DARK ENERGY AS A FIXED POINT OF THE EINSTEIN YANG-MILLS HIGGS EQUATIONS

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Facts

- The current expansion of the Universe is accelerated
- The acceleration started “recently”
- General Relativity mathematically can explain why: cosmological constant
- $\Lambda$ is way too small to make sense in QFT

Ways out

- Acceleration is just a local effect
- Acceleration is due to “exotic” d.o.f. in the matter sector (quintessence, etc)
- Standard General Relativity does not work at the largest scales ($f(R)$, etc)
- There are fundamental, global fields to take in account beyond gravity
"Accelerated expansion corresponds to the only stable, asymptotic fixed point of the Yang-Mills Higgs Einstein equations on a homogeneous and isotropic cosmological spacetime"

"Dark energy as a fixed point of the Einstein Yang-Mills Higgs Equations"
M. Rinaldi, JCAP 1510 (2015) 10, 023
e-Print: arXiv:1508.04576

"Higgs Dark Energy"
M. Rinaldi, Class.Quant.Grav. 32 (2015) 045002
e-Print: arXiv:1404.0532

"The dark aftermath of Higgs inflation"
e-Print: arXiv:1309.7332
“Spintessence” (Boyle et al., PLB 2002).

Complex scalar field in $U(1)$ potential: the $U(1)$ charge, rescaled by cosmological expansion, acts as a time-dependent cosmological constant.

$$\ddot{R} + 3H \dot{R} + V'(R) = \frac{Q^2}{a^6 R^3}$$

$Q$ depends on the phase: slow spinning gives quintessence, large spinning gives spintessence.

The model is very unstable and decays into Q-balls (à la Coleman) except for designer (unnatural) potentials.

It gives an interesting dark matter candidate.
“Higgs inflation” (Bezrukov et al., PLB 2008).

Jordan frame Lagrangian with non-minimally coupled Higgs doublet

\[ L_{\text{tot}} = L_{\text{SM}} - \frac{M^2}{2} R - \xi H^\dagger HR \]

including Yang-Mills fields!

Einstein frame and unitary gauge: flat potential, good inflationary dynamics.

\[ S_E = \int d^4x \sqrt{-\hat{g}} \left\{ - \frac{M_P^2}{2} \hat{R} + \frac{\partial_{\mu} \chi \partial^{\mu} \chi}{2} - U(\chi) \right\} \quad U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 + \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \right)^{-2} \]

Unitary gauge has been chosen = Yang-Mills fields neglected

Kaiser et al. studied multifield dynamics for the Higgs in this context.

Ignoring YM is OK during inflation … but what happens at low energy?
Higgs inflation model rewritten:

\[ \frac{\mathcal{L}_J}{\sqrt{g}} = \left( \frac{M_p^2}{2} + \xi \mathcal{H}^\dagger \mathcal{H} \right) R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{1}{4} F^2 - V(\mathcal{H}^\dagger \mathcal{H}) \]

At low energy Jordan frame \( \approx \) Einstein frame:

\[ \frac{\mathcal{L}_E}{\sqrt{g}} \approx \frac{M_p^2}{2} R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{1}{4} F^2 - V(\mathcal{H}^\dagger \mathcal{H}) \]

The question is whether in this regime the classical dynamics is affected by Gauge Fields + Higgs complex multiplet.
The full model

Full Einstein Yang-Mills + Higgs + matter fluid action:

\[
L = \sqrt{|\det g|} \left[ \frac{M^2}{2} R - \frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a - \frac{1}{2} (D_\mu \Phi^a)(D^\mu \Phi^a) - V(\Phi^a \Phi^a) \right] + L_m
\]

where:

- \( F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu \quad a = 1, 2, 3 \)
- \( D_\mu \Phi^a = \partial_\mu \Phi^a + g\varepsilon^{abc} A^b_\mu \Phi^c \)
- \( V = \frac{\lambda}{4} (\Phi^2 - \Phi_0^2)^2 \)

\( L_m \) = perfect fluid Lagrangian (cold dark matter + radiation)

We choose a \( SO(3) \) representation and we impose isotropy and homogeneity:

\[
A_0^a = 0, \quad A_i^a = f(t) \delta_i^a
\]
EQUATIONS OF MOTION

\[
H^2 = \frac{1}{3M^2} \left[ \frac{3f^2}{2a^2} + \frac{3g^2f^4}{2a^4} + \frac{\dot{\Phi}^2}{2} + \frac{g^2f^2\Phi^2}{a^2} + V + \rho_m + \rho_r \right]
\]

\[
\dot{H} = -\frac{1}{2M^2} \left[ \frac{2f^2}{a^2} + \frac{2g^2f^4}{a^4} + \dot{\Phi}^2 + \frac{2g^2f^2\Phi^2}{3a^2} + \rho_m + \frac{4\rho_r}{3} \right]
\]

\[
\dot{\rho}_m = -3H\rho_m \quad \dot{\rho}_r = -4H\rho_r \quad V = \frac{\lambda}{4} (\Phi^2 - \Phi_0^2)^2
\]

\[
\ddot{f} + H\dot{f} + \frac{2g^2f^3}{a^2} + \frac{2g^2f\Phi^2}{3} = 0
\]

\[
\ddot{\Phi}^a + 3H\dot{\Phi}^a + \frac{2g^2f^2\Phi^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) = 0, \quad a = 1, 2, 3
\]
**Equations of motion**

\[
H^2 = \frac{1}{3M^2} \left[ \frac{3f^2}{2a^2} + \frac{3g^2f^4}{2a^4} + \frac{\dot{\Phi}^2}{2} + \frac{g^2f^2\Phi^2}{a^2} + V + \rho_m + \rho_r \right]
\]

\[
\dot{H} = -\frac{1}{2M^2} \left[ \frac{2f^2}{a^2} + \frac{2g^2f^4}{a^4} + \dot{\Phi}^2 + \frac{2g^2f^2\Phi^2}{3a^2} + \rho_m + \frac{4\rho_r}{3} \right]
\]

\[
\dot{\rho}_m = -3H \rho_m, \quad \dot{\rho}_r = -4H \rho_r
\]

\[
f + H\dot{f} + \frac{2g^2f^3}{a^2} + \frac{2g^2f\Phi^2}{3} = 0
\]

\[
\ddot{\Phi}^a + 3H\dot{\Phi}^a + \frac{2g^2f^2\Phi^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) = 0, \quad a = 1, 2, 3.
\]

Note that $\dot{H} \leq 0$ can vanish for $a \to \infty$

\[
V = \frac{\lambda}{4}(\Phi^2 - \Phi_0^2)^2
\]

Not good for slow-roll:

\[
\frac{1}{V} \frac{dV}{d\Phi_a} \to \infty
\]

Late-time acceleration is possible only in the “ultra slow-roll” regime:

\[
|\ddot{\Phi}| = |\dot{\Phi}| \ll 1
\]

\[
\frac{2g^2f^2\dot{\Phi}^a}{a^2} + \lambda\Phi^a(\Phi^2 - \Phi_0^2) \simeq 0
\]

M. Rinaldi - Trento U.
Dynamical System Analysis (1)

Define the new dimensionless variables (‘ = derivative wrt $N = \ln a$):

\[
\begin{align*}
x &= \frac{f'}{\sqrt{2}aM}, & y &= \frac{gf^2}{\sqrt{2}MHa^2}, & v &= \frac{1}{MH}\sqrt{\frac{V}{3}}, & r &= \frac{1}{MH}\sqrt{\frac{\rho_r}{3}}, \\
l &= \frac{\sqrt{2}Ma}{f}, & w^i &= \frac{gf\Phi^i}{\sqrt{3}MaH}, & z^i &= \frac{(\Phi^i)'}{\sqrt{6}M}, & i &= 1, 2, 3,
\end{align*}
\]

Deceleration parameter and effective equation of state:

\[
q = -1 - \frac{H'}{H} = \frac{1}{2}(1 + x^2 + y^2 + r^2 - 3v^2 + 3z^2 - w^2) \quad \omega_{\text{eff}} = \frac{1}{3}(2q - 1)
\]

Energy densities:

\[
\begin{align*}
\Omega_m &\equiv \frac{\rho_m}{3M^2H^2} = 1 - (x^2 + y^2 + z^2 + w^2 + v^2 + r^2) \\
\Omega_r &\equiv r^2, \\
\Omega_{\text{de}} &\equiv x^2 + y^2 + z^2 + w^2 + v^2
\end{align*}
\]

\[1 = \Omega_{\text{de}} + \Omega_r + \Omega_m\]
Dynamical System Analysis (2)

System of equations:

\[ l' = l(1 - lx), \]
\[ x' = (q - 1)x - l(w^2 + 2y^2), \]
\[ y' = y(q - 1 + 2xl), \]
\[ r' = (q - 1)r, \]
\[ v' = v(q + 1) + \alpha l(w_1 z_1 + w_2 z_2 + w_3 z_3), \]
\[ w_i' = w_i(q + lx) + \sqrt{2}l y z_i, \quad i = 1, 2, 3, \]
\[ z_i' = (q - 2)z_i - lw_i(\sqrt{2}y + \alpha v), \quad i = 1, 2, 3, \]

(Note that the system depends on the ratio \( \alpha \) only. In SM \( \alpha = 0.96 \))

There are 5 classes of fixed points corresponding to:

**UNSTABLE**

\[ q = \begin{pmatrix} 2 \\ 1 \\ 1/2 \\ 0 \end{pmatrix} \]

**STABLE**

\[ q = -1 \]
Figure 1. Evolution of the deceleration parameter $q(N)$ and the corresponding effective equation of state $\omega_{\text{eff}}(N)$. The initial conditions are given by (4.1) at $N=0$, corresponding to the black vertical line. We note that the evolution of the system mimics the transition between a remote past stiff matter-dominated Universe and a final dark energy-dominated Universe, passing through two transient phases of radiation and matter domination respectively.

As explained above, in principle we do not have clear indications on the value of $\omega_{\text{eff}}$. However, we can relate it to physical observables. By using the definitions of the variables $l(N)$, $y(N)$, and $v(N)$, we find that

$$M_H^2 = y l^2 p^2 g^2,$$

(4.2)

$$2M_0^2 = 6 p^2 v y l^2.$$

The first relation implies that, at the present time, and with the initial conditions (4.1), the coupling $g$ must be very small. With $H = 1.4 \times 10^{-42}$ GeV and $M = 2.4 \times 10^{18}$ GeV, and $\omega_{\text{eff}} = 1$, we find $g \ll 10^{54}$. For $\omega_{\text{eff}} = 1$, this implies an even smaller $g$, of the order $g \ll 10^{109}$.

The second relation instead gives an estimate of the displacement from the vacuum value of the Higgs field, which corresponds to $\omega_{\text{eff}} = 10^6$ with (4.1).

If we wish to increase the value of $g$ and $\omega_{\text{eff}}$, we need to increase of several orders of magnitude the initial value of $l(N)$ (on the contrary, the value of $y(N)$ cannot be larger than unity because of the constraints (3.5) and (4.7)). In turn, this implies that the initial value of $x(N)$ must be carefully chosen so that the first equation of (3.8) does not yield large derivatives. This means, essentially, that $x \approx 1/l$ over all the integration range. All
Dynamical System Analysis (4)

Initial conditions at \( N = 0 \):

\[
\begin{aligned}
x &= 10^{-9}, & y &= 10^{-8}, & v &= 0.83, & r &= 10^{-2}, \\
z_1 &= 10^{-11}, & z_2 &= 2 \times 10^{-11}, & z_3 &= 3 \times 10^{-11}, & l &= 4 \times 10^{-8}, \\
w_1 &= 10^{-9}, & w_2 &= 4 \times 10^{-10}, & w_3 &= 2 \times 10^{-9}, \\
\end{aligned}
\]
**LINEARISED SYSTEM**

We linearise and solve the system around the stable fixed point:

\[
\omega_{\text{eff}} = -1 - \frac{w_0^2}{3a^2} - \frac{2v_0}{a^3} + \frac{x_0^2 + y_0^2 + r_0^2}{3a^4} - \frac{u_0^2 - z_0^2}{a^6}
\]

we impose the conditions

\[
\left. \frac{d\omega_{\text{eff}}}{da} \right|_{a=1} = \left. \frac{d^2\omega_{\text{eff}}}{da^2} \right|_{a=1} = \left. \frac{d^3\omega_{\text{eff}}}{da^3} \right|_{a=1} = 0
\]

we find that

\[
\omega_{\text{eff}} = -1 - \frac{w_0^2}{27} = -1 - \frac{g^2 f_0^2 \Phi_0^2}{3M^2 H_0^2}
\]

therefore this model predicts phantom dark energy at the present time!
**LINEARISED SYSTEM**

- If we compare with data (WiggleZ DE Survey)

\[ \omega_0 = -1.080 \pm 0.135 \]

- assume the Standard Model parameter \( g = 1/2 \),
- assume that the system is close to the potential minimum

\[ \Phi \simeq \Phi_0 \simeq 246 \text{ GeV} \]

we can fix the values of the current gauge field to

\[ f_0 \simeq 7 \times 10^{-26} \text{ GeV} \]

work in progress…
CONCLUSIONS

• Dark Energy is natural in Yang-Mills Higgs Einstein gravity
• It is a mathematical solution and the only stable fixed point
• Standard potential and couplings
• Other fixed points correspond to matter/radiation domination
• There are trajectories linking Early Universe to DE dominated Universe
• The model makes a strong prediction: phantom dark energy today

THINGS TO DO

• Perturbations
• Improve forecasts, including early DE
• Connect to Early Universe (Higgs inflation?)
• Quantum corrections …