Primordial perturbations in a bouncing Universe with quintessence

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Abstract

In this work we investigate the features of the primordial power spectrum when it arises from a contracting phase in the context of a bouncing Universe.

1. Introduction

We consider a toy model in which the Universe is dominated by a scalar field with an exponential potential, further referred as the quintessence component. This choice is motivated by known results in the literature showing that such a potential yields dust-like matter in the asymptotic past and asymptotic future [1, 2], implying the generation of an almost scale invariant spectrum for large scale modes [3], but can also exhibit a dark energy behavior in between. The dynamical system analysis of the background equations shows that the scalar field experiences an effective equation of state of dark energy type either in the contracting phase or in the expanding phase of a quantum bouncing model, but not in both. The first scenario is an exercise about how a quintessence field playing the role of dark energy could add new features in the power spectrum if it was present in a contracting phase. The second is closer to realistic cosmological models where dark energy is present in a contracting phase, but can also exhibit a dark energy behavior in between.

2. Statement of the problem

We consider a FLRW flat Universe dominated by a canonical scalar field with the potential \( V = V(\phi) \). The coupled Friedmann and Klein-Gordon equations are \( (\ddot{x}^2 + 1) \equiv D^2 = 8\pi G \rho \) and \( \phi = -\nabla^2 \phi + V(\phi) \).

Those equations can be recast in term of the variables:

\[
x = a - \frac{\dot{a}^2}{H^2}, \quad y = \frac{\phi}{\sqrt{V}}.
\]

which are constrained by \( x^2 + y^2 = 1 \), because of the quintessence domination. Defining \( \lambda \equiv \frac{2x}{\sqrt{V}} \), the system reads:

\[
x' = 2x(1 - x^2) + y \left( 1 - \lambda^2 \right) \quad \text{and} \quad y' = 2y \left( 1 - \lambda^2 \right) \quad \text{for both scenarios.}
\]

This system accounts for both expansion when \( y > 0 \) and contraction when \( y < 0 \). Since the quintessence is a canonical scalar field, its density and pressure are given by \( \rho = H^2 x^2 \) and \( p = \frac{1}{3} \left( 3x^2 - 1 \right) \). The effective equation of state, \( w = \frac{p}{\rho} \), satisfies:

\[
w = x^2 - 1/3.
\]

We can see from the previous equation that \( w \) will assume different values along the universe’s history. Specifically, when \( x^2 \leq 1/3 \), the quintessence behaves as a dark energy.

The perturbative Einstein’s equation for this scenarios are written in terms of the Mukhanov-Sasaki variable \( \xi \) [4]:

\[
\frac{\ddot{\xi}}{a^2} + 2H\dot{\xi} + \left( \frac{k^2}{a^4} - \frac{1}{6} \right) \xi = 0
\]

in Fourier space the perturbed equations reduces:

\[
v_x = 1 - \frac{k^2}{a^2} - \frac{1}{6} v_{\xi} = 0
\]

3. Background dynamics

The dynamical system analysis of the coupled equations 4 and 5, shows the existence of four critical points. In the next table we show the critical points and the behavior of quintessence effective equation of state.

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<th>( \lambda )</th>
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The stability analysis shows that: the first two critical points are unstable in the expansion phase and attractors in the contraction phase; the third critical point is an attractor and exists in the expansion phase; and the last critical point is an unstable point of the contraction phase.

Figure 1: Contraction in the lower quadrants and expansion in the upper quadrants for \( \lambda = +7 \).

As we are interested in studying the primordial perturbations we have to take into account that they emerge in the far past of the Universe’s history. For that purpose it is very convenient that we set \( \lambda = +\sqrt{7} \), because it gives a Minkowski vacuous at \( t \to -\infty \), since the quintessence will behave like dust in the unstable critical point. In that environment we already know the initial spectrum and it is given by:

\[
v(k, \eta) = \frac{\xi^2}{\sqrt{V}}
\]

We can see from the fig.1 that, as the Universe approaches stable critical point excursions that can provide more insight on the primordial power spectrum. For that purpose is clear calculate the amplitude effect and the running in the wave number for the spectral index due to the contraction phase in both cases. If those new features are not completely ruled out by the current data, more realistic models can be supported. In particular, considering other energy content to interact with the quintessence.

4. Perturbations dynamics

Taking the classical background behavior we can compute the term \( \nabla^2 \phi + V(\phi) \) which, as we can see from equation 9, holds all the information on the dynamics of the perturbation.

Figure 4: Numerical solutions for the wave equation potential \( U \).

As soon as we are able to prescribe a bounce solution we can clearly calculate the amplitude effect and the running in the wave number for the spectral index due to the contraction phase in both cases. If those new features are not completely ruled out by the current data, more realistic models can be supported. In particular, considering other energy content to interact with the quintessence.

5. Conclusion

In this first approach we examined two new bounce Universe universes that can provide more insight on the primordial power spectrum that emerges from a contraction phase. One of them addresses the problem of the presence of a dark energy era during contraction (case 1) and the other is an interesting model when the dark energy are only present in the expansion phase. The quantum bounce solution can be reached through a canonical quantization in the framework of the de Broglie-Bohm formulation of quantum mechanics.

Acknowledgments

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References