# A new instability to black-hole spin precession 

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Geneva
2. A timescale hierarchy
3. Effective potentials arxiv:1506.03492 PRD 92:064016 arXiv:1411.0674 PRL 114:081103
4. Up-down instability
arXiv:1506.09116 PRL 115:141102
5. From $\mathbf{G R}$ to astro


## Spinning BH binaries

## Mass measurements




Spin measurements


## Astrophysical and relativistic inspiral

Gravitational waves are efficient below

$$
a_{\mathrm{GW}}=1.2 \times 10^{11}\left(\frac{t_{\mathrm{GW}}}{1.4 \times 10^{10} \mathrm{yr}}\right)^{1 / 4}\left(\frac{M}{M_{\odot}}\right)^{3 / 4} \mathrm{~cm}
$$

$\sim 10 R_{\odot}$ stellar-mass BHs
$\sim 0.01$ pc supermassive BHs
supermassive BHs: accretion discs
stellar-mass BHs: SN kicks, tides


Bardeen \& Petterson 1975; King et al 2005
Bogdanovic et al 2007; Lodato and DG 2012,
Miller and Krolik 2013, DG et al 2015b
Can astrophysics align the spins?

## Aligned configurations

Astrophysical processes may drive binaries here

## $\varlimsup_{S_{1}}^{4} \overbrace{S_{2}}^{4}$ <br> up-up




Do aligned binaries stay aligned in the gravitational-wave driven inspiral?

## 1. Orbital motion $\longrightarrow t_{\text {torb }} \propto\left(r / r_{g}\right)^{3 / 2}$ <br> Kepler's third law

2. Spin \& orbital-plane precession $\quad t_{\mathrm{pre}} \propto\left(r / r_{g}\right)^{5 / 2}$
3. GW emission and inspiral
if (Post-)Newtonian $r \gg r_{g}=G M / c^{2}$ : timescale hierarchy

## Orbit $\ll$ Precession

Common practice in binary dynamics

- periastron precession
- osculating orbital elements
- variation of constants


## Averaging the average

## Standard orbit-averaged PN dynamics.

The relative orientation of the three momenta is described by 4 variables $r, \theta_{1}, \theta_{2}, \Delta \Phi$


Precession


Let's freeze GW emission

- Separation $r$ varies on $t_{\mathrm{RR}}$
- Also $J=\left|\mathbf{L}+\mathbf{S}_{\mathbf{1}}+\mathbf{S}_{\mathbf{2}}\right|$ varies on $t_{\mathrm{RR}}$
- Effective spin is constant (at least) at 2PN!

Damour 2001: Racine 2008

$$
\xi \equiv M^{-2}\left[(1+q) \mathbf{S}_{1}+\left(1+q^{-1}\right) \mathbf{S}_{2}\right] \cdot \hat{\mathbf{L}}
$$

Double-spin precession
is (actually) a 1D problem!

$$
S=\left|\mathbf{S}_{\mathbf{1}}+\mathbf{S}_{\mathbf{2}}\right|
$$

## On the shoulders of giants



## Kepler's two-body problem

## What you do:

- One effective particle: 3D
- 3D to 2D problem:
$L$ is a constant of motion!
- Energy is constant: 2 D to 1D
- Effective potential


## What you get:

- Solutions are Kepler's orbits
- A lot of understanding

Integrating $G M m / r^{2}$ to get a bunch of points along an orbit or... knowing that that curve is an ellipse!

## Effective potentials for spin precession



Integrating the PN eq. to get a bunch of points on a precession cone or... knowing the shape of that cone!

What you do:

- 4D to 2D problem: GW are frozen, $r$ and $J$ are constant,
- Further constant of motion, effective spin: 2D to 1D
- Effective potentials for BH binary spin precession


## What you get:

- Analytical solutions
- A lot of understanding


## How it works:

- Geometrical constraints: only within the loop!
- All configurations once you fixed q, spins mag., $r$ and $J$
- Evolution in $S$ is precession
- Spin-orbit resonances.


## Aligned configurations

 How about these configurations?

All these configurations are solutions of the PN equations at any separation, but... are they stable?


## New precessional instability!

Up-down binaries are
unstable to spin precession at separations $r_{\text {ud }}<r<r_{u d+}$
$r_{\mathrm{ud} \pm}=\frac{\left(\sqrt{\chi_{1}} \pm \sqrt{q \chi_{2}}\right)^{4}}{(1-q)^{2}} M$

## Precession if

there are many $\mathbf{S}$ for fixed $\mathbf{J}$ and $\xi$


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## Exciting prediction

$$
f_{\mathrm{ud} \pm} \simeq 6.4 \times 10^{4} \mathrm{~Hz}\left(M / M_{\odot}\right)^{-1}(1-q)^{3} /\left(\sqrt{\chi_{1}} \pm \sqrt{q \chi_{2}}\right)^{6}
$$

For sensible masses, the onset of the instability is in the LIGO and eLISA band!

cf. Klein et al. 2013 for the waveform model
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