# Global simulations of magnetized disks in non-ideal GRMHD: connecting small and large scale phenomena

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28th Texas Symposium
Geneva, 17th December 2015

# Outline

# Accretion disks and magnetic fields

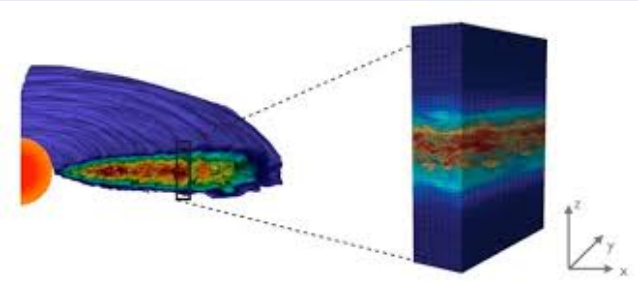
- Accretion on compact objects is commonly retained the most plausible mechanism to power up a list of astrophysical systems (AGNs, GRBs, X-Ray Binaries, etc...).
- Ordered magnetic fields on large scales are a fundamental part of many processes related to accretion disks:
  - Relativistic Jets in AGNs (McKinney and Blandford, 2009)
  - Blandford-Znajek mechanism (Blandford and Znajek, 1977)
  - MRI (Balbus and Hawley, 1998)
- Magnetized plasmas are prone to develop turbulent behavior on small scales.

On large scales one can have in principle dissipation and dynamo action.

# Numerical simulations

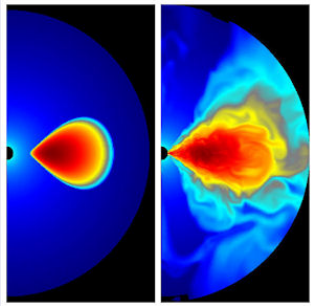
## Shearing box

- Cartesian box placed corotating with the flow.
- Coriolis and centrifugal forces combined with gravity in an effective potential.
- Good description of the local behavior of the plasma in thin accretion disks (Shakura and Sunyaev, 1973).
- Major tool to study MRI.
- Doesn't allow a global description of the disk.



## Global

- Full description of the disk, including formation of relativistic jets and winds.
- Possible evolution on time-scales much larger than the dynamical one.
- Closer chance to provide observationally testable models.
- Typical length-scales of the turbulent instabilities not resolved.



(Gammie et al., 2003)

# Closure schemes

To connect small and large scales one can adopt a specific closure scheme in GRMHD:

- Ohmic resistivity in neutron star mergers (Dionysopoulou et al., 2015).
- Sub-grid dynamo in thick accretion disks with radiation field (Sądowski et al., 2015).

Our approach focuses on a covariant formulation of Ohm's law involving both resistivity and mean-field dynamo action.

# Non-ideal effects in GRMHD

# Mean field dynamo in Classical MHD

Consider a resistive plasma with large-scale fields and small-scale fluctuations:

$$\begin{aligned} \boldsymbol{V}(\boldsymbol{x},t) &= \boldsymbol{V}_0(\boldsymbol{x},t) + \boldsymbol{v}(\boldsymbol{x},t) \\ \boldsymbol{B}(\boldsymbol{x},t) &= \boldsymbol{B}_0(\boldsymbol{x},t) + \boldsymbol{b}(\boldsymbol{x},t) \end{aligned}$$

The induction equation for the mean magnetic field reads (Moffatt, 1978):

$$\partial \boldsymbol{B}_0/\partial t = \boldsymbol{\nabla} \times (\boldsymbol{V}_0 \times \boldsymbol{B}_0) + \eta \nabla^2 \boldsymbol{B}_0 + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}$$

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{v} \times \boldsymbol{b} \rangle \simeq \alpha \boldsymbol{B}_0 - \beta \boldsymbol{\nabla} \times \boldsymbol{B}_0$$

$$\Downarrow$$

$$\partial \boldsymbol{B}_0/\partial t = \boldsymbol{\nabla} \times (\boldsymbol{V}_0 \times \boldsymbol{B}_0) + (\eta + \beta) \nabla^2 \boldsymbol{B}_0 + \boldsymbol{\nabla} \times (\alpha \boldsymbol{B}_0)$$

- Increase of the magnetic resistivity
- Generation of currents parallel to the magnetic field

# The $\alpha\Omega$ dynamo

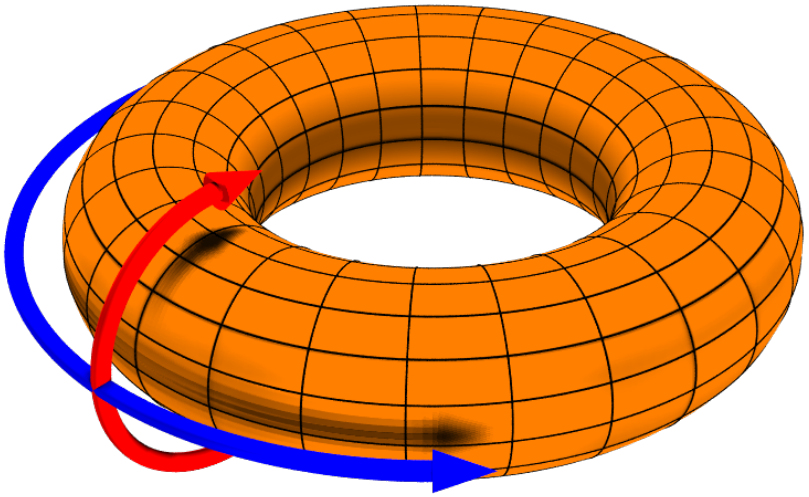
Considering poloidal and toroidal components:

$$\partial A/\partial t + (\boldsymbol{U}_P \cdot \boldsymbol{\nabla})A = \alpha B + \eta \nabla^2 A$$
$$\partial B/\partial t + (\boldsymbol{U}_P \cdot \boldsymbol{\nabla})B = (\boldsymbol{B}_P \cdot \boldsymbol{\nabla})U + \eta \nabla^2 B$$

- $\alpha$ *Effect*: toroidal field $\Rightarrow$ toroidal currents (mean field dynamo) $\Rightarrow$ poloidal field
- $\Omega$ *Effect*: poloidal field $\Rightarrow$ stretching (differential rotation) $\Rightarrow$ toroidal field

**Closed Dynamo Cycle**

$$\boldsymbol{B}_P \Rightarrow \boldsymbol{B}_T \Rightarrow \boldsymbol{J}_T \Rightarrow \boldsymbol{B}_P$$



# Resistive MHD in General Relativity

# General Relativistic Maxwell's equations

Line element (ADM form):

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\alpha_{lapse} \neq \alpha_{dyn} \; !$$

Faraday's Law:

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{B}\right) + \boldsymbol{\nabla} \times (\alpha\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}) = 0$$

Ampere's Law:

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{E}\right) + \boldsymbol{\nabla} \times (-\alpha\boldsymbol{B} + \boldsymbol{\beta} \times \boldsymbol{E}) = -(\alpha\boldsymbol{J} - q\boldsymbol{\beta})$$

$$\begin{aligned} \boldsymbol{\nabla} \cdot \boldsymbol{B} &= 0 \\ \boldsymbol{\nabla} \cdot \boldsymbol{E} &= q \end{aligned}$$

# Ohm's law

## Fully covariant Ohm's law (in the fluid reference frame)

For a resistive plasma (Palenzuela et al., 2009) with dynamo action (Bucciantini and Del Zanna, 2013):

$$e^\mu = \eta j^\mu + \xi b^\mu \quad \text{with} \quad \xi \equiv -\alpha_{dyn}$$

$$\Downarrow$$

$$\Gamma[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - (\boldsymbol{E} \cdot \boldsymbol{v})\boldsymbol{v}] = \eta(\boldsymbol{J} - q\boldsymbol{v}) + \xi\Gamma[\boldsymbol{B} - \boldsymbol{v} \times \boldsymbol{E} - (\boldsymbol{B} \cdot \boldsymbol{v})\boldsymbol{v}]$$

Classical limit ($\Gamma = 1$, $|\boldsymbol{v}| \ll 1$, $|\boldsymbol{E}| \ll |\boldsymbol{B}|$):

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{J} + \xi \boldsymbol{B}$$

# Evolution equation for $\boldsymbol{E}$

Computing $\boldsymbol{J}$ from Ohm's law and replacing in Ampere's law we get:

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{E}\right) = \boldsymbol{\nabla}\times(\alpha\boldsymbol{B}-\boldsymbol{\beta}\times\boldsymbol{E})+(\alpha\boldsymbol{v}-\boldsymbol{\beta})q+\frac{1}{\eta}R(\boldsymbol{E},\boldsymbol{B},\boldsymbol{v},\xi)$$

**Stiff equation $\Rightarrow$ numerical instability**

Terms $\propto \eta^{-1}$ can evolve on time scales $\tau_\eta \ll \tau_h$ (time scale of hyperbolic part).

$$\Downarrow$$

Implementation of IMEX Schemes (Pareschi and Russo, 2005).

# Kinematic Dynamo in Magnetized Disks (Bugli et al., 2014)

# Magnetized Thick Disks (Komissarov, 2006)

Momentum-Energy tensor for a magnetized plasma:

$$T^{\mu\nu} = (w + b^2)u^\mu u^\nu + \left(p + \frac{b^2}{2}\right) g^{\mu\nu} - b^\mu b^\nu$$

Angular momentum and angular velocity:

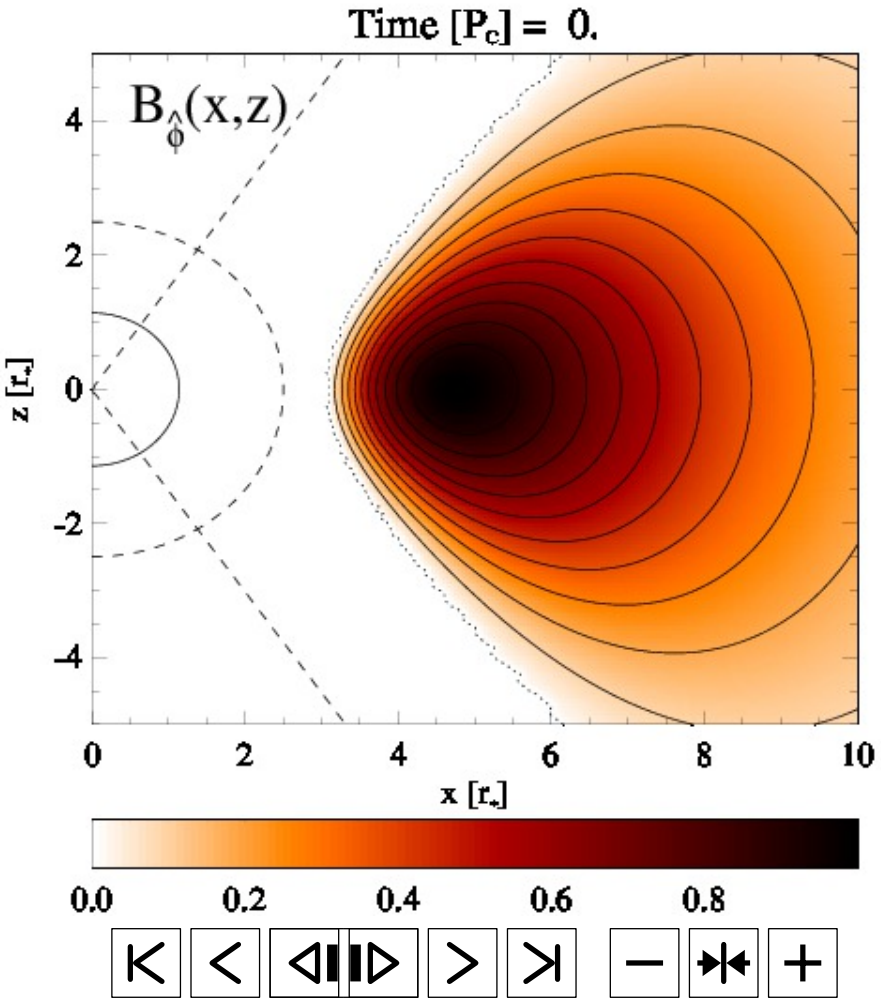
$$l = -\frac{u_\phi}{u_t}, \qquad \Omega = \frac{u^\phi}{u^t},$$

Momentum-Energy conservation:

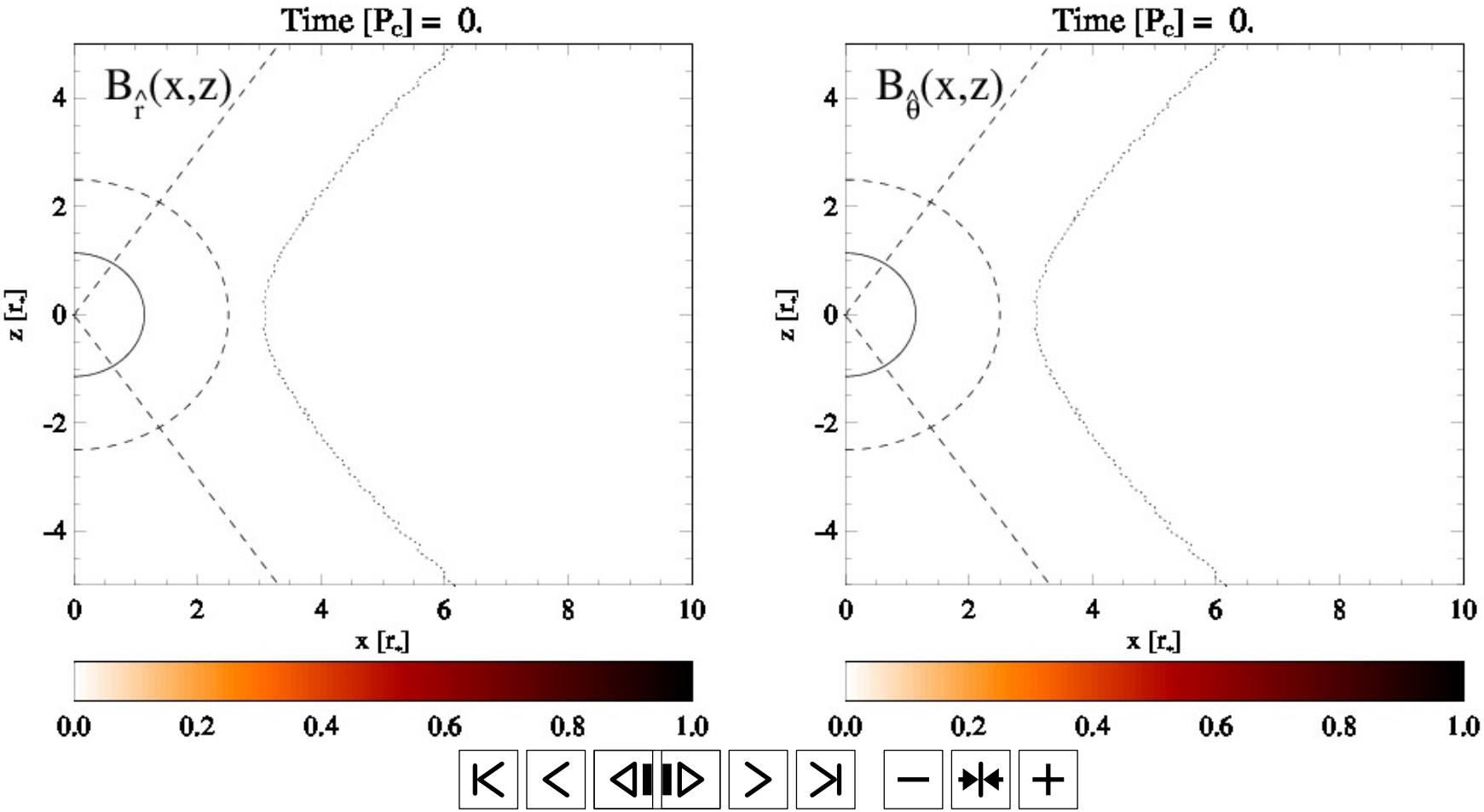
$$W - W_{\rm in} + \frac{\kappa}{\kappa - 1}\frac{p}{w} + \frac{\zeta}{\zeta - 1}\frac{p_m}{w} = 0$$

$$W = \ln|u_t| + \int_l^{l_\infty} \frac{\Omega \mathrm{d}l}{1 - \Omega l}$$

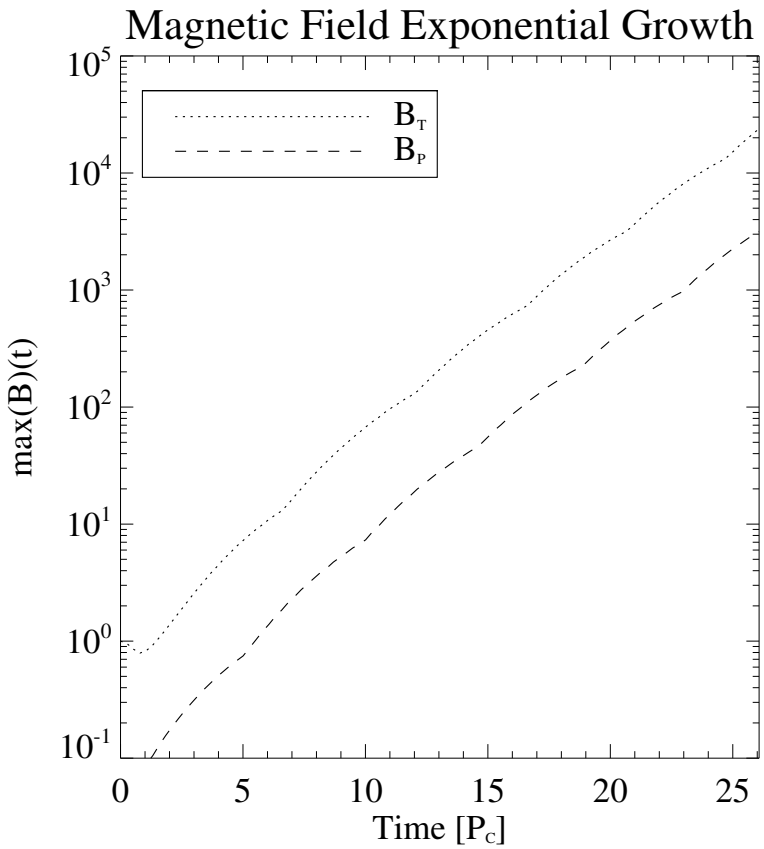
# The $\alpha\Omega$ dynamo: toroidal component

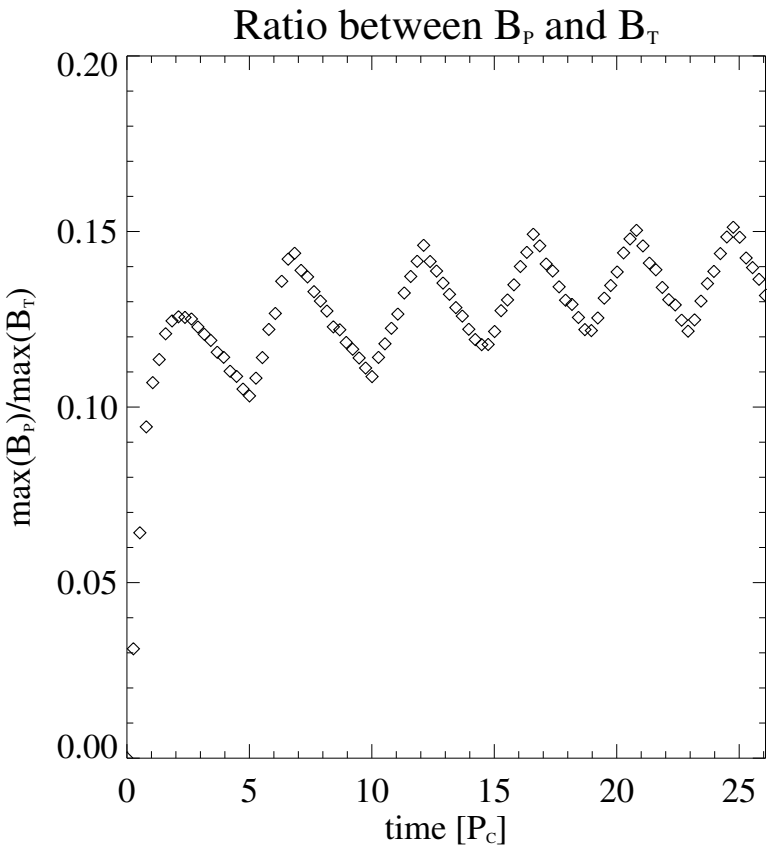


# The $\alpha\Omega$ dynamo: poloidal components

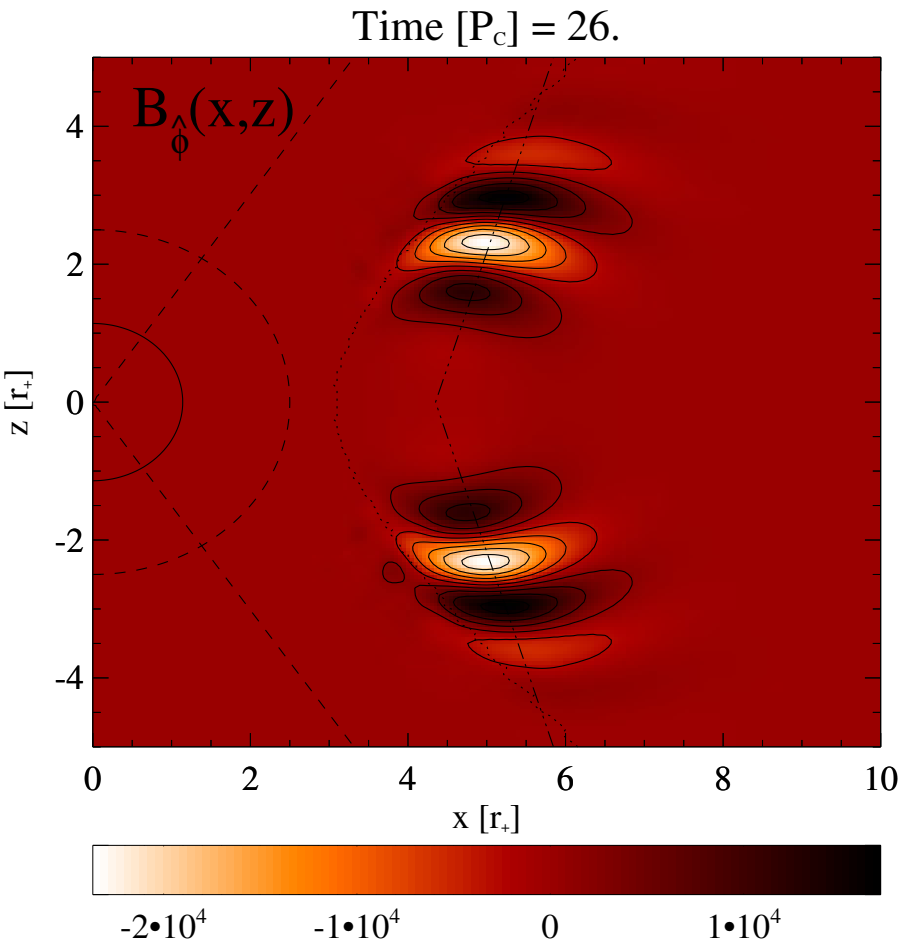


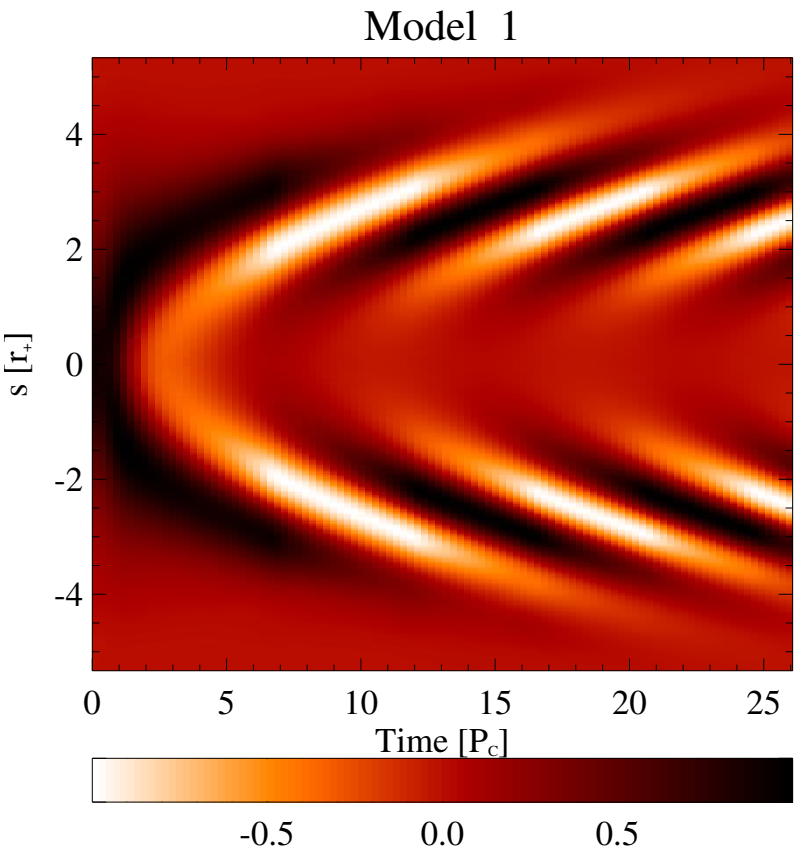
# Growth rate and ratio $B_P/B_T$



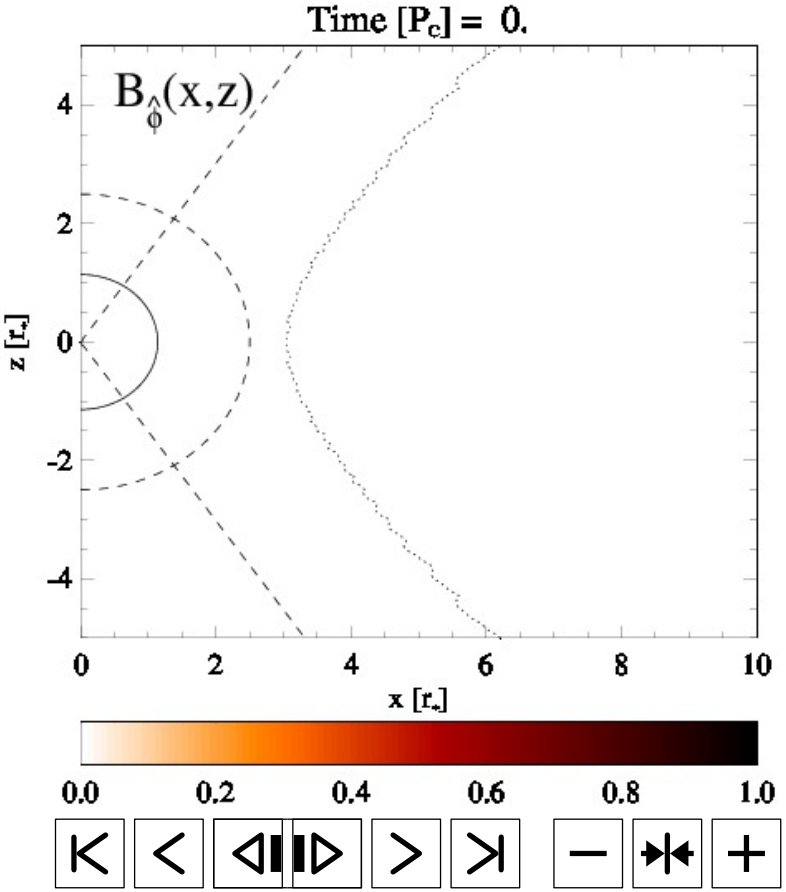



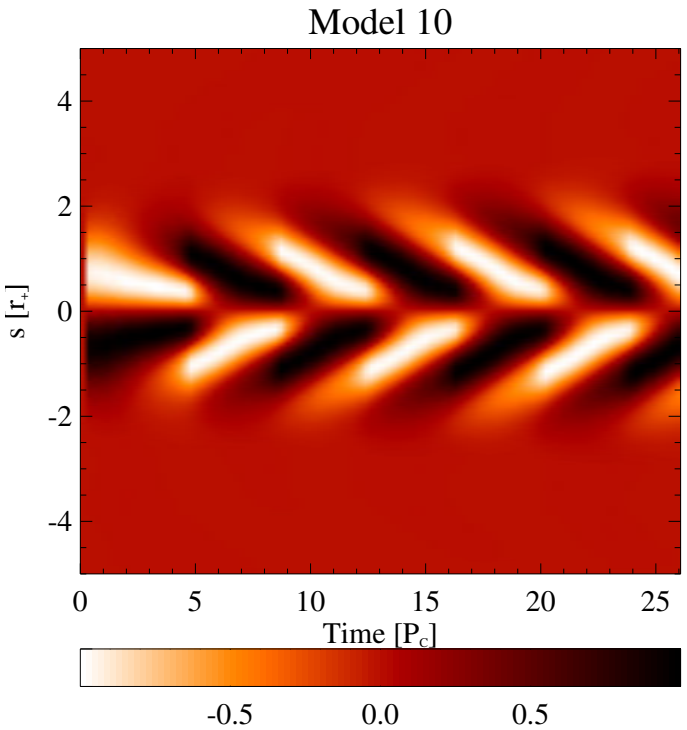
# Butterfly Diagram



# Other models

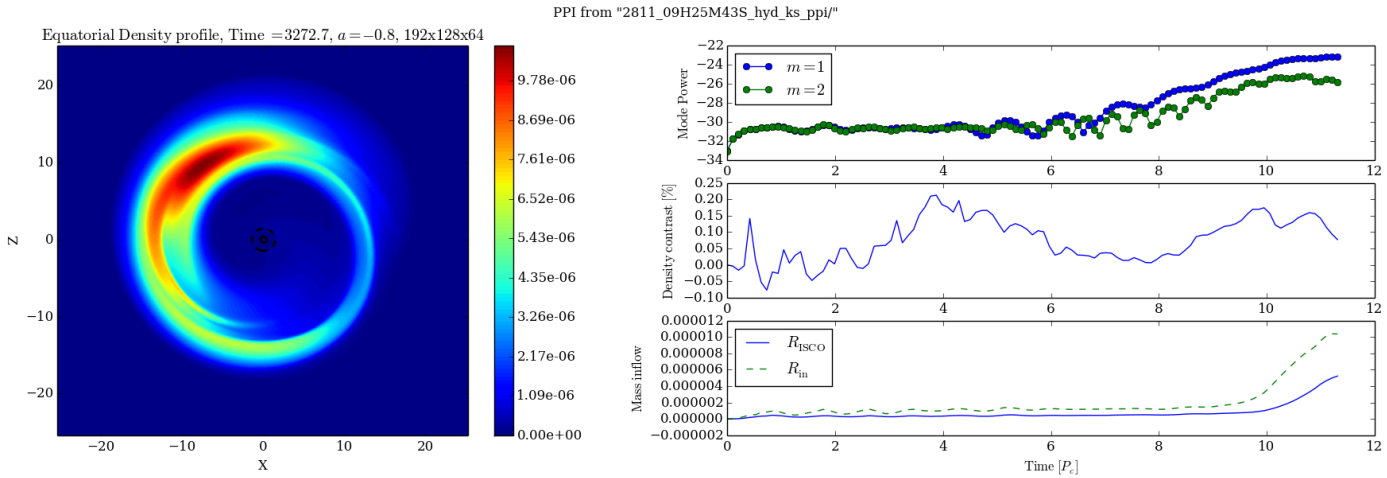


# Current developments and perspectives

# 3D thick disk models

- Constant-$l$ tori are most susceptible to develop Papaloizou-Pringle instability (PPI) (De Villiers and Hawley, 2002; Mewes et al., 2015).




- Adding a magnetic field affects the growth of the instability, mainly due to the action of the MRI.
- Magnetic resistivity may therefore play a significant role in the development of global modes.

The $m = 1$ is the fastest growing mode $\Rightarrow$ need for the whole azimuthal range $[0, 2\pi]$ $\Rightarrow$ computationally expensive.

- Multidimensional `MPI` domain-decomposition.
- Parallel I/O via `Hdf5-MPI` standard.

# Relativistic *ideal* tearing mode

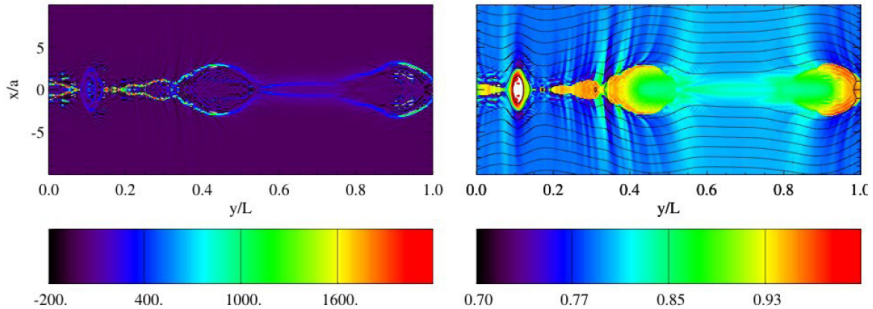
- Magnetic reconnection ⇒ conversion from magnetic to kinetic energy, particle acceleration in magnetically dominated systems.
- Sweet-Parker model and tearing instability provide reconnection rates too slow to explain the astrophysical observations.

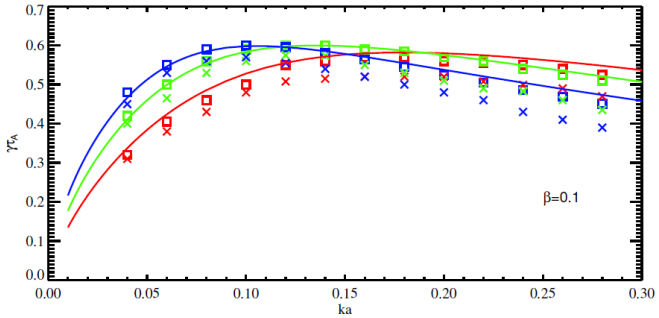
## Ideal tearing mode (Pucci and Velli, 2014)

Critical threshold in the current sheet aspect ratio $a/L \sim S^{-1/3}$, beyond which tearing modes evolve on fast macroscopic time-scales.

## Numerical results

- Analysis well verified in 2D resistive MHD simulations (Landi et al., 2015).
- To be extended to the resistive RMHD regime.



# Conclusions

## Covariant mean-field dynamo closure

Covariant Ohm's law for a resistive plasma with mean-field dynamo action, to include small-scale turbulent behavior and go beyond the MHD approximation.

## Current applications

- Kinematic axissymmetric $\alpha\Omega$ dynamo action in thick disks.
- Stability of 3D magnetized tori in resistive GRMHD.
- Relativistic *ideal* tearing mode.

## Future Perspectives

- $\alpha$-quenching prescription: beyond the linear phase to estimate the disk back-reaction on the magnetic field (Brandenburg and Subramanian, 2005).
- Spatial profiles for $\eta$ and $\xi$: more realistic connections to shearing box simulations. (Gressel, 2010)

# Merci pour votre attention!

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