Global simulations of magnetized disks in non-ideal GRMHD: connecting small and large scale phenomena

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Outline

1 Introduction
   - Accretion disks and magnetic turbulence
   - Numerical simulations

2 Non-ideal GRMHD
   - $\alpha\Omega$ Dynamo
   - Resistive GRMHD

3 Kinematic Dynamo in Magnetized Disks
   - Disk model
   - Study of the $\alpha\Omega$ dynamo

4 Current developments and perspectives
Accretion disks and magnetic fields

- **Accretion on compact objects** is commonly retained as the most plausible mechanism to power up a list of astrophysical systems (AGNs, GRBs, X-Ray Binaries, etc...).

- **Ordered magnetic fields on large scales** are a fundamental part of many processes related to accretion disks:
  - Relativistic Jets in AGNs (McKinney and Blandford, 2009)
  - Blandford-Znajek mechanism (Blandford and Znajek, 1977)
  - MRI (Balbus and Hawley, 1998)

- **Magnetized plasmas are prone to develop turbulent behavior** on small scales.

On large scales one can have in principle **dissipation** and **dynamo action**.
Numerical simulations

Shearing box

- Cartesian box placed corotating with the flow.
- Coriolis and centrifugal forces combined with gravity in an effective potential.
- Good description of the local behavior of the plasma in thin accretion disks (Shakura and Sunyaev, 1973).
- Major tool to study MRI.
- Doesn’t allow a global description of the disk.

Global

- Full description of the disk, including formation of relativistic jets and winds.
- Possible evolution on time-scales much larger than the dynamical one.
- Closer chance to provide observationally testable models.
- Typical length-scales of the turbulent instabilities not resolved.

(Gammie et al., 2003)
To connect small and large scales one can adopt a specific closure scheme in GRMHD:

- Ohmic resistivity in neutron star mergers (Dionysopoulou et al., 2015).
- Sub-grid dynamo in thick accretion disks with radiation field (Sądowski et al., 2015).

Our approach focuses on a covariant formulation of Ohm’s law involving both resistivity and mean-field dynamo action.
Non-ideal effects in GRMHD
Mean field dynamo in Classical MHD

Consider a resistive plasma with large-scale fields and small-scale fluctuations:

\[ V(x, t) = V_0(x, t) + v(x, t) \]
\[ B(x, t) = B_0(x, t) + b(x, t) \]

The induction equation for the mean magnetic field reads (Moffatt, 1978):

\[ \frac{\partial B_0}{\partial t} = \nabla \times (V_0 \times B_0) + \eta \nabla^2 B_0 + \nabla \times \mathcal{E} \]
\[ \mathcal{E} = \langle v \times b \rangle \simeq \alpha B_0 - \beta \nabla \times B_0 \]

\[ \downarrow \]

\[ \frac{\partial B_0}{\partial t} = \nabla \times (V_0 \times B_0) + (\eta + \beta) \nabla^2 B_0 + \nabla \times (\alpha B_0) \]

- Increase of the magnetic resistivity
- Generation of currents parallel to the magnetic field
The $\alpha\Omega$ dynamo

Considering **poloidal** and **toroidal** components:

\[
\frac{\partial A}{\partial t} + (U_P \cdot \nabla) A = \alpha B + \eta \nabla^2 A \\
\frac{\partial B}{\partial t} + (U_P \cdot \nabla) B = (B_P \cdot \nabla) U + \eta \nabla^2 B
\]

- $\alpha$ **Effect**: toroidal field $\Rightarrow$ toroidal currents (mean field dynamo) $\Rightarrow$ poloidal field
- $\Omega$ **Effect**: poloidal field $\Rightarrow$ stretching (differential rotation) $\Rightarrow$ toroidal field

**Closed Dynamo Cycle**

$B_P \Rightarrow B_T \Rightarrow J_T \Rightarrow B_P$
Resistive MHD in General Relativity
General Relativistic Maxwell’s equations

Line element (ADM form):

\[ ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \]

\[ \alpha_{lapse} \neq \alpha_{dyn} ! \]

Faraday’s Law:

\[ \gamma^{-1/2} \partial_t \left( \gamma^{1/2} B \right) + \nabla \times (\alpha E + \beta \times B) = 0 \]

Ampere’s Law:

\[ \gamma^{-1/2} \partial_t \left( \gamma^{1/2} E \right) + \nabla \times (-\alpha B + \beta \times E) = -(\alpha J - q\beta) \]

\[ \nabla \cdot B = 0 \]
\[ \nabla \cdot E = q \]
Ohm’s law

Fully covariant Ohm’s law (in the fluid reference frame)

For a resistive plasma (Palenzuela et al., 2009) with dynamo action (Bucciantini and Del Zanna, 2013):

\[ e^\mu = \eta j^\mu + \xi b^\mu \quad \text{with} \quad \xi \equiv -\alpha_{\text{dyn}} \]

\[ \Gamma[E + v \times B - (E \cdot v)v] = \eta(J - qv) + \xi \Gamma[B - v \times E - (B \cdot v)v] \]

Classical limit \((\Gamma = 1, |v| \ll 1, |E| \ll |B|)\):

\[ E + v \times B = \eta J + \xi B \]
Evolution equation for $E$

Computing $J$ from Ohm’s law and replacing in Ampere’s law we get:

$$\gamma^{-1/2} \partial_t \left( \gamma^{1/2} E \right) = \nabla \times (\alpha B - \beta \times E) + (\alpha v - \beta) q + \frac{1}{\eta} R(E, B, v, \xi)$$

Stiff equation $\Rightarrow$ numerical instability

Terms $\propto \eta^{-1}$ can evolve on time scales $\tau_\eta \ll \tau_h$ (time scale of hyperbolic part).

Implementation of IMEX Schemes (Pareschi and Russo, 2005).
Kinematic Dynamo in Magnetized Disks (Bugli et al., 2014)
Magnetized Thick Disks (Komissarov, 2006)

Momentum-Energy tensor for a magnetized plasma:

\[ T^{\mu\nu} = (w + b^2)u^\mu u^\nu + \left(p + \frac{b^2}{2}\right)g^{\mu\nu} - b^\mu b^\nu \]

Angular momentum and angular velocity:

\[ l = -\frac{u_\phi}{u_t}, \quad \Omega = \frac{u_\phi}{u_t}, \]

Momentum-Energy conservation:

\[ W - W_{\text{in}} + \frac{\kappa}{\kappa - 1} \frac{p}{w} + \frac{\zeta}{\zeta - 1} \frac{p_m}{w} = 0 \]

\[ W = \ln |u_t| + \int_1^{l_\infty} \frac{\Omega dl}{1 - \Omega l} \]
The $\alpha\Omega$ dynamo: toroidal component
The $\alpha\Omega$ dynamo: poloidal components

\begin{align*}
\text{Time } [P] &= 0, \\
B_r(x,z) \\
\text{Time } [P] &= 0, \\
B_\theta(x,z)
\end{align*}
**Growth rate and ratio** $B_P/B_T$

**Magnetic Field Exponential Growth**

- **$B_T$**
- **$B_P$**

**Ratio between $B_P$ and $B_T$**

- $\max(B_P)/\max(B_T)$

![Graph showing magnetic field exponential growth and ratio between $B_P$ and $B_T$.](image)
Butterfly Diagram

Time \([P_c] = 26\).

\[ B_\phi (x, z) \]

Model 1

Time \([P_c] \)

\([-2 \times 10^4, -1 \times 10^4, 0, 1 \times 10^4]\)
Other models

\[ B_\phi(x, z) \]

Time [Pc] = 0.

Model 10

\[ s \in [r_*] \]

\[ z \in [r_*] \]

\[ x \in [r_*] \]
Current developments and perspectives
**3D thick disk models**

- Constant-\(l\) tori are most susceptible to develop Papaloizou-Pringle instability (PPI) (De Villiers and Hawley, 2002; Mewes et al., 2015).

- Adding a magnetic field affects the growth of the instability, mainly due to the action of the MRI.
- **Magnetic resistivity** may therefore play a significant role in the development of global modes.

The \(m = 1\) is the fastest growing mode \(\Rightarrow\) need for the whole azimuthal range \([0, 2\pi]\) \(\Rightarrow\) computationally expensive.

- Multidimensional **MPI** domain-decomposition.
- Parallel I/O via **Hdf5-MPI** standard.
Relativistic *ideal* tearing mode

- Magnetic reconnection $\Rightarrow$ conversion from magnetic to kinetic energy, particle acceleration in magnetically dominated systems.
- Sweet-Parker model and tearing instability provide reconnection rates too slow to explain the astrophysical observations.

**Ideal tearing mode (Pucci and Velli, 2014)**

Critical threshold in the current sheet aspect ratio $a/L \sim S^{-1/3}$, beyond which tearing modes evolve on fast macroscopic time-scales.

**Numerical results**

- Analysis well verified in 2D resistive MHD simulations (Landi et al., 2015).
- To be extended to the resistive RMHD regime.
Conclusions

Covariant mean-field dynamo closure

Covariant Ohm’s law for a resistive plasma with mean-field dynamo action, to include small-scale turbulent behavior and go beyond the MHD approximation.

Current applications

- Kinematic axissymmetric $\alpha \Omega$ dynamo action in thick disks.
- Stability of 3D magnetized tori in resistive GRMHD.
- Relativistic ideal tearing mode.

Future Perspectives

- $\alpha$-quenching prescription: beyond the linear phase to estimate the disk back-reaction on the magnetic field (Brandenburg and Subramanian, 2005).
- Spatial profiles for $\eta$ and $\xi$: more realistic connections to shearing box simulations. (Gressel, 2010)
Merci pour votre attention!


References II


