Primordial perturbations in a bouncing Universe with quintessence

Anna Paula Bacalhau, Nelson Pinto-Neto

Centro Brasileiro de Pesquisas Fisicas
Xavier Sigaud, 150, Rio de Janeiro, Brazil
anna@cbpf.br, nelsonpn@cbpf.br

Abstract

In this work we investigate the features of the primordial power spectrum when it arises from a contracting phase in the context of a bouncing Universe.

1. Introduction

We consider a toy model in which the Universe is dominated by a scalar field with an exponential potential, further on referred as the quintessence component. This choice is motivated by known results in the literature showing that such scalar field can behave like dust in the asymptotic past and asymptotic future [1, 2], implying the generation of an almost scale invariant spectrum for large scale modes [3], but can also exhibit a dark energy behavior in between. The dynamical system analysis of the background equations shows that the scalar field experiences an effective equation of state of dark energy type either in the contracting phase or in the expanding phase of a quantum bouncing model, but not in both. The first scenario is an exercise about how a quintessence field playing the role of dark energy could add new features in the power spectrum if it was present in a contracting phase. The second is closer to realistic cosmological models where dark energy is present in the expanding phase, but is absent in the contracting phase. Both deserve attention, and are first approximations to the development of realistic approaches to adress the problem of structure formation in bounce cosmologies with dark energy.

2. Statement of the problem

We consider a FLRW flat Universe dominated by a canonical scalar field with the potential $V = V_0 e^{-\kappa\lambda\phi}$. The coupled Friedmann and Klein-Gordon equations are ($\kappa^2 = 8\pi G$):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) \tag{1}$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{dV}{d\phi}. \tag{2}$$

Those equations can be recast in term of the variables:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \quad \text{and} \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \tag{3}$$

those variable are constrained by $x^2 + y^2 = 1$, because of the quintessence dominance. Defining $' = \frac{d}{d(lna)}$, the system reads:

$$x' = -3x(1-x^2) + \lambda\sqrt{\frac{3}{2}}y^2 \quad \text{and} \tag{4}$$

$$y' = xy\left(3x - \lambda\sqrt{\frac{3}{2}}\right). \tag{5}$$

This system accounts for both expansion when $y > 0$ and contraction when $y < 0$. Since the quintessence is a canonical scalar field, its density and pressure are given by $\rho = \frac{\dot{\phi}^2}{2} + V$ and $p = \frac{\dot{\phi}^2}{2} - V$. The effective equation of state, $p = w\rho$, satisfies:

$$w = x^2 - y^2 \tag{6}$$

We can see from the previous equation that w will assume different values along the universe's history. Specifically, when $x^2 < -\frac{1}{3} + y^2$, the quintessence behaves as a dark energy.

The perturbative Einstein's equation for this scenarios are written in terms of the Mukhanov-Sasaki variable [4]:

$$v \equiv \sqrt{\rho_{,X}}a\left(\delta\phi + \frac{\phi^{|(0)}}{\mathcal{H}}\Psi\right) \tag{7}$$

Here we are using $^{|} \equiv \frac{d}{d\eta}$, where η is the conformal time; $\mathcal{H} = \frac{a^{|}}{a}$ and $X = \dot{\phi}$; $\delta\phi$ and Ψ is the the field and the metric scalar perturbations. Using the definition

$$z \equiv \frac{a^2(\rho + p)^{\frac{1}{2}}}{c_s\mathcal{H}} \tag{8}$$

in Fourier space the perturbed equations reduces:

$$v_k^{||} - (k^2 - \frac{z^{||}}{z})v_k = 0 \tag{9}$$

3. Background dynamics

The dynamical system analysis of the coupled equations 4 and 5, shows the existence of four critial points. In the next table we show the critial point and the behavior of quintessence effective equation of state.

x	y	w
-1	0	1
1	0	1
$\frac{\lambda}{\sqrt{6}}$	$-\sqrt{1-\frac{\lambda^2}{6}}$	$\frac{1}{3}(\lambda^2-3)$
$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	$\frac{1}{3}(\lambda^2-3)$

The stability analysis shows that: the first two critical points are unstable in the expansion phase and attractors in the contraction phase; the third critical point is an attractor and exists in the expansion phase; and the last critical point is an unstable point of the contraction phase.

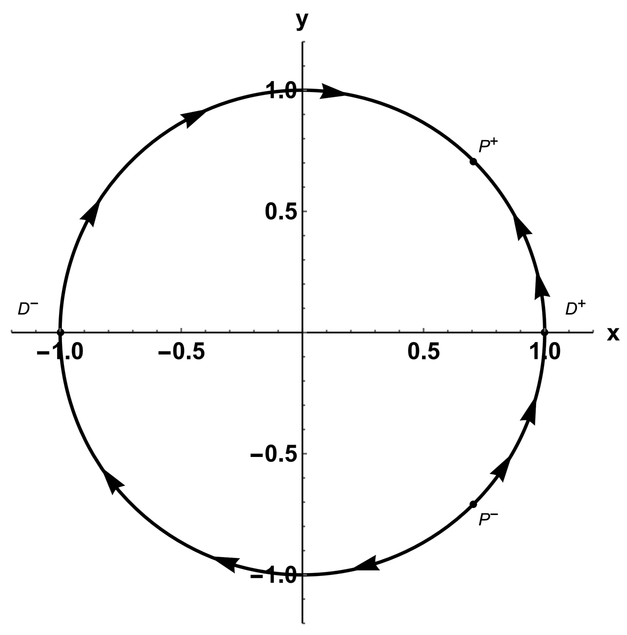


Figure 1: *Contraction in the lower quadrants and expansion in the upper quadrants for $\lambda = \sqrt{3}$.*

As we are interested in studying the primordial perturbations we have to take into account that they emerge in the far past of the Universe's history. For that purpose is very convenient that we set $\lambda = \sqrt{3}$, because it gives a Minkowski vacuum when $t \to -\infty$ since the quintessence will behave like dust in the unstable critical point. In that environment we already know the initial spectrum and it is given by:

$$v(k, \eta) = \frac{1}{\sqrt{k}}e^{ik\eta}. \tag{10}$$

We can see from the fig.1 that, as the Universe approachs eighter one of the first two critical points in table 1 (labled $D^{\pm}$ in fig. 1), the effective equation of state w is of the stiff matter, $w = 1$. In what concerns the general relativity, it means that there is a singularity when $t \to 0^{\pm}$ and the phase space are not connected as it looks like in fig.1. In fact, if we track the qualitative behavior of the phase space in terms of the variables H and $\dot{\phi}$, fig. 2, we see that there is only two possible histories for the Universe: eighter we have a contraction with an dark energy era followed by an expansion without a dark energy era, case 1, fig.3(a); or a contraction without a dark energy era followed by an expansion with an dark energy era, case2, fig.3(b).

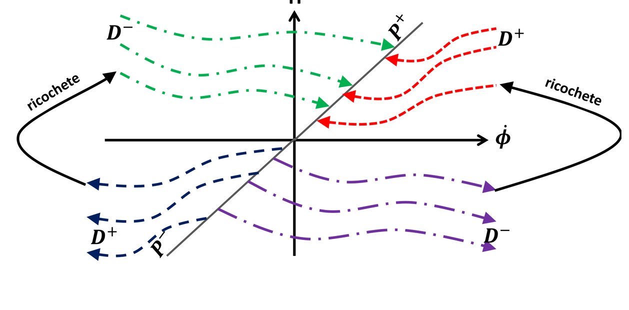
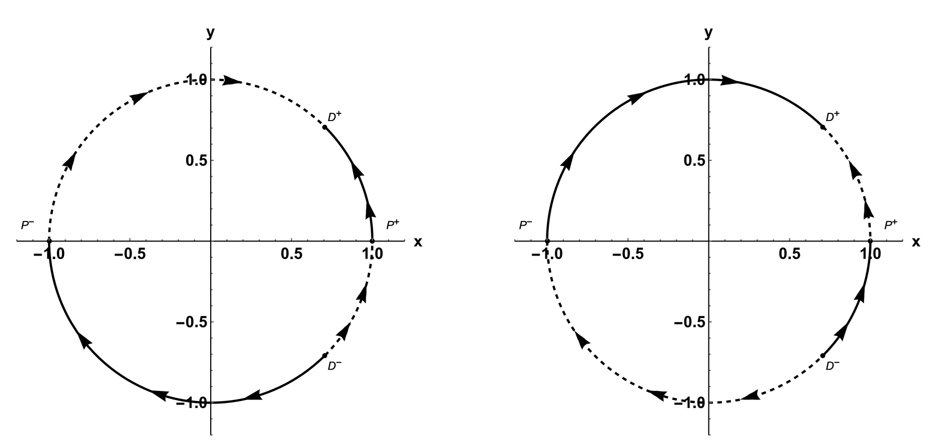


Figure 2: *Sketch of the behavior of trajectories in the $\dot{\phi}xH$ plane. Because trajectories can not cross, two mutually excludent scenarios arises.*



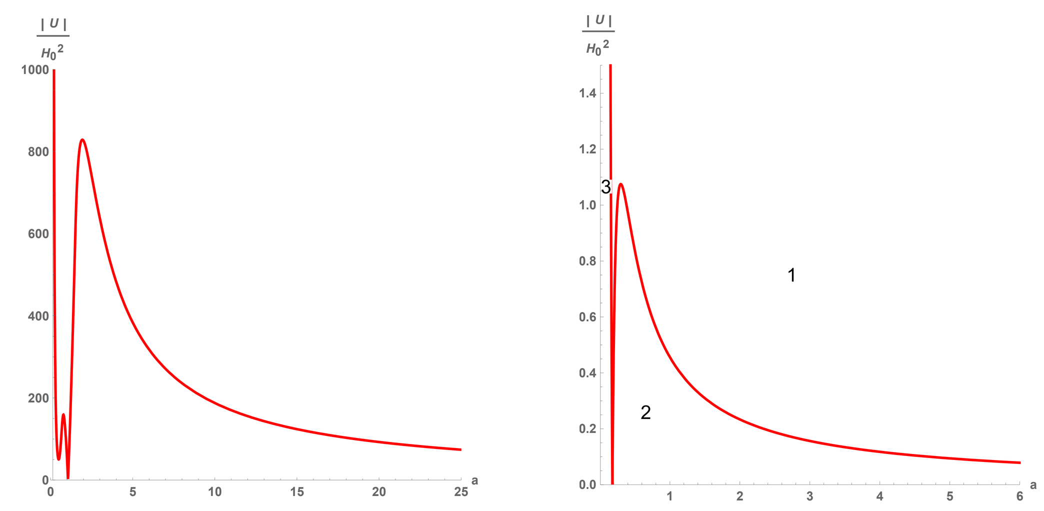
(a) Case 1 (b) Case 2

Figure 3: *Two mutually excludents histories for the Universe*

It clear that whatever happen near the singularity should be driven by a new physics in order to provide a bounce. In the previous works by [5], the canonical quantization of gravity is used within the de Broglie-Bohm formulation of quantum mechanics to construct bohmian trajectories for the scale factor. the find a non-singular bounce due to the quantum corrections. In essence, the canonical field quantized in [5] has a limiting behavior around the bounce of an effective stiff matter, which can be prescribe for our case. So, even thought we haven't quantize our model yet, we expect that will be possible to construct a complete bounce background for both scenarios.

4. Perturbations dynamics

Taking the classical background behavior we can compute the term $U \equiv \frac{z^{||}}{z}$, which, as we can see from equation 9, holds all the information on the dynamics of the perturbation.



(a) With a dark energy era (b) Without the dark energy era

Figure 4: *Numerical solutions for the wave equation potential U.*

As soon as we are able to prescribe a bounce solution we can clearly calculate the amplitude effect and the running in the wave number for the spectral index due to the contraction phasein both cases. If those new features are not completely ruled out be the current data, more realistic models can be supported, for example, considering other energy content to interact with the quintessence.

5. Conclusion

In this first approach we exhibited two new bounce Universes that can provide more insight on the primordial power spectrum that emerges from a contraction phase. One of them addresses the problem of the presence of a dark energy era during contraction (case 1) and the other is an interesting model when the dark energy are only present in the expansion phase. The quantum bounce solution can be reached through a canonical quantization in the framework of the de Broglie-Bohm formulation of quantum mechanics.

Acknowledgments

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