Are outflows and jets rotation-powered? If so, where are their “power stations” and “roots”?
Frame dragging, Unipolar induction and Jets in Kerr black hole magnetospheres

Toward Constructing “Power Station” for Outflows and Jets

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Acknowledgement
References
1. Introduction

1.1 Scenario in the 1980s

- **Extraction of hole’s spin energy**: Blandford-Znajek process (1977), Phinney’s jet model (1982), Macdonald & Thorne’s “3+1” formalism (82), Thorne et al.’s “The Membrane Paradigm” (86). Invited a criticism of causality violation.

- **Existence of a battery in the horizon of a magnetized Kerr hole.**

- **By no-hair theorem, we can draw out no information from under the Kerr hole’s horizon, except mass M and angular momentum J.**

The image in the 1980s

Single circuit

1.2 Unipolar Induction battery for pulsars

A perfectly conducting sphere, rotating with $\Omega$ about the direction of magnetization $M$

$$\text{EMF} = B_0 \Omega a^3/2c = \Omega M/ca$$

$$M = B_0 a^3/2$$

= total magnetic moment

between two field lines $\psi_1$ and $\psi_2$
pinned down at the neutron star

It is $\Omega_F$ that relates unipolar induction to EMF.
1.3 Purpose

- **Purpose**: By coupling frame dragging effect with unipolar induction in flat space, construct a strong power station consisting of not only a pair of batteries for driving currents but strong voltage drop in-between for particle creation in a non-causal location.

- Revive the BZ process, Phinney’s jet model, and the Membrane Paradigm.
2.1 Black Hole Electrodynamics

- We use “3+1” formalism in Boyer-Lindquist coordinates. GR effects are condensed in
  $\alpha$: lapse function and $0 \leq \alpha \leq 1$
  $\omega$: frame-dragging angular velocity. $\Omega_H \geq \omega \geq 0$

- FIDucial Observer FIDOs, resident in absolute space circulating with $\omega$.

- FIDO-measured FLAV: $\Omega_{F\omega} = \Omega_F - \omega$
  Key quantity embodying the coupling of frame dragging with unipolar induction.

- Assume presence of copious particles threaded by large-scale magnetic field lines with the angular velocity $\Omega_F$ constant from $S_H$ to $S_{\infty}$. 
The freezing-in, force-free conditions

\[ E = -(v/c) \times B, \quad \rho_e E + (j/c) \times B = 0 \]

where

\[ E \cdot B = j \cdot E = 0, \quad j_p = \rho_e \nu_p = -\frac{1}{\alpha \frac{dI}{d\Psi}} B_p \]

\[ B_p = -\frac{1}{2\pi \omega} (t \times \nabla \Psi), \quad B_t = -\frac{2I}{\omega \alpha c}, \]

where \( \Psi = \) constant \( \Rightarrow \) “field-streamline”

\[ I = \) constant \( \Rightarrow \) “current line”

\[ E_p = -\frac{\Omega_{Fw}}{2\pi \alpha c} \nabla \Psi, \quad S_{EM} = \frac{\alpha c}{4\pi} (E \times B) = \frac{\Omega_{Fw} I(\Psi)}{2\pi \alpha c} B_p \]

No MHD-acceleration. Current-field-streamlines are equipotentials in the force-free domains.

Charge-separated plasma. \( \rho_e = -en(\cdot) \) or \( = +en(+) \)
3.1 Eigenvalue Problem due to Criticality-Boundary Conditions

- Two integral functions of $\Psi$
  - $\Omega_F(\psi)$: field line angular velocity/potential gradient
  - $I(\psi)$: current function/field angular momentum flux

- $S_{ff\infty}$ and $S_{ffH}$: two membranes terminating “force-free” domains. The “criticality condition” from wind theory fixes $I_{out}(\psi)$ and $I_{in}(\psi)$ for outflow, inflow.

- $S_N$: the interface between the two, outer and inner force-free domains. The “boundary condition” for continuity of angular momentum flux to be continuous across $S_N$, i.e. $I_{out}(\psi) = I_{in}(\psi)$, fixes $\Omega_F(\psi)$. 
3.2 Eigenvalues $I_{\text{out}}(\Psi), I_{\text{in}}(\Psi)$ and $\Omega_f(\Psi)$

Criticality condition for ingoing/outgoing winds

$$I(\Psi) = \begin{cases} 
\frac{1}{2}(\Omega_H - \Omega_F)(B_p \varpi^2)_{\text{ffH}} \equiv I_{\text{in}}, & \text{at } S_{iF} \approx S_{\text{ffH}}, \quad \omega \approx \Omega_H, \\
\frac{1}{2} \Omega_F (B_p \varpi^2)_{\text{ff}\infty} \equiv I_{\text{out}}, & \text{at } S_{oF} \approx S_{\text{ff}\infty}, \quad \omega \approx 0.
\end{cases}$$

Equivalent to Ohm’s law on resistive membranes $S_{\text{ffH}}$ and $S_{\text{ff}\infty}$

Boundary condition $I_{\text{in}} = I_{\text{out}}$ at $S_N$, $\omega = \Omega_F$

$$\Omega_F = \frac{\Omega_H}{1 + \zeta}, \quad \zeta \equiv \frac{(B_p \varpi^2)_{\text{ff}\infty}}{(B_p \varpi^2)_{\text{ffH}}}.$$
3.3 FIDO-measured field line angular velocity and gravito-electric potential

\[ A^{(4)} = (A_0 \omega, 0, 0, A_\phi) : \text{ Kerr space 4-potential} \]
\[ A_0 \omega = A_0 + V : \text{ Kerr space scalar potential} \]
\[ A_0 : \text{ Flat-space scalar potential} \]
\[ V : \text{ Gravito-electric potential} \]
\[ A_\phi = \Psi / 2\pi : \text{ Magnetic potential} \]
\[ \Omega_F = -2\pi c (dA_0 / d\Psi) : \]
\[ \omega = 2\pi c (dV / d\Psi)_\ell : \]
\[ \Omega_{F\omega} = -2\pi c (dA_0\omega / d\Psi)_\ell = \Omega_F - \omega : \]
4.1 Angular Momentum and Energy Fluxes

- $S_J$: Angular momentum flux, extracted from the hole by the surface torque.
- $S_{SD}$: Spin-down energy flux form the hole due to the frame dragging effect, related to $S_J$.
- $S_{EM}$: Poynting ElectroMagnetic energy flux
- $S_E$: Total energy flux

$$S_J = \frac{I(\Psi)}{2\pi \alpha c} B_p, \quad S_{SD} = \omega S_J, \quad S_{EM} = \Omega_F \omega S_J,$$

$$S_E = S_{SD} + S_{EM} = \Omega_F S_J$$

$$\Leftrightarrow \omega + (\Omega_F - \omega) = \Omega_F$$
Angular momentum loss of the hole by the surface torque to $S_{\text{ffH}}$

$$\frac{dJ}{dt} = - \int (\alpha I_{\text{ffH}} / c \times B_p) \cdot \omega t \, dA$$
$$= - \frac{1}{2\pi c} \int I(\Psi) \, d\Psi = - \int \alpha S_J \cdot dA$$

Angular momentum flux

$$S_J = \frac{I(\Psi)}{2\pi \alpha c} B_p$$

Spin-down energy flux due to the frame dragging effect

$$S_{\text{SD}} = \omega S_J = \frac{\omega I}{2\pi \alpha c} B_p$$

Loss of the hole’s rotational energy

$$\int_{\text{ffH}} \alpha S_{\text{SD}} \cdot dA = \frac{\Omega_H}{2\pi c} \int I(\Psi) \, d\Psi = -\Omega_H \frac{dJ}{dt}$$
4.3 Three modes of energy fluxes

Variation of energy fluxes with $\omega$ along each field line

$$\Omega_F = \omega + (\Omega_F - \omega)$$

- **Spin-down energy flux due to frame dragging**
- **Total energy flux**
- **Poynting flux**

**Outer domain**

**Inner domain**
4.4 Interface $S_N$ between two domains

- **Outer domain**: outgoing pulsar-type wind, $\Omega_{FW} > 0$
  \[ E_p < 0, \quad \nu_p = j_p / \rho_e > 0 \]

- **Inner domain**: ingoing anti-pulsar-type wind with $\Omega_{FW} < 0$, i.e. turned-outside-in toward the hole
  \[ E_p > 0, \quad \nu_p = j_p / \rho_e < 0 \]

- **At $S_N$: $\omega = \Omega_F$**
  \[ E_p \lesssim 0, \quad \text{for } \omega \lesssim \Omega_F \]
  \[ \rho_e \to 0, \quad \nu_p = j_p / \rho_e \to \pm \infty \quad \text{for } \omega \to \Omega_F \]

- Inevitable breakdown of force-freeness

- Stationary particles / currents sources needed in-between

- We show existence of a pair of unipolar induction batteries and voltage drop $\Delta V$
  \[ \Delta V = \mathcal{E}_{\text{out}} - \mathcal{E}_{\text{in}} = -\frac{\Omega_H}{2\pi c} \Delta \psi \]
5.1 Double Circuits with Magnetized Gap in-between

- DC double circuit
- $= \text{EMF}$
- + volume-current lines in Force-Free domains
- + Impedances on the Resistive Membranes, $S_{ff\infty}$ and $S_{ffH}$, dissipating Surface-Currents
- + Surface Lorenz torque extracting the hole's AM and rotational energy through $S_{ffH}$ and transferring them to particles on $S_{ff\infty}$.
- One can describe the wind domains by superposition of infinite number of nested circuits.
5.2 Faraday integral along circuits

\[ \mathcal{E}_C = \int_C \alpha \mathbf{E}_p \cdot d\mathbf{l} = - \int_C \Omega_F \omega \nabla \Psi \cdot d\mathbf{l} \]

\[ \mathbf{E}_p = - \frac{\Omega_F \omega}{2\pi \alpha c} \nabla \Psi \]

- Resistive Horizon Membrane
- Integral path
- Inner Domain
- Outer Domain
- Resistive Astrophys. loads

\[ \alpha \mathbf{E}_p \cdot d\mathbf{l} = 0 \quad \text{along } \Psi_{1/2} \]
5.3 EMF's for a pair of circuits, and Voltage Drop hidden under $S_N$

\[ \mathcal{E}_{\text{out}} = - \frac{1}{2\pi c} \int_{\Psi_1}^{\Psi_2} \Omega_F(\Psi) d\Psi, \]

\[ \mathcal{E}_{\text{in}} = + \frac{1}{2\pi c} \int_{\Psi_1}^{\Psi_2} [\Omega_H - \Omega_F(\Psi)] d\Psi, \]

\[ \mathcal{E}_{\text{out}} - \mathcal{E}_{\text{in}} = \Delta V = - \frac{\Omega_H}{2\pi c} \Delta \Psi \]

Note that

\[ \Delta \Omega_{F\omega} \equiv (\Omega_{F\omega})_\infty - (\Omega_{F\omega})_H \]

\[ = \Omega_F - [(\Omega_H - \Omega_F)] \]

\[ = \Omega_H \]
The results for $\mathbf{EMF}$’s are the same for $\Omega_{F\omega}$ and $\overline{\Omega_{F\omega}}$

\[
\mathcal{E}_{\text{out/in}} = \int_{C_{\text{out/in}}} \Omega_{F\omega} \nabla \Psi \cdot d\ell \\
= \int_{C_{\text{out/in}}} \overline{\Omega_{F\omega}} \nabla \Psi \cdot d\ell
\]

in the curved space

in the pseudo-flat space

\[
\overline{\Omega_{F\omega}} = \begin{cases} 
\Omega_{F} & \uparrow ; 0 \leq \omega < \Omega_{F} : \text{pulsar.type} \\
0 & ; \omega = \Omega_{F} : \text{gap} \\
-(\Omega_{H} - \Omega_{F}) & \downarrow ; \Omega_{F} < \omega \leq \Omega_{H} : \text{anti.pulsar.type}
\end{cases}
\]

because current-field-streamlines are equipotentials, and $E_p = 0$ on $S_N$.

This allows us to present the hole’s magnetosphere with a pair of a pulsar and an anti-symmetric pulsar (the outside-turned-in version).
### 6.2 Two “Virtual” Magnetized Rotators

Ingoing wind with \(-\Omega_F\) to Inner Domain,

\[ \Delta \Omega_{F\omega} = (\Omega_{F\omega})_\infty - (\Omega_{F\omega})_H = \Omega_F - [-(\Omega_H - \Omega_F)] = \Omega_H \]

Outgoing wind with \(\Omega_F\) to Outer Domain.
6.3 Simple Image of Double Circuits

\[ I_{in}(\Psi_1) \quad J_p \quad I_{out}(\Psi_1) \]

\[ I_{ffH} \]

\[ \rho_e > 0 \]

\[ -(\Omega_H - \Omega_F) \]

\[ \Omega_F \]

\[ \rho_e < 0 \]

\[ I_{ff\infty} \]

\[ \Psi_1 \]

\[ \Psi_C \]

\[ \Psi_2 \]

\[ \omega = \Omega_H \]

\[ \omega = \Omega_F \]

\[ \omega = 0 \]

Gap \( \Delta V \)
7.1 New Gap Physics?

(1) Two oppositely directed UPI batteries due to two oppositely directed **magnetized** rotators seem to be existent back-to-back under $S_N$.

(2) Voltage drop $\Delta V$ between inner and outer circuits is **stationary, stable, strong enough to pair-create particles**.

$$\Delta V = \varepsilon_{\text{out}} - \varepsilon_{\text{in}} = -\frac{\Omega_H}{2\pi c} \Delta \Psi$$

(3) How large should the particle density be, to **pin down magnetic field lines and to fix $\Omega_f$** by the local value $\omega_N$?

(4) What is magnetized matter under $S_N$? (Cf. neutron stars' matter of pinning field lines down under the surface.)

(5) No need of resorting to invoking vacuum $E_{||}$ and pair-creation discharge for particle supply, as in the previous models.

7.2 What do we expect to take place?

Simple Image of Magnetized Gap

- Plasma source under $S_N$ with $\omega \approx \Omega_F$
- Pinning magnetic fluxes at the plasma source.
- Fixing $\Omega_F = \omega(\ell_N)$

Pair-creation

- Voltage Difference between the Dual Batteries
  \[ \Delta V = -\left(\Omega_H/2\pi c\right)(\Psi_2 - \Psi_1) \]
- Outer Battery: $\Omega_F$
- Inner Battery: $-(\Omega_H - \Omega_F)$

BH-UPI
8. Estimate of Location $S_N$, $\Delta V$ and Power

Location of “Power Station”

\[ \omega = \Omega_F \simeq (1/2) \Omega_H, \]
\[ r \simeq 1.6 r_H \]

Voltage Drop at $S_N$

\[ \Delta V = \frac{\Omega_H}{2\pi c} \Delta \Psi \]
\[ \simeq \frac{1}{2\pi c} \frac{a}{2M r_H} B_n \pi r_H^2 \]
\[ \simeq (10^{20} \text{volts}) \left( \frac{a}{M} \right) \left( \frac{M}{10^9 M_\odot} \right) \left( \frac{B_n}{10^4 G} \right) \]

Power

\[ P_H = \frac{1}{2\pi c} \int \Omega_F(\Psi)I(\Psi) d\Psi \]
\[ \simeq 2 \times (10^{45} \text{erg/sec}) \left( \frac{a}{M} \right)^2 \left( \frac{M}{10^9 M_\odot} \right)^2 \left( \frac{B_n}{10^4 G} \right)^2 \]
9. Summary

- By collaboration of frame dragging and unipolar induction, the hole’s magnetosphere has a double structure, divided by interface $S_N$ into two domains, outer S-C and inner G-R domains, with outflow and inflow. Strong magnetized gap with voltage drop is concealed under $S_N$, with a secret Power Station for launching outflow, which will be accelerated to a large-scale jet.

- Large-scale jets will be a manifestation of the frame-dragging effect, coupled with unipolar induction.

- The present model is an extended version of S. Phinney’s jet model (1982).
10. Concluding Remarks

(i) “Power Station” at the Magnetized Gap will be a product of collaboration of Frame Dragging with Unipolar Induction.

(ii) Physics of Magnetized Gap is awaiting further elucidation.

(iii) Observations will be expected to provide some firm evidence for this model.
I love Switzerland, the Alps and Matterhorn.

I. Okamoto
43 years ago.
At the summit of Matterhorn.

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