \\
z_i' &= (q-2) z_i - l w_i(\sqrt{2} y + \alpha v)\,, \quad i = 1,2,3\,,
\end{aligned}
$$

There are 5 classes of fixed points corresponding to:

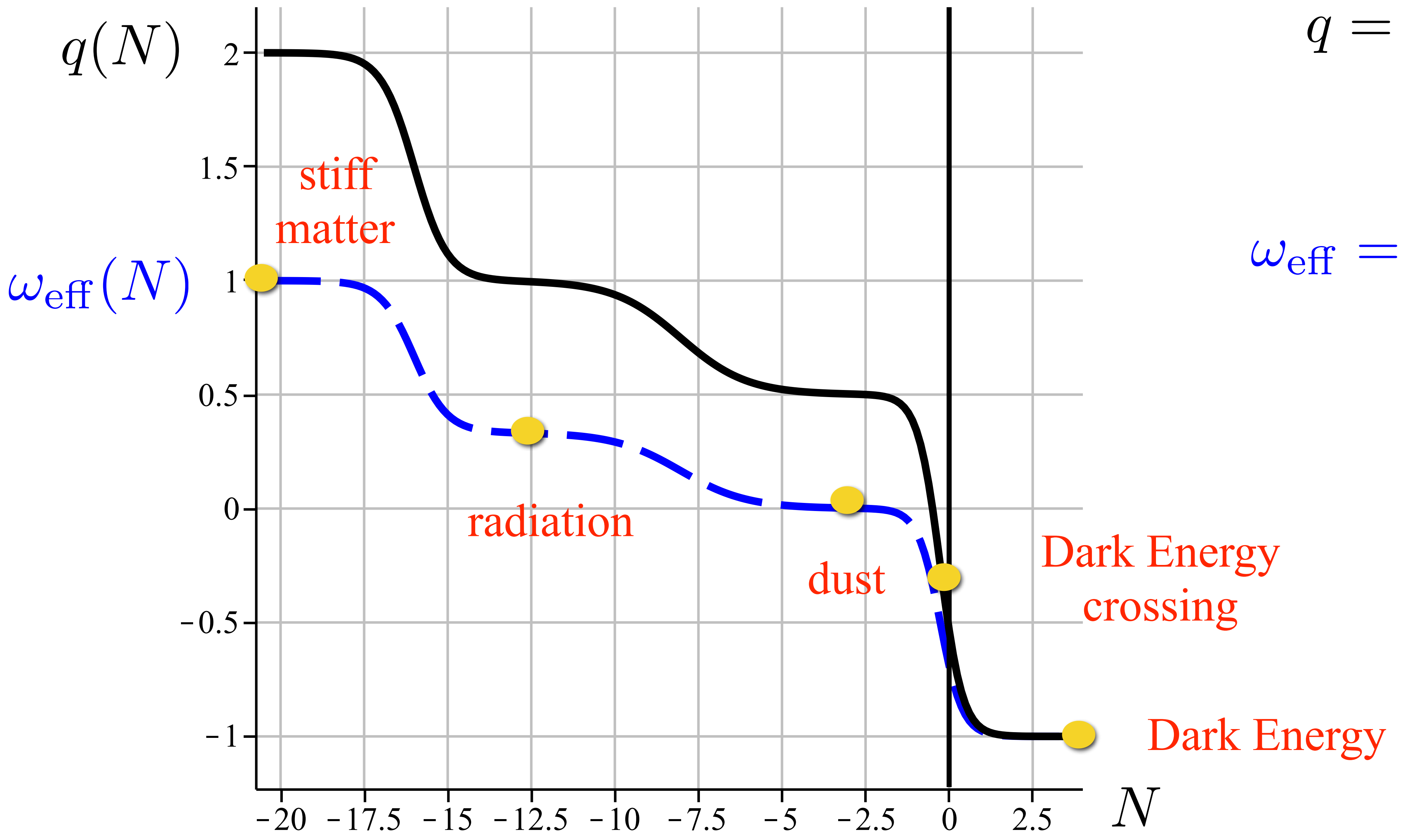
UNSTABLE

$$q = \begin{pmatrix} 2 \\ 1 \\ 1/2 \\ 0 \end{pmatrix}$$

STABLE

$$q = -1$$

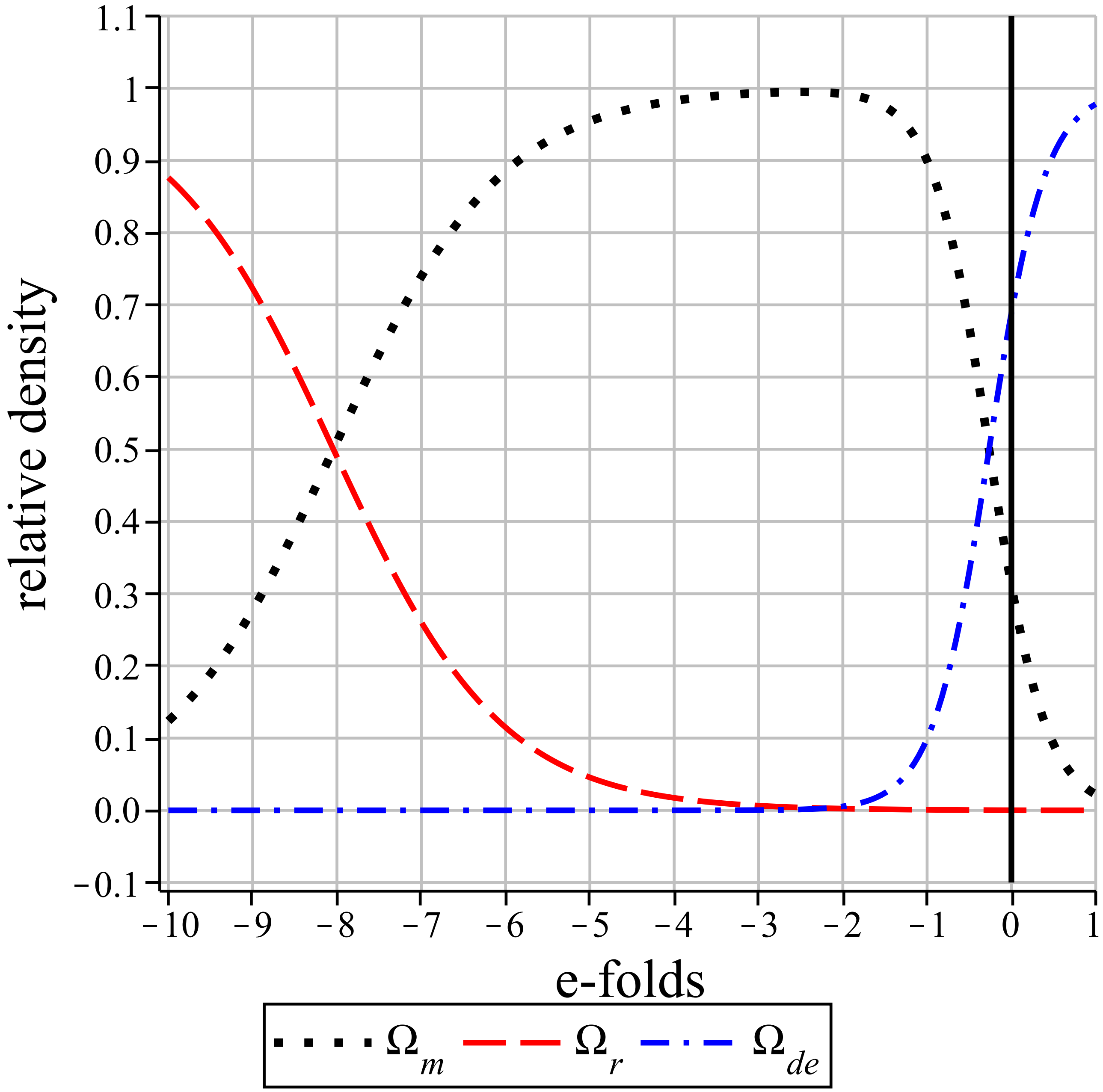
DYNAMICAL SYSTEM ANALYSIS (3)

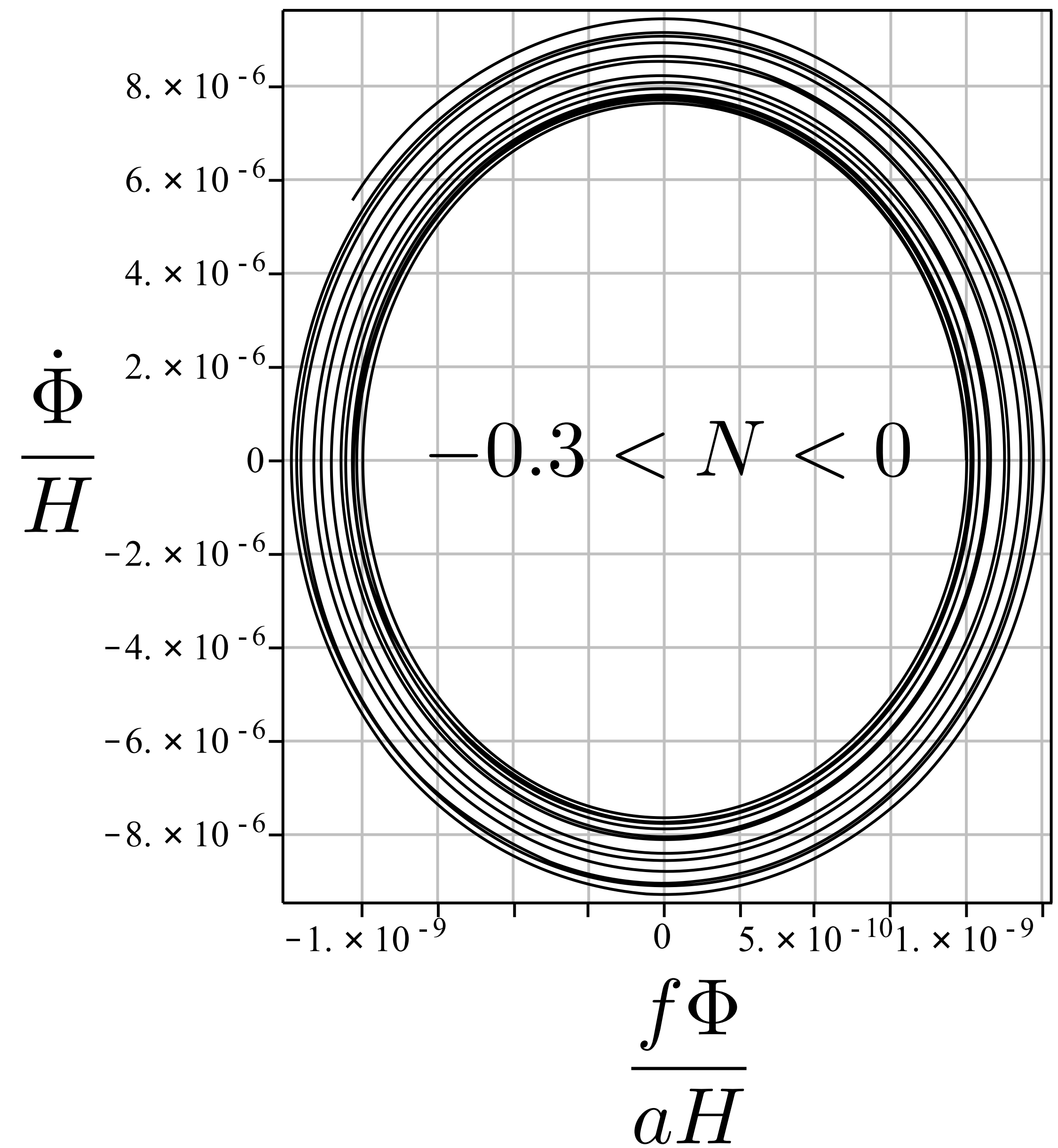


$$q = -1 - \frac{H'}{H}$$

$$\omega_{\rm eff} = \frac{1}{3}(2q-1)$$

Dynamical System Analysis (4)





Initial conditions at N = 0 :

$$\begin{cases} x = 10^{-9}\,, & y = 10^{-8}\,, & v = 0.83\,, & r = 10^{-2}\,, \\ z_1 = -10^{-11}\,, & z_2 = 2\times 10^{-11}\,, & z_3 = 3\times 10^{-11}\,, & l = 4\times 10^{-8}\,, \\ w_1 = 10^{-9}\,, & w_2 = 4\times 10^{-10}\,, & w_3 = 2\times 10^{-9}\,, & \end{cases}$$

LINEARISED SYSTEM

We linearise and solve the system around the stable fixed point:

$$\omega_{\rm eff} = -1 - \frac{w_0^2}{3a^2} - \frac{2v_0}{a^3} + \frac{x_0^2 + y_0^2 + r_0^2}{3a^4} - \frac{v_0^2 - z_0^2}{a^6}$$

we impose the conditions

$$\left.\frac{d\omega_{\rm eff}}{da}\right|_{a=1} = \left.\frac{d^2\omega_{\rm eff}}{da^2}\right|_{a=1} = \left.\frac{d^3\omega_{\rm eff}}{da^3}\right|_{a=1} = 0$$

we find that

$$\omega_{\rm eff} = -1 - \frac{w_0^2}{27} = -1 - \frac{g^2 f_0^2 \Phi_0^2}{3M^2 H_0^2}$$

therefore this model predicts phantom dark energy at the present time!

Linearised System

- If we compare with data (WiggleZ DE Survey)

$$\boxed{\omega_0 = -1.080 \pm 0.135}$$

- assume the Standard Model parameter ***g = 1/2***,
- assume that the system is close to the potential minimum

$$\Phi \simeq \Phi_0 \simeq 246 \ \ \mathrm{GeV}$$

we can fix the values of the current gauge field to

$$f_0 \simeq 7 \times 10^{-26} \ \ \mathrm{GeV}$$

work in progress…

Conclusions

- Dark Energy is natural in Yang-Mills Higgs Einstein gravity
- It is a mathematical solution and the only stable fixed point
- Standard potential and couplings
- Other fixed points correspond to matter/radiation domination
- There are trajectories linking Early Universe to DE dominated Universe
- The model makes a strong prediction: phantom dark energy today

Things To Do

- Perturbations
- Improve forecasts, including early DE
- Connect to Early Universe (Higgs inflation?)
- Quantum corrections …