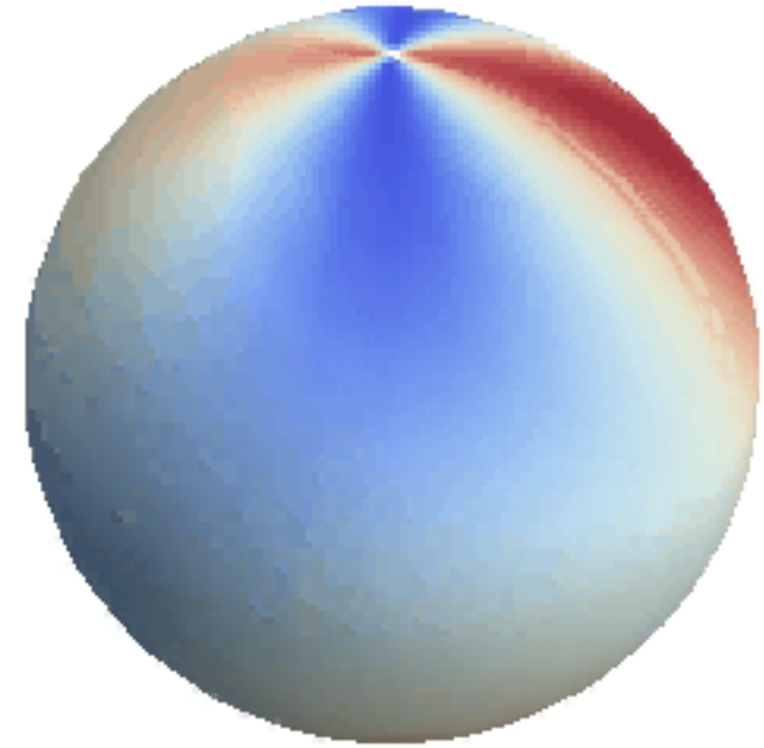
Accurate Phenomenological Waveform Models for BH Coalescence in the Frequency Domain



Goal: synthesize inspiral-merger-ringdown models of the complete WF of Compact Binary Coalescence from pN, NR, BH perturbation theory, self-force, …

S. Husa, Universitat de les Illes Balears
28th Texas Symposium, 12/2015

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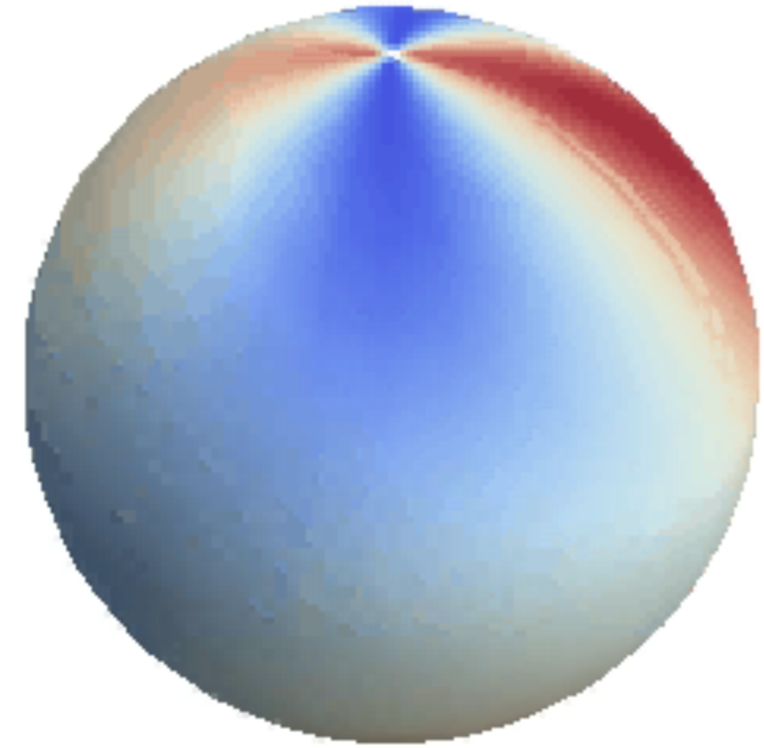

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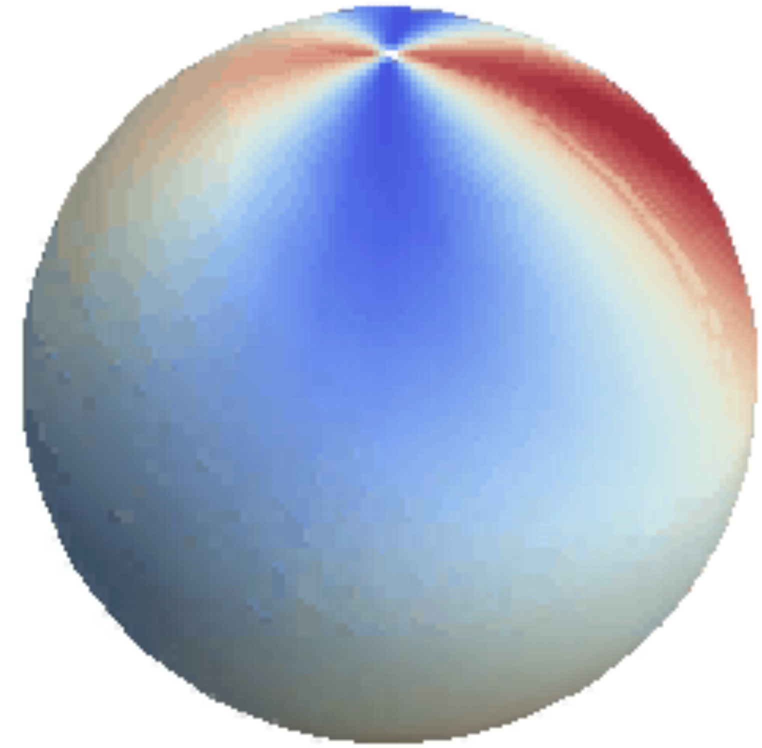

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Frequency-domain gravitational waves from non-precessing black-hole binaries -
I. New numerical waveforms and anatomy of the signal
II. A phenomenological model for the advanced detector era

arXiv:1508.07250, SH, S Khan, M Hannam, M Pürrer, F Ohme, X Jiménez Forteza, A Bohé

arXiv:1508.07253, S Khan, SH, M Hannam, F Ohme, M Pürrer, X Jiménez Forteza, A Bohé

New work with X Jiménez Forteza & D Keitel

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Motivation

- Optimal analysis of data from GW detectors relies on matched filtering with accurate template waveforms.

$$\langle h_1, h_2 \rangle = \max_{\phi_0, t_0} 4\Re \int_{f_1}^{f_2} \frac{\tilde{h}_1(f)\,\tilde{h}_2^*(f)}{S_n(f)}\, df$$

SNR: $\rho = ||h||$ $\qquad$ $\mathcal{M} = 1 - \langle h_1, h_2 \rangle / (||h_1||\,||h_2||)$

- 2005 breakthrough in NR: Pretorius, NASA Goddard/Brownsville
  - short time scale to explore consequences for GW data analysis
- Applications of waveforms:
  - Injections
  - Searches + Bayesian parameter estimation

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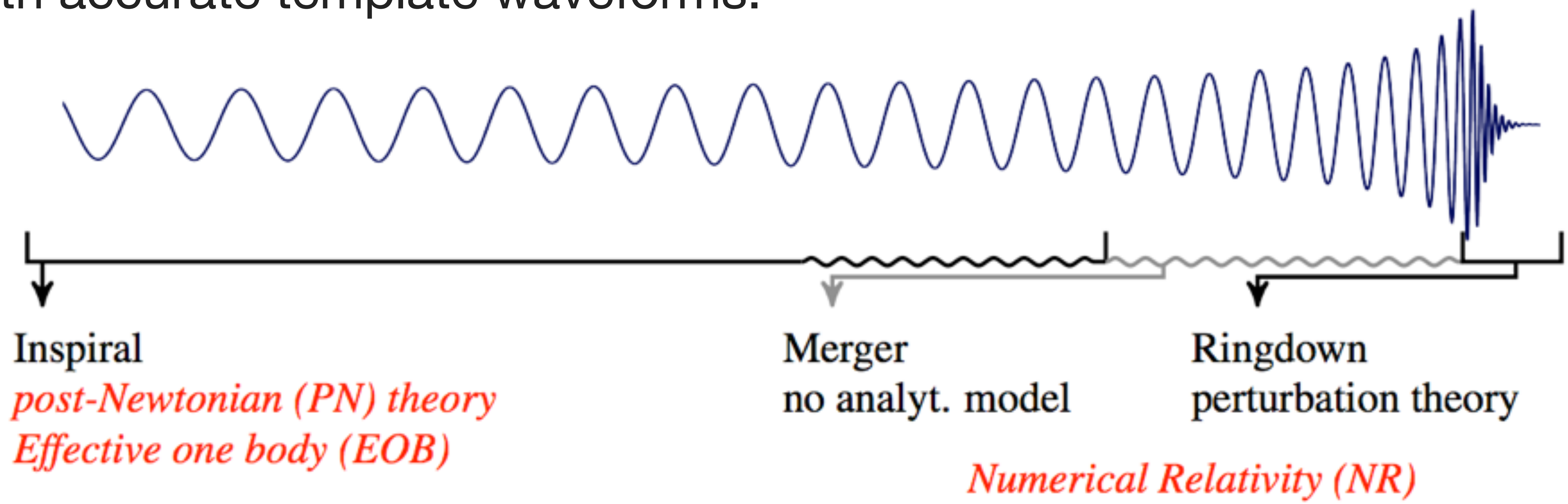

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Phenomenological modelling of IMR waveforms

- Key “design” ideas [alternative choices: **E**ffective **O**ne **B**ody]
  - “phenomenological”: minimal assumptions - look at waveforms and describe what we see. [EOB-model]
  - Frequency domain: matched filter calculations in Freq. domain [time domain]
  - Explicit expression in terms of elementary functions -> fast, simple [ODEs + optional ROM acceleration]

- Minimal ingredients:
  - PN approximate to describe low frequencies: uncalibrated EOB
  - Set of NR WFs: SXS + BAM
  - Prediction for BH remnant:

    New fits for final mass & spin -> QNM freq.

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Talk about l=|m|=2 mode only!

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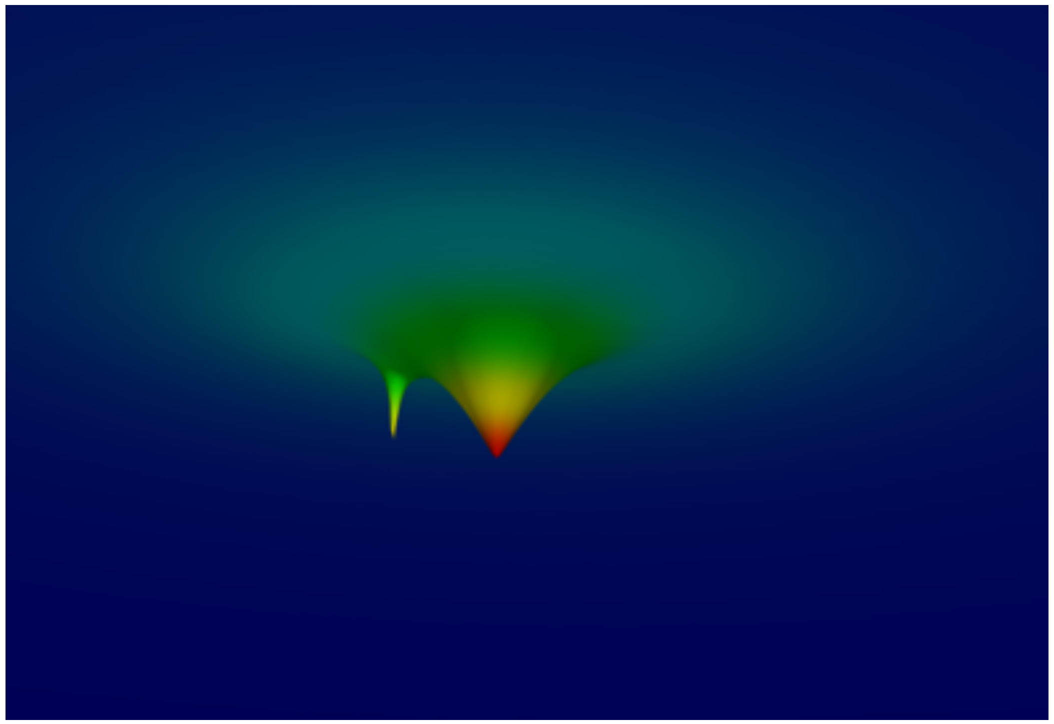
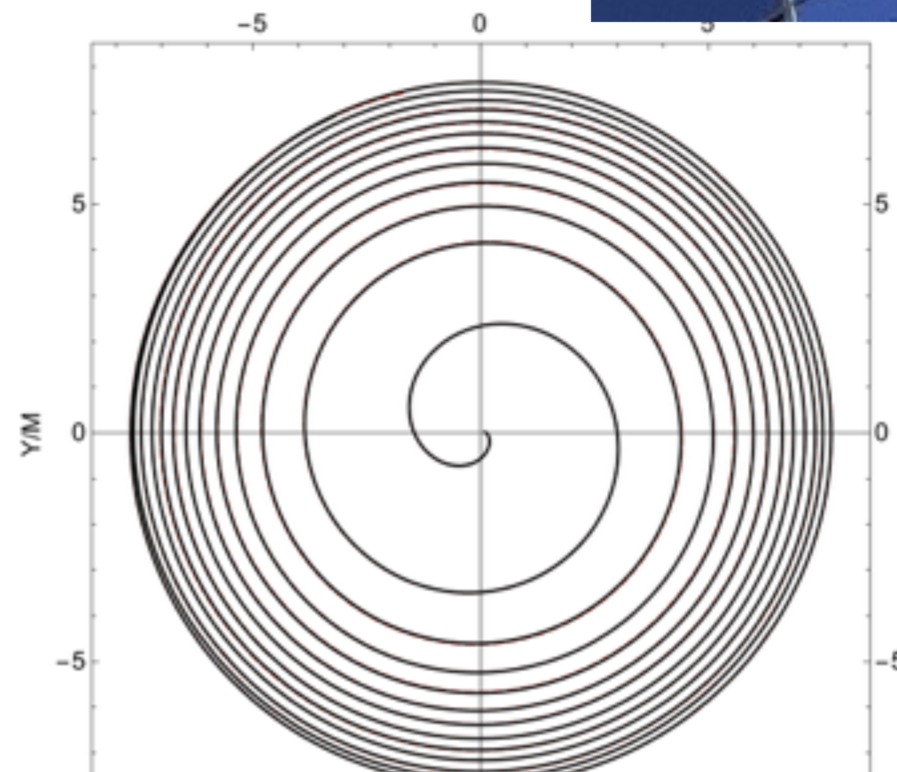
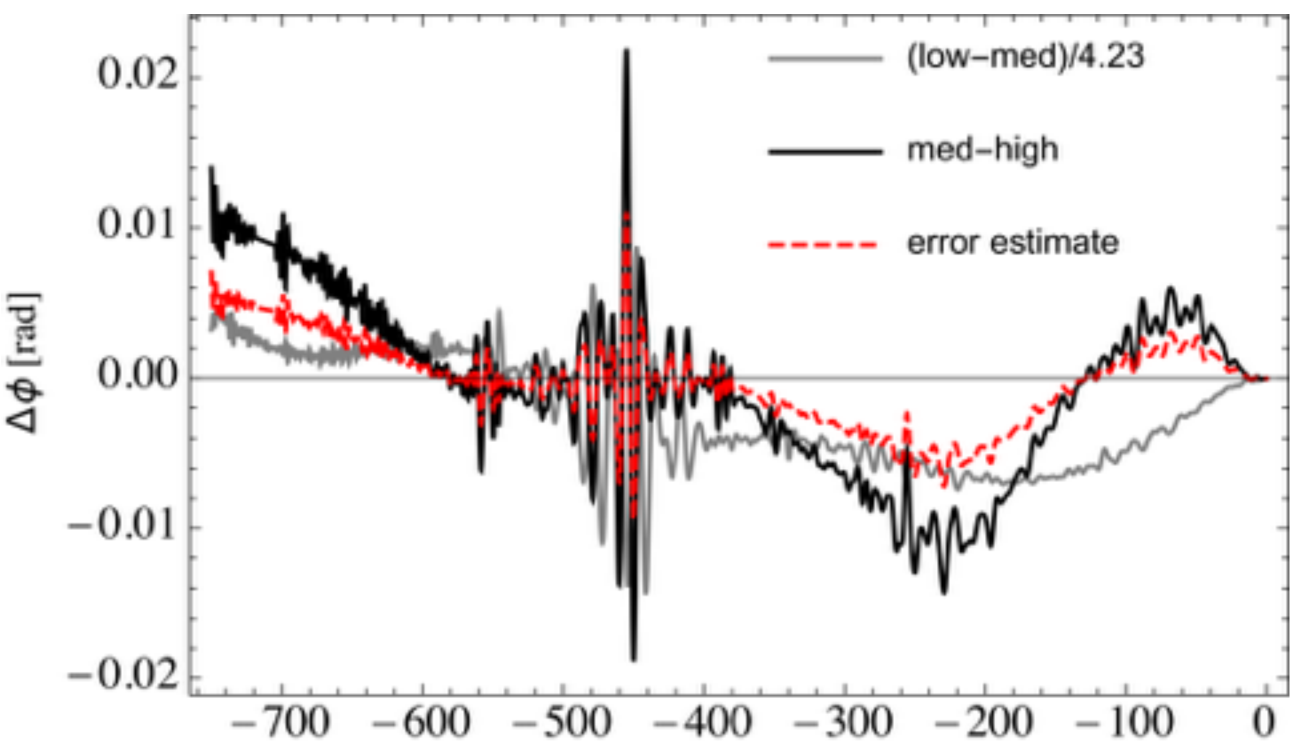
New NR Waveforms: m_1/m_2 = 4, 8,18

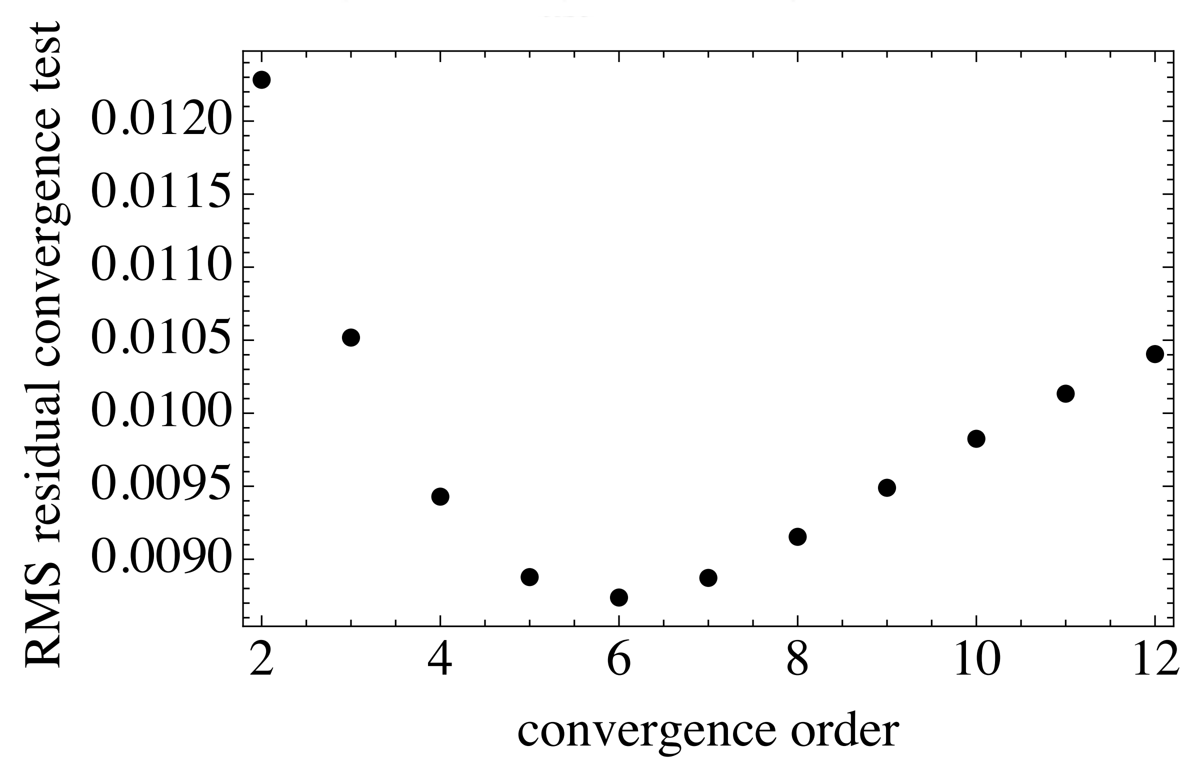
- BAM code: "moving puncture" finite difference mesh refinement





2 resolutions for q=18, spin 0.4





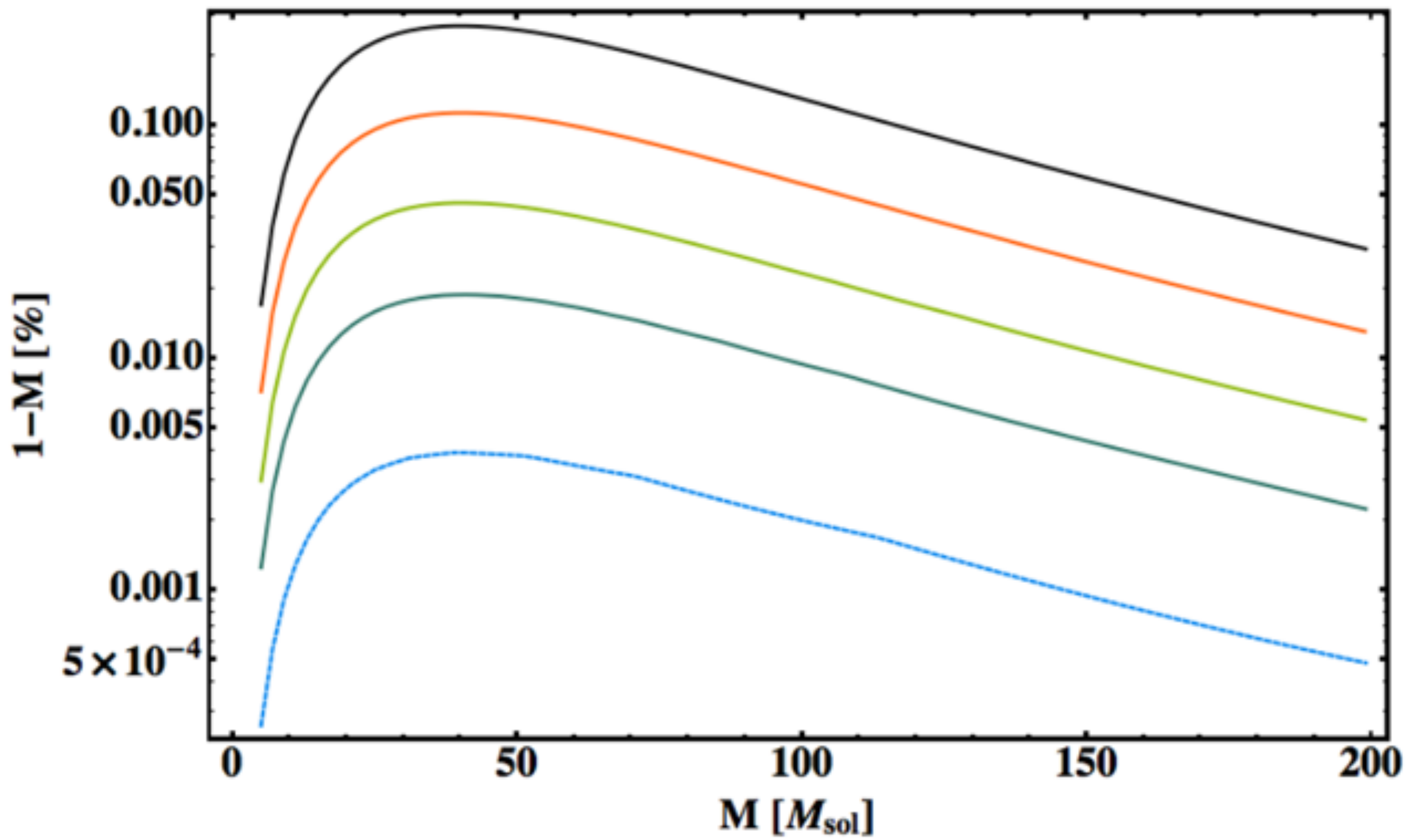


FIG. 3: Mismatch errors due to finite-radius waveform extraction for the 120-point simulations of the same $q = 4$ case as in Fig. 2. Mismatches are between the $R_{ex} = 100\,M$ waveform and those extracted at $R_{ex} = \{50, 60, 70, 80, 90\}\,M$ (from top to bottom).

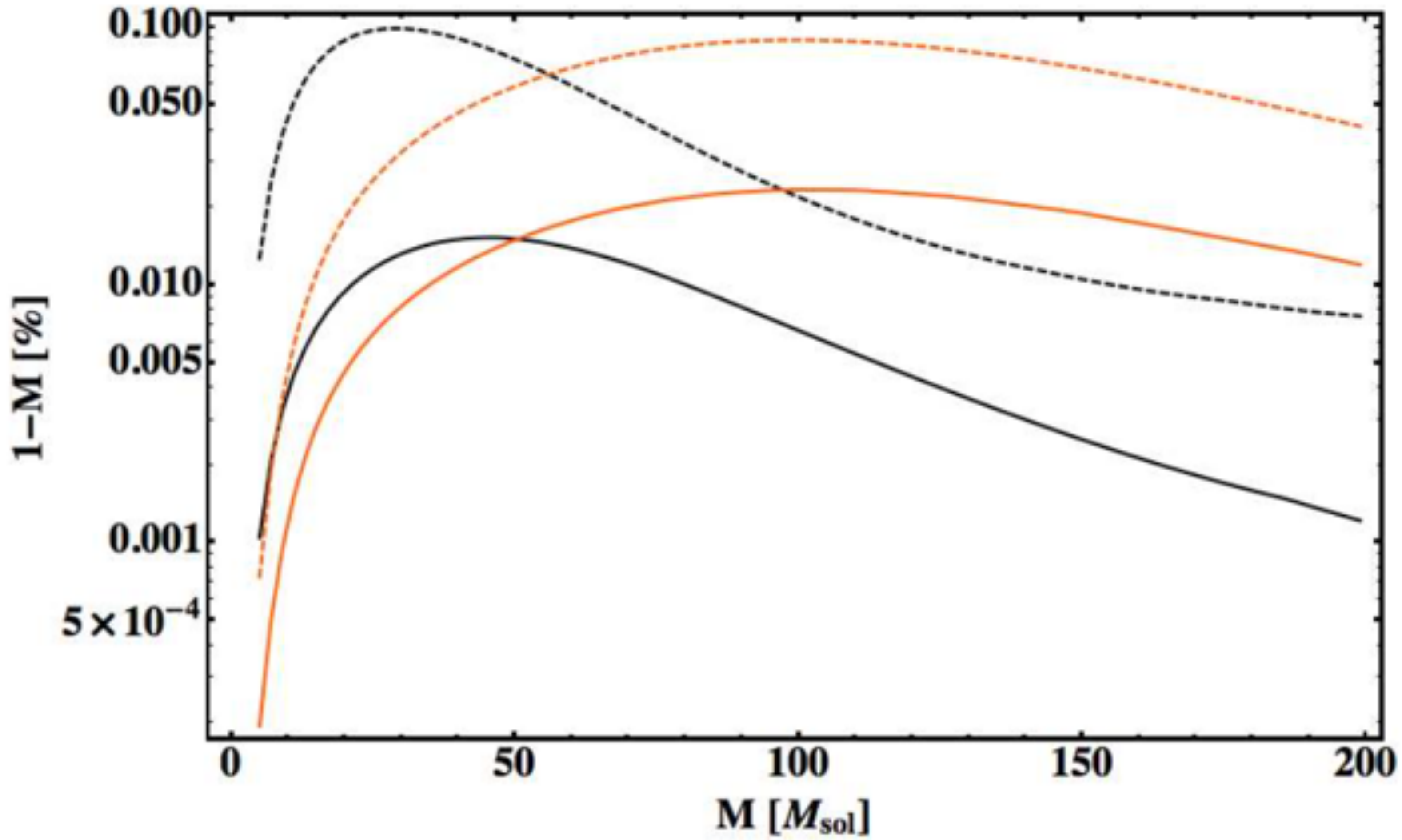
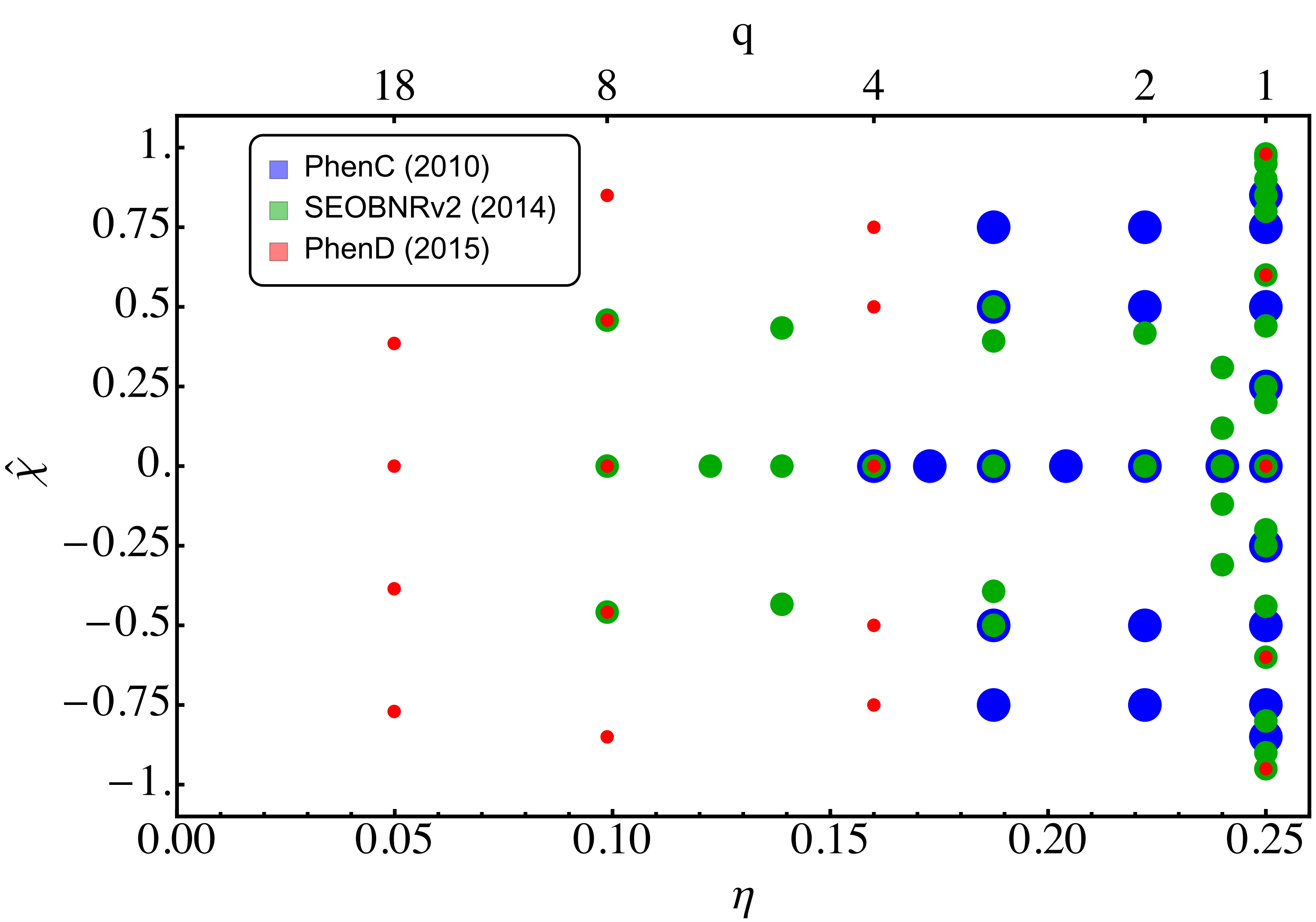


FIG. 2: Mismatch error due to numerical resolution, for the $q = 4$, $\chi_1 = \chi_2 = \hat{\chi} = 0.75$ (black lines) and non-spinnning $q = 18$ simulations (orange lines). The solid black line shows the mismatch between waveform $q = 4$ 112- and 96-point simuations, and the dashed black line shows the mismatch between the 96- and 80-point simulations. For the $q = 18$ configuration, the solid orange line shows the mismatch between the 144- and 120-point simulations, and the dashed orange line shows the mismatch between the 144- and 96-point simulations (see text).

NR Waveforms: SXS catalogue + new BAM WFs

- BAM code: “moving puncture” finite difference mesh refinement, BSSN formulation of Einstein Equations



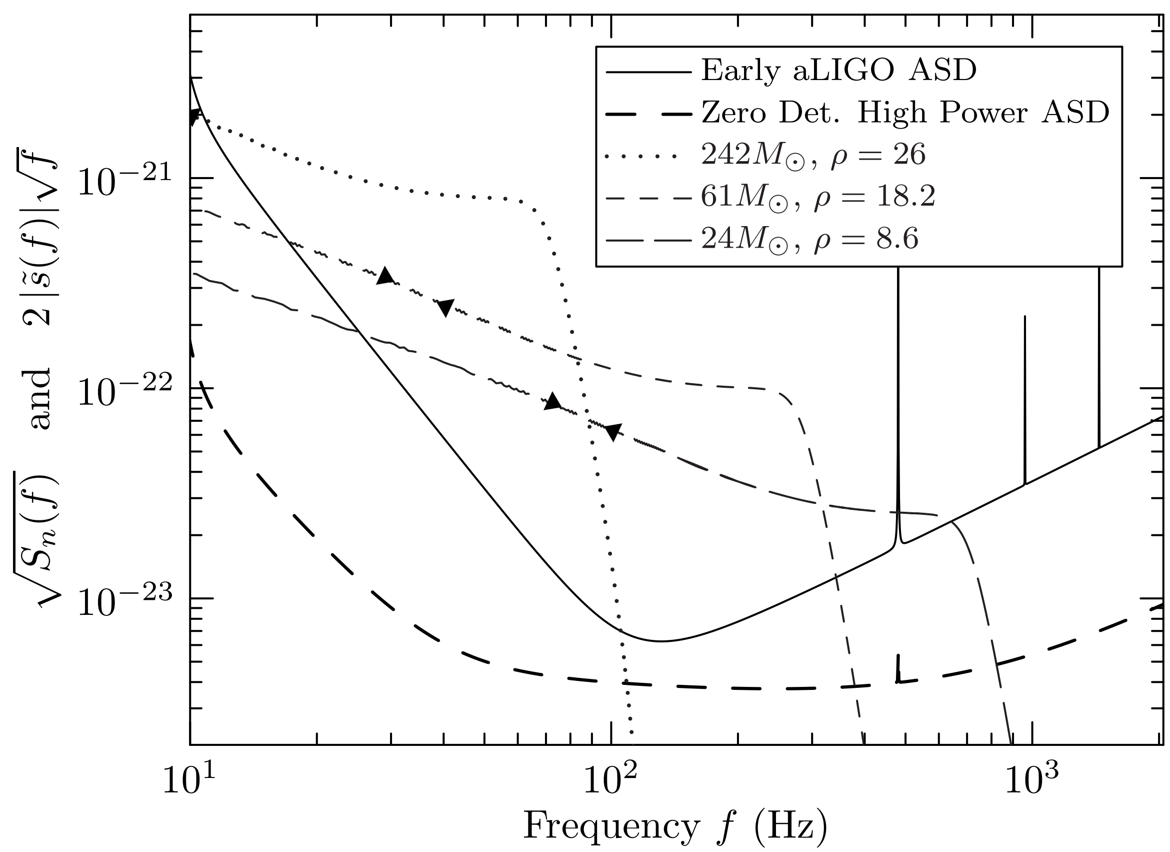
$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Split WFs into calibration & verification data sets:

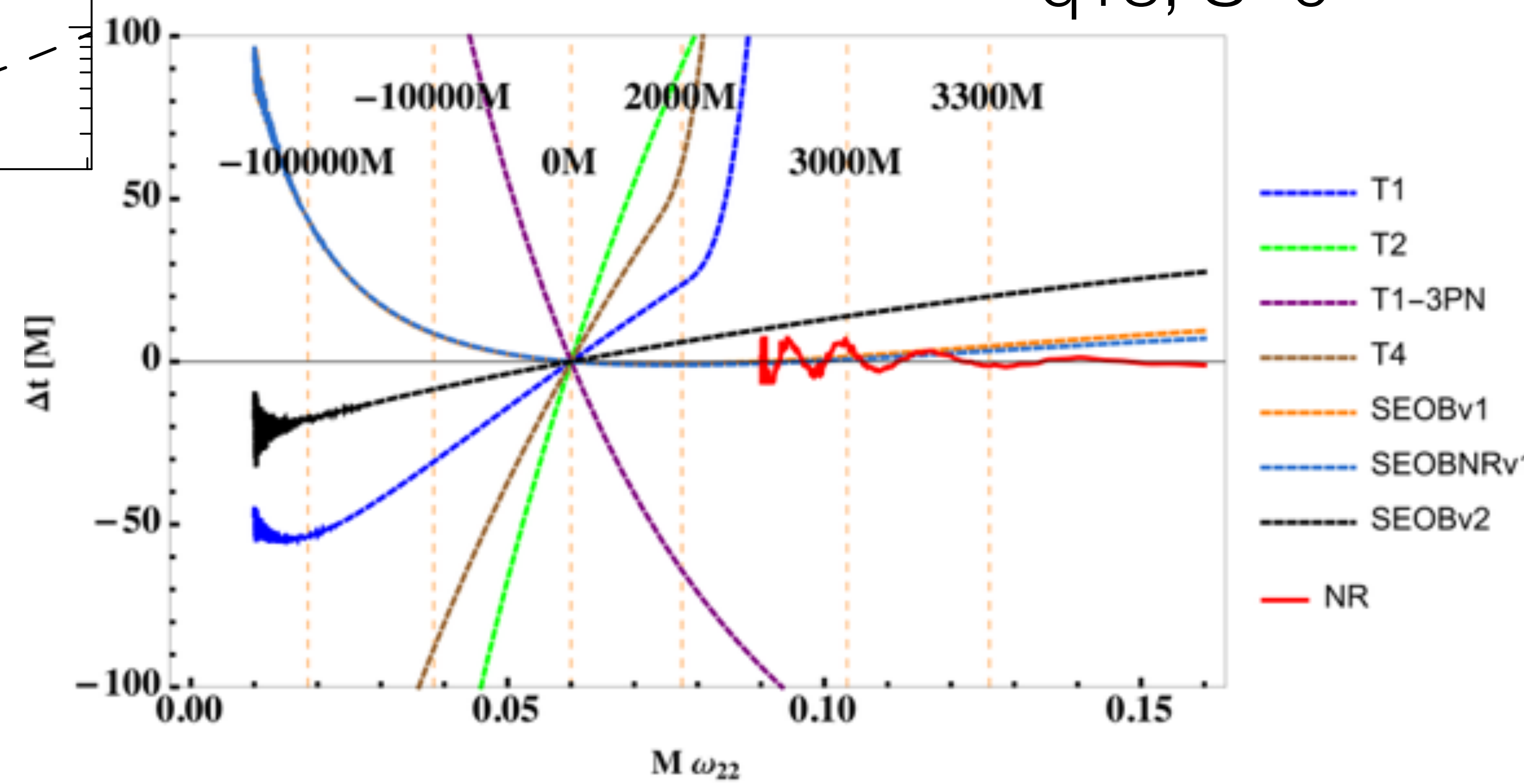
- calibration:
  10 BAM, 9 SXS
- verification:
  23 SXS, 6 BAM

Choice of inspiral approximate: uncalibrated SEOB

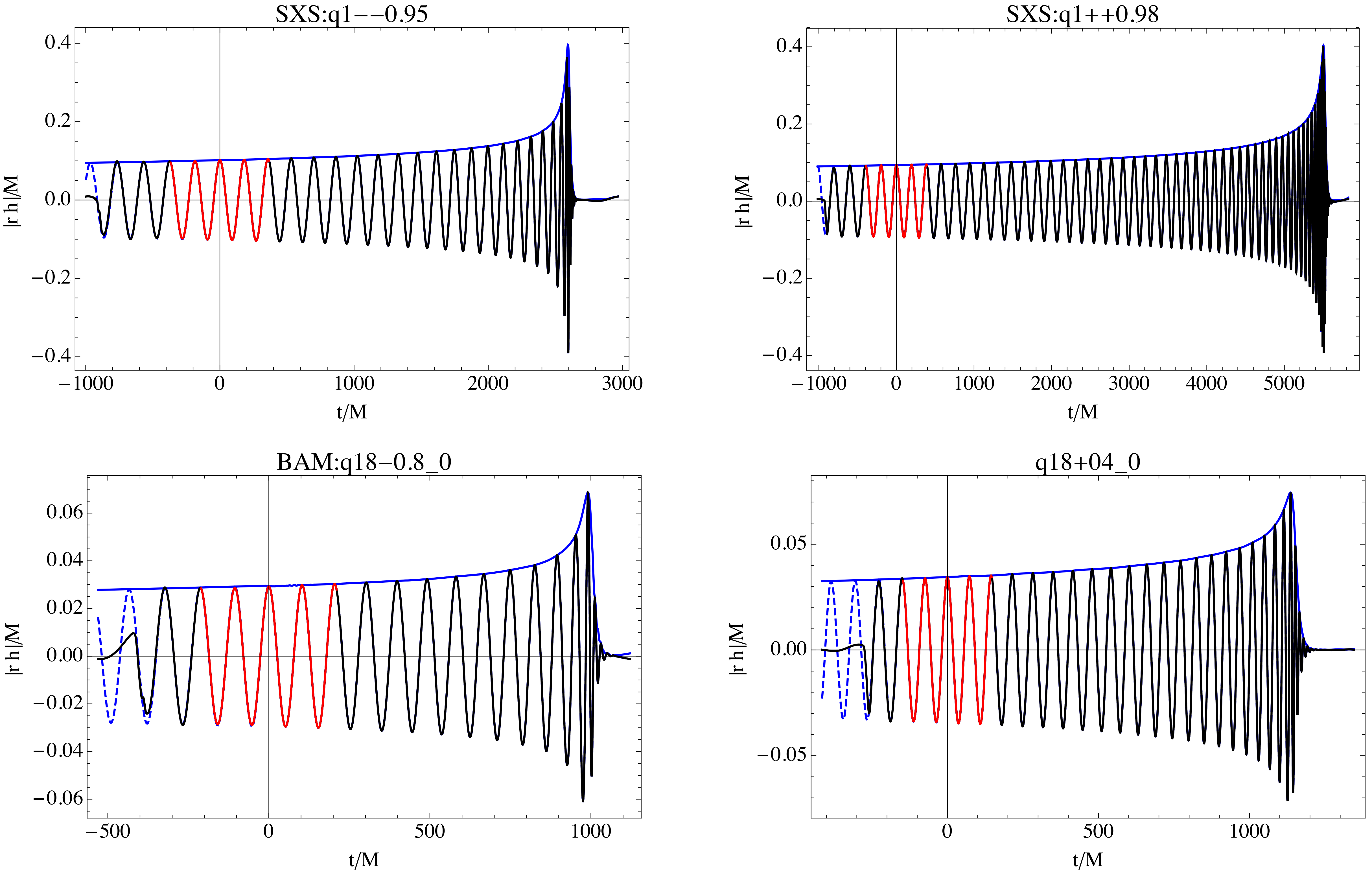
- Compare PN approximants in hybridization procedure -> decide for uncalibrated SEOBNRv2.



q18, S=0



Hybrid waveforms: corner cases



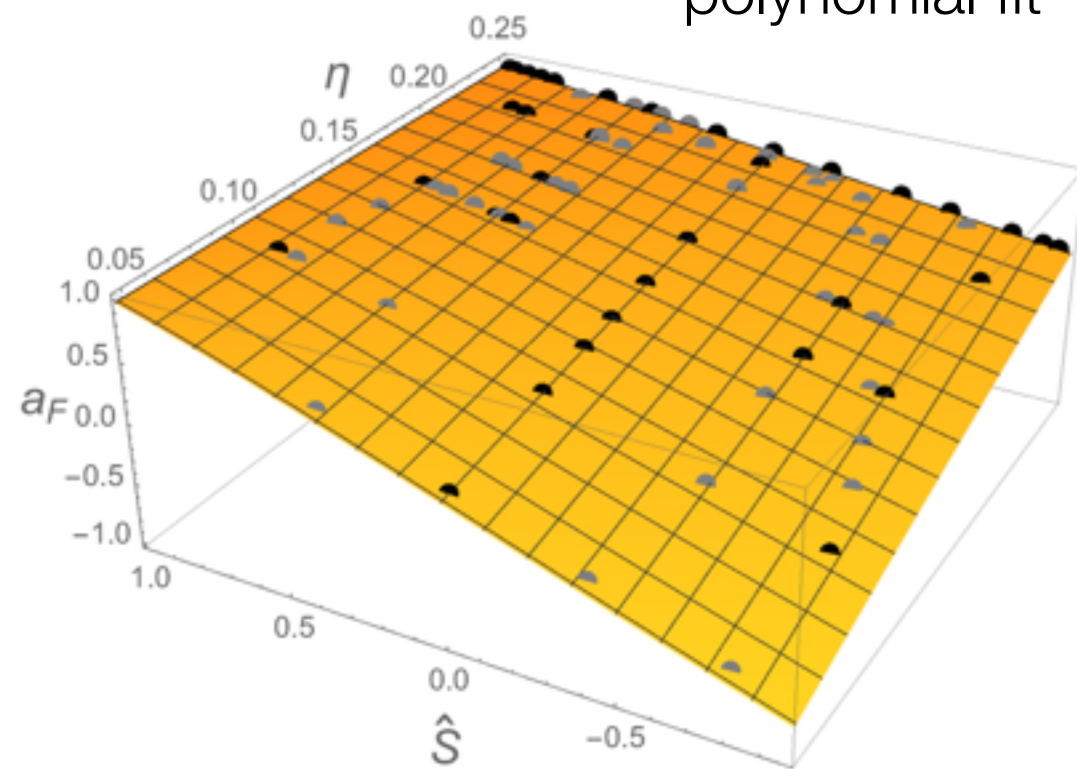
Final state

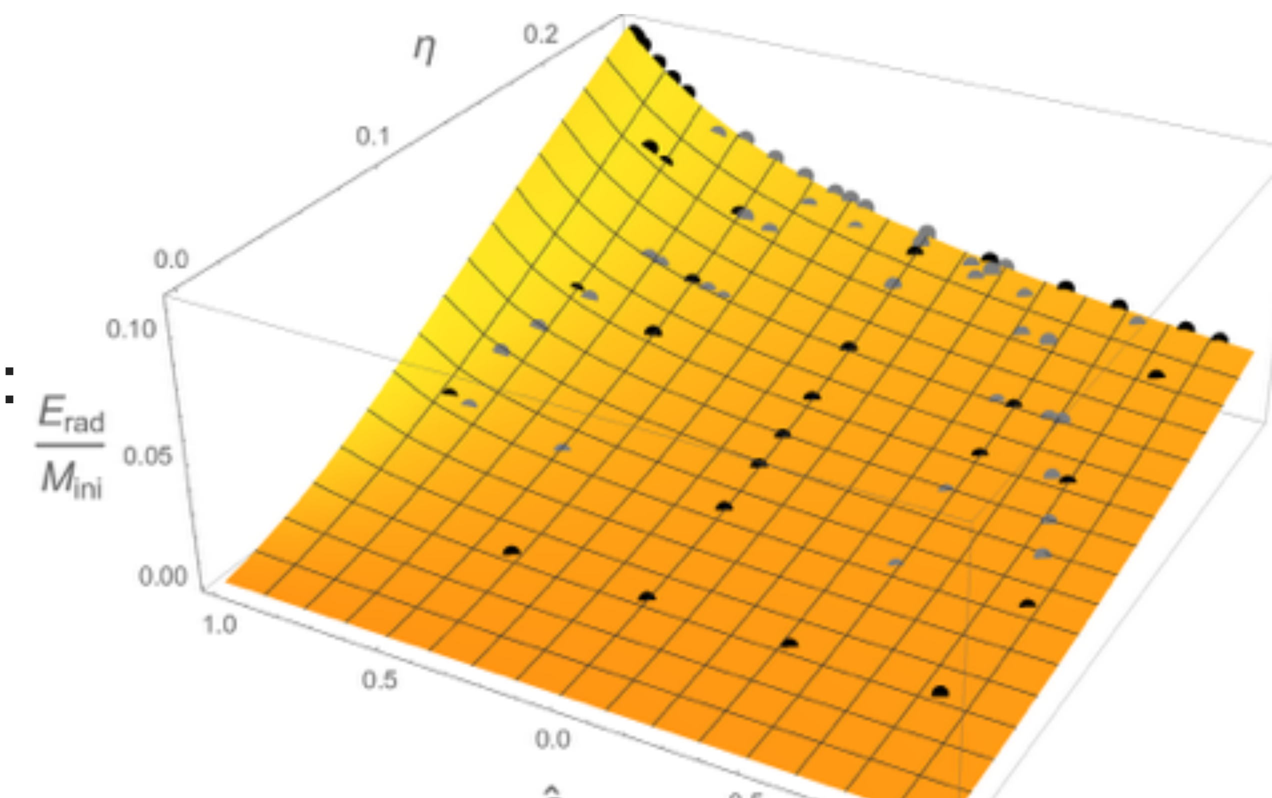
- Kerr BH perturbation theory -> complex frequencies of spheroidal harmonic QNMs, functions of final mass & final spin.
- Amplitudes and relative phases of different harmonics computed in NR.
- SXS, RIT + BAM q≦18 data =>
  Effective spin fits for final spin & radiated energy (final mass)

$$\hat{S} = \frac{m_1^2\chi_1 + m_2^2\chi_2}{m_1^2 + m_2^2}$$

- Hierarchical fitting approach by subspaces:
- no spin / equal mass / full

polynomial fit





rational function fit

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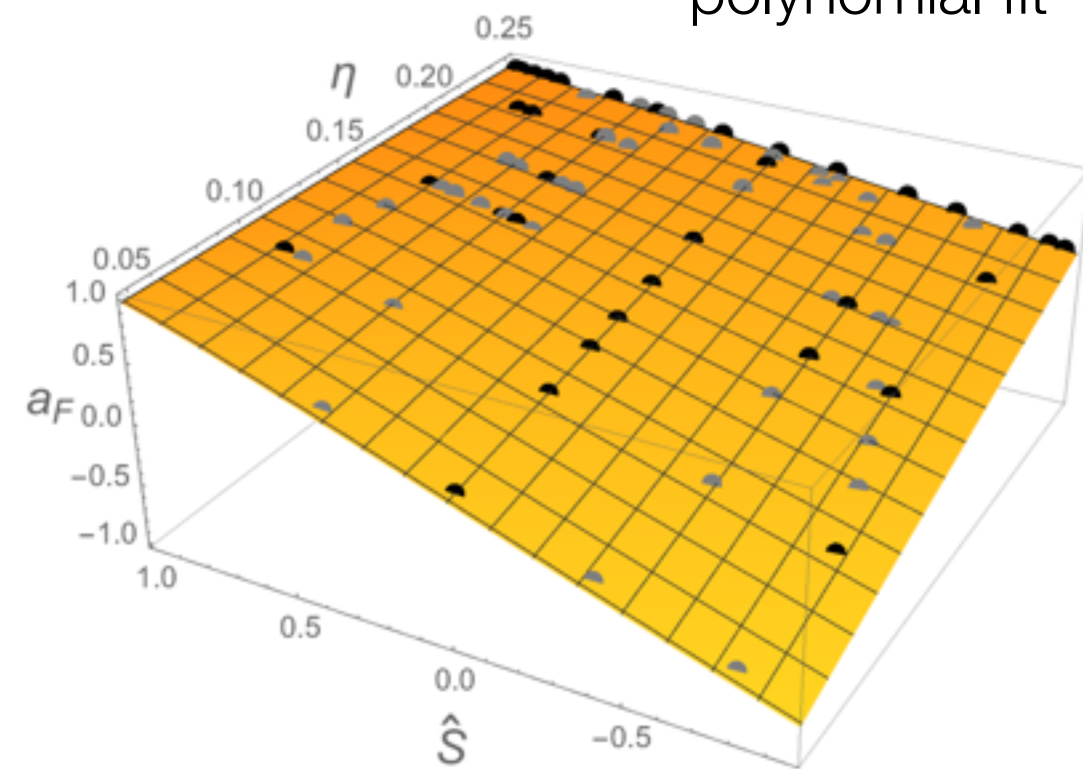
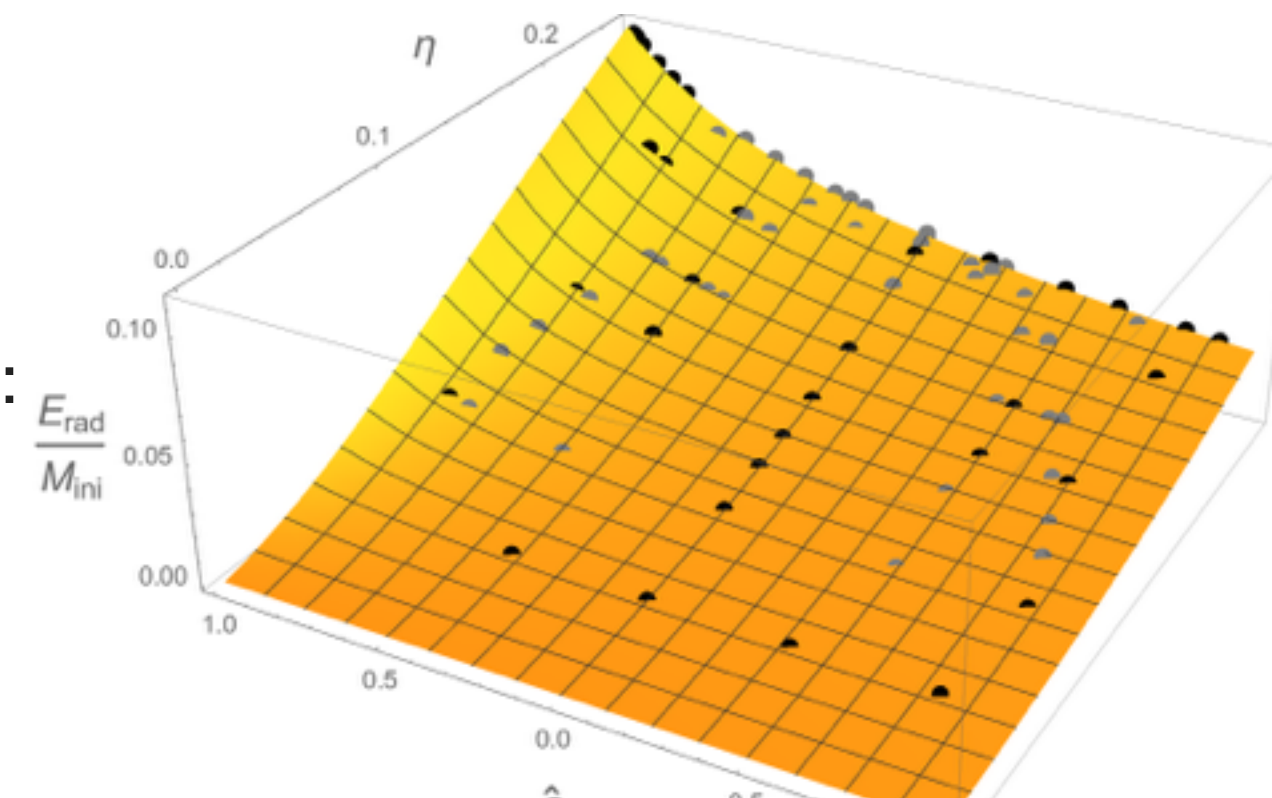




rational function fit

Need more high spin data points

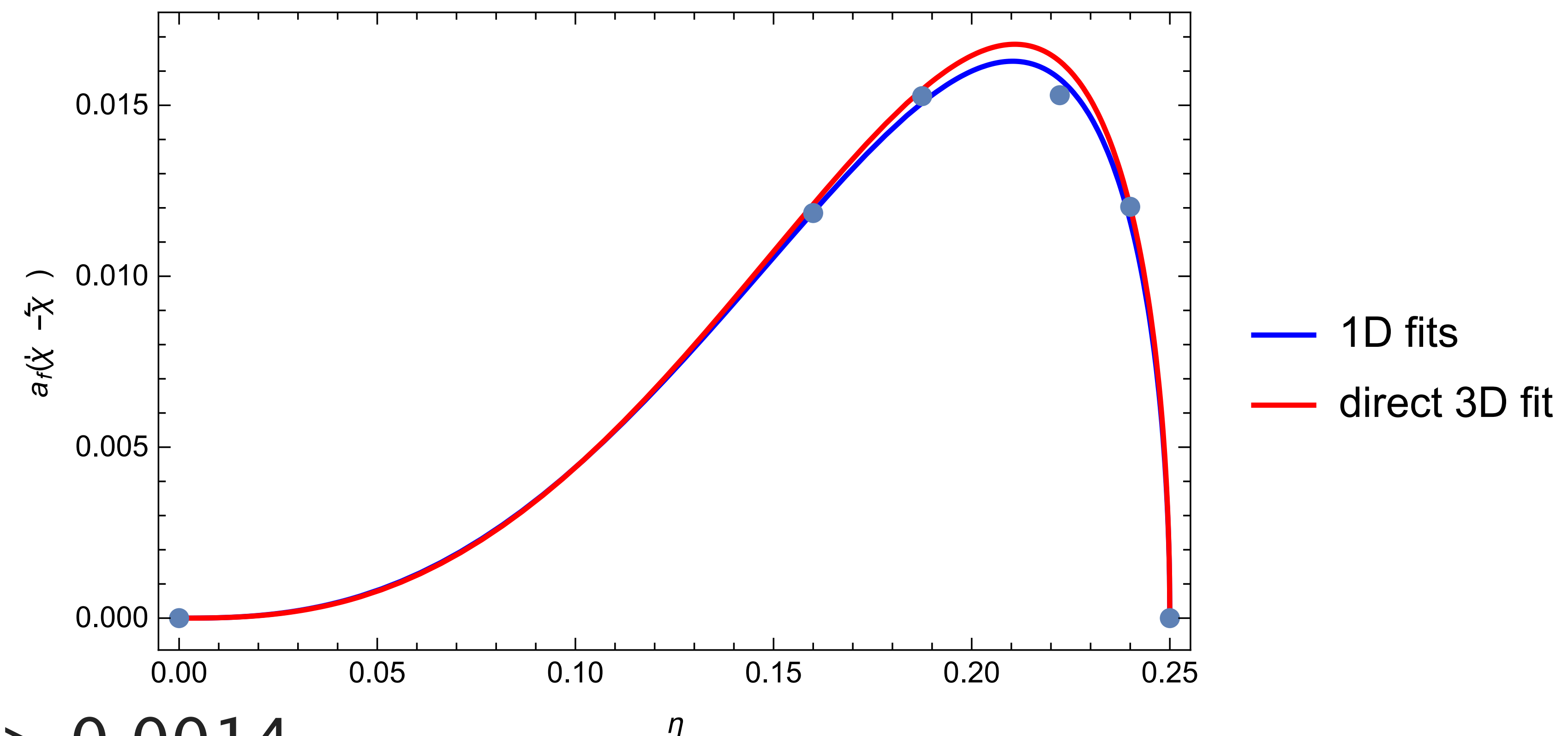
Update on final state results: unequal spins

- Final spin - extension to unequal spins:

$$a_f = a_f^{Eq} + f(\eta)(\chi_1 - \chi_2)$$

- Guess ansatz for f(η) from inspecting data:
  - At fixed η, difference with equal spin fit well approximated by plane -> determine coefficients, plot in 1D.

$$f(\eta) = a_0\, \eta^p (1 - 4\eta)^q$$

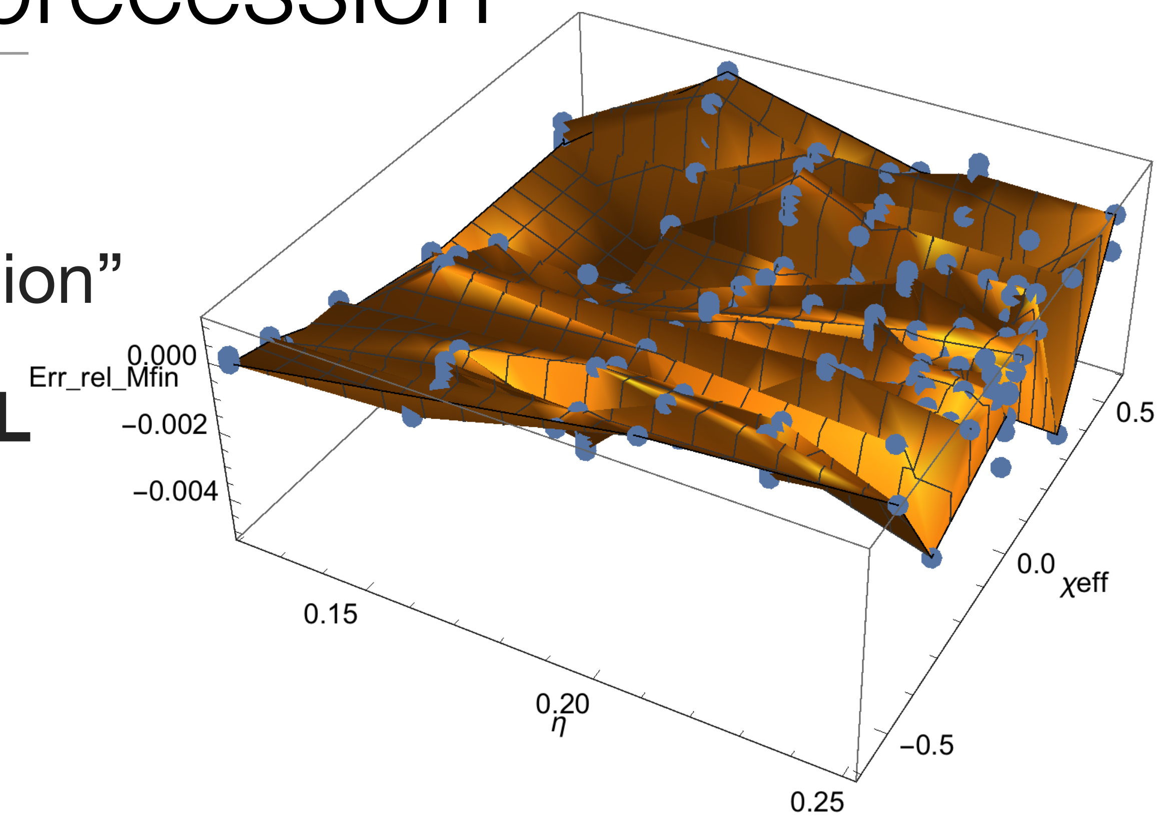


  - Compare with fit to full data set.

  - RMS error: 0.0075 -> 0.0014
  - similar for radiated energy.

Update on final state results: precession

- Based on PhenomP approximation:
  - emission in comoving frame = “no precession”
  - preserve total spin projections unto || & $\perp$ **L**
  - => radiated energy should depend only weakly on precession. ✅



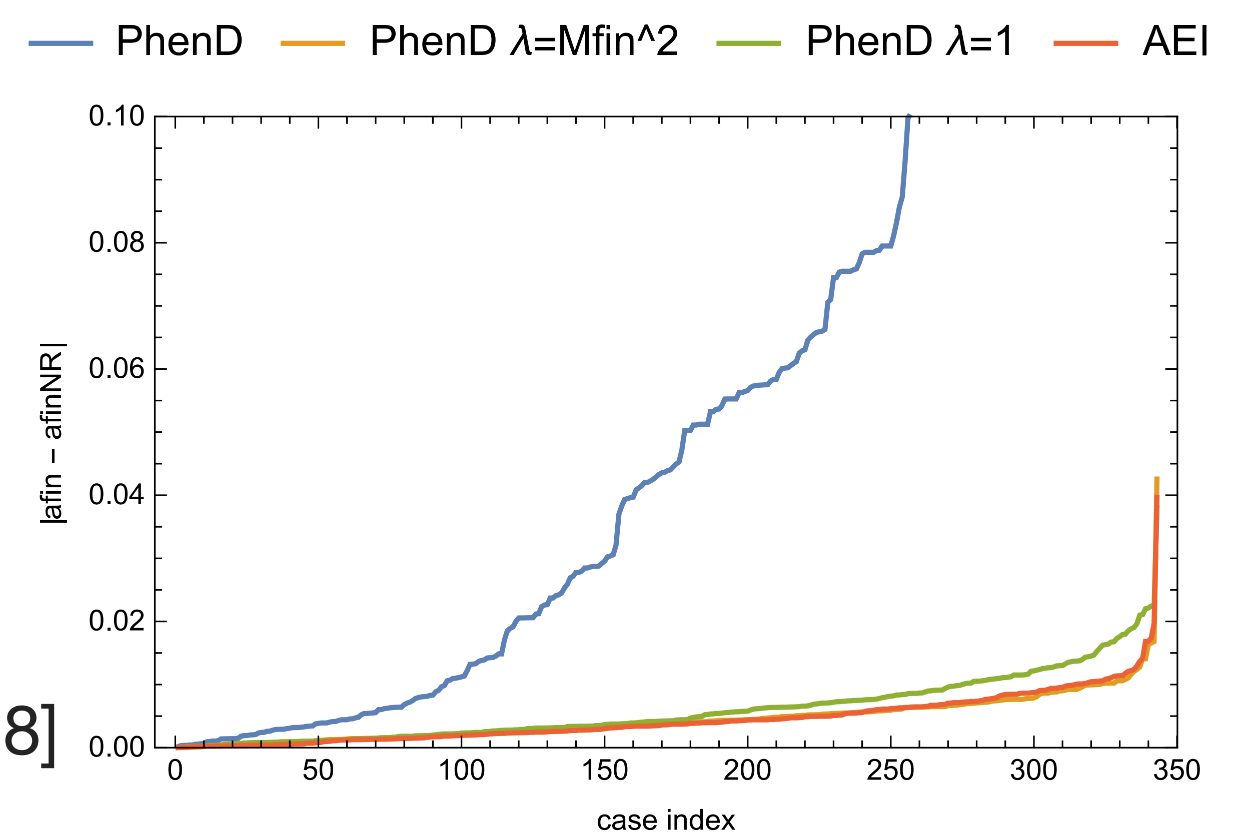
- Final spin:

$$|a_{fin}| = \sqrt{S_\perp^2 \frac{\lambda}{M_{fin}^2}^2 + a_{fin}^{||}{}^2}$$

  - choose “fudge parameter”

$$\lambda = M_{fin}^2$$

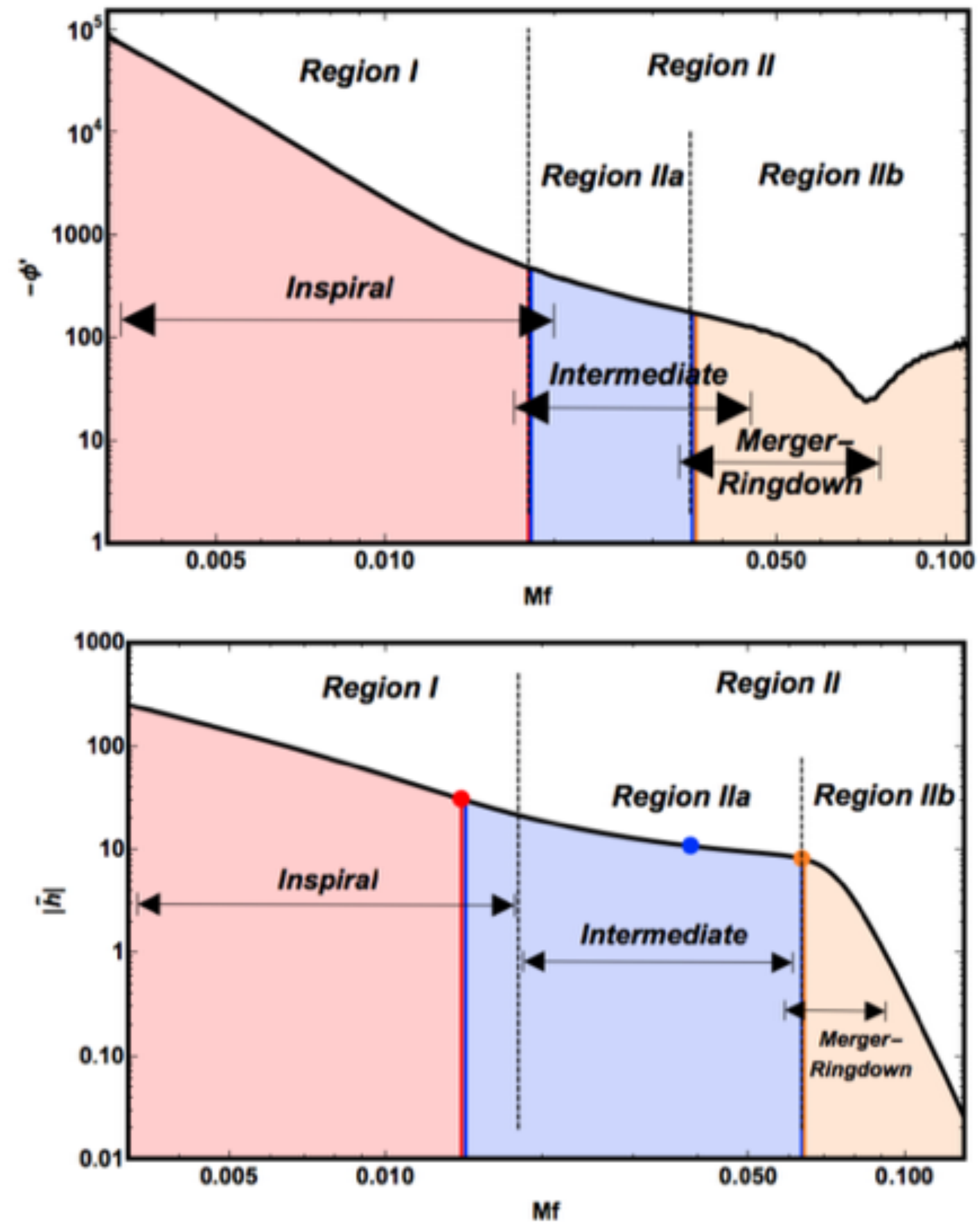
  - as in 2007 AEI fit [Rezzolla+,PRD78,2008]



Splitting into amplitude/phase & frequency regions

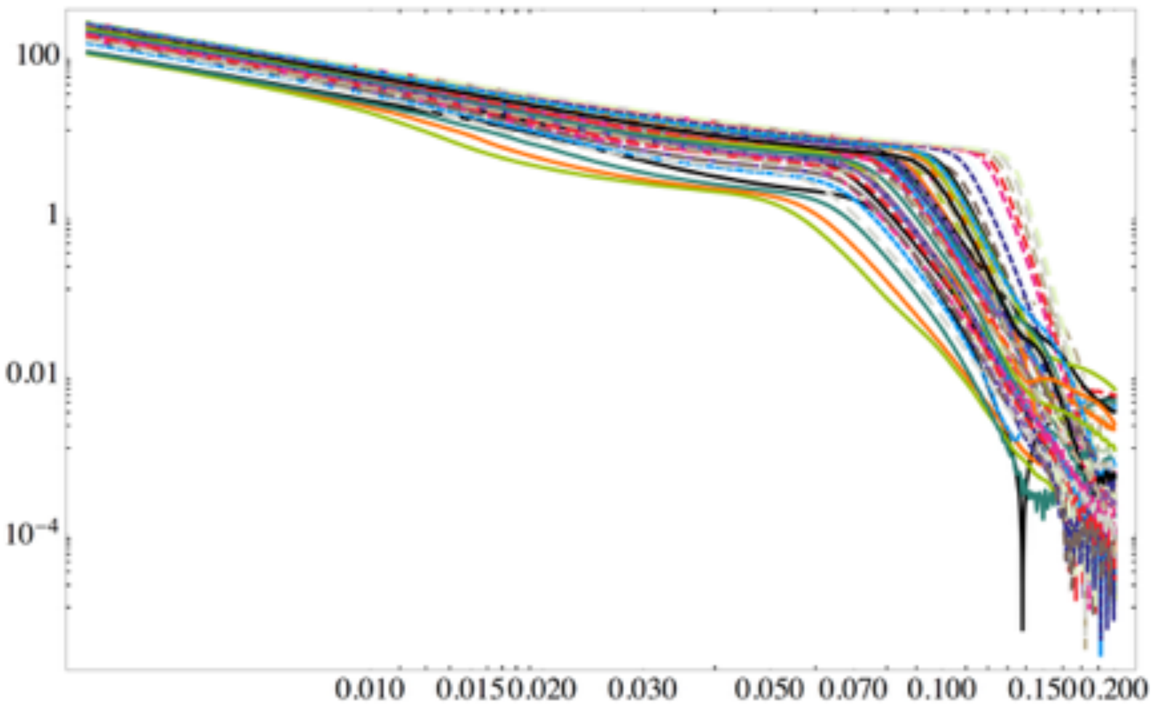
Divide and conquer:

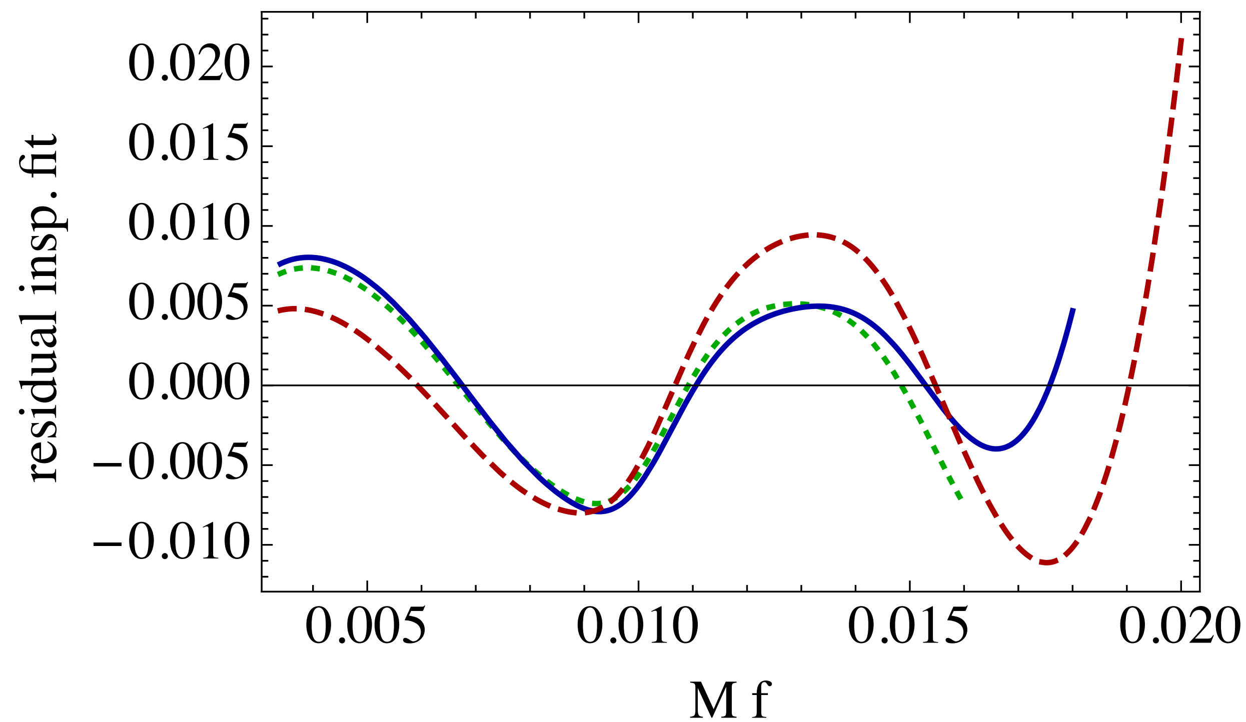
- Split waveform into amplitude and phase, model simple non-oscillatory functions.
- Simplicity of modelling increases with the number of frequency-regions.
- Simplest: tens of points, cubic spline.
- Our choice - 3 regions:
    - inspiral (use PN intuition)
    - merger-ringdown (use QNM intuition)
    - intermediate



Amplitude inspiral model

$$Mf \leq 0.018: \quad h_{\text{insp}} = \text{PN} + \alpha f^{7/3} + \beta f^{8/3} + \gamma f^3$$





- For each WF fit for α, β, γ
- PN terms have alternate signs, converge slowly
  -> represent curve by 3 equispaced data points
- Parameterize (η, χ_{eff}) parameter space and interpolate with polynomial.

P. Ajith, Phys. Rev. D 84, 084037 (2011)

$$\eta = \frac{m1m2}{(m1+m2)^2}, \quad \chi_{eff} = \frac{m_1\chi_1 + m_2\chi_2}{m_1+m_2} - \frac{76}{113}\frac{1}{2}(\chi_1+\chi_2)\eta$$

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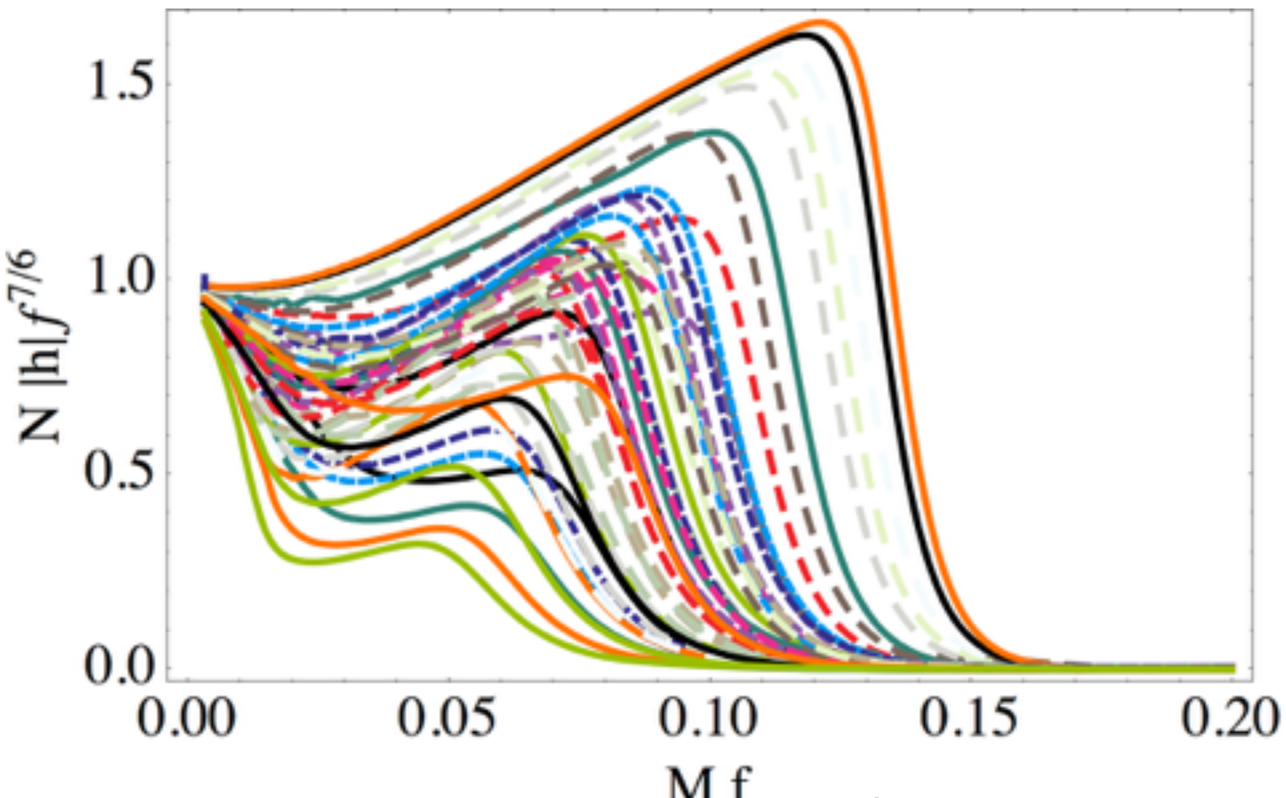

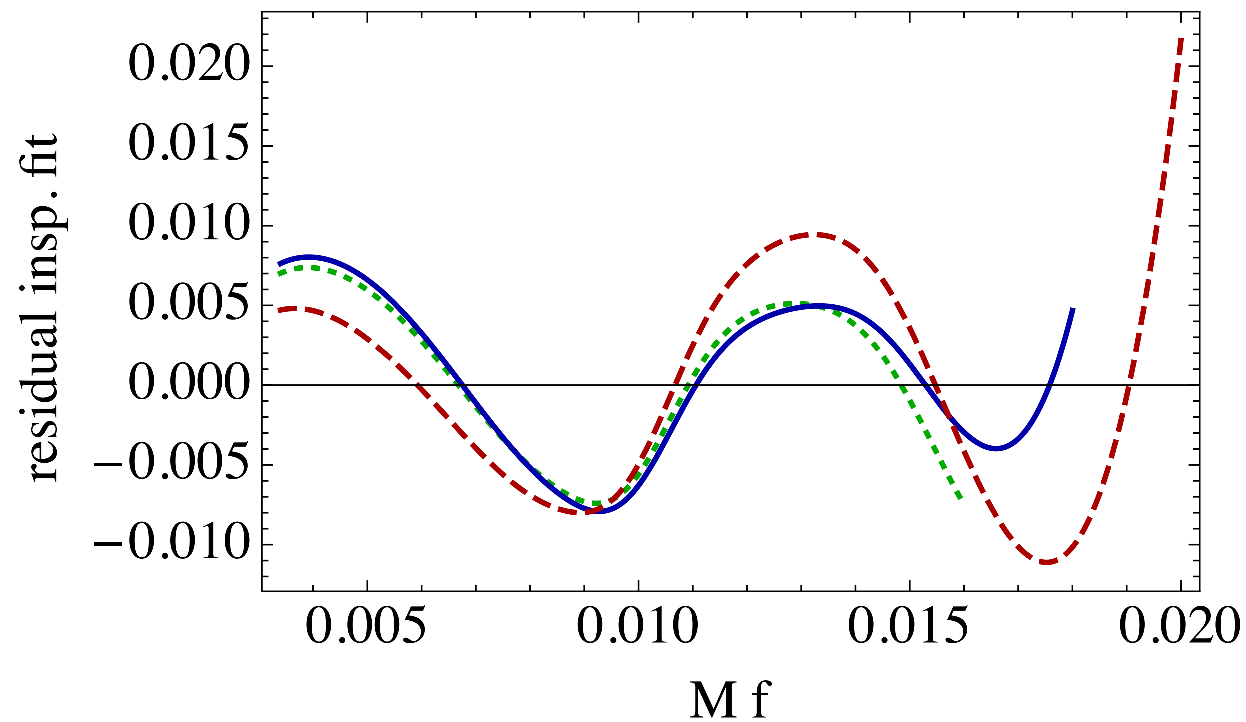

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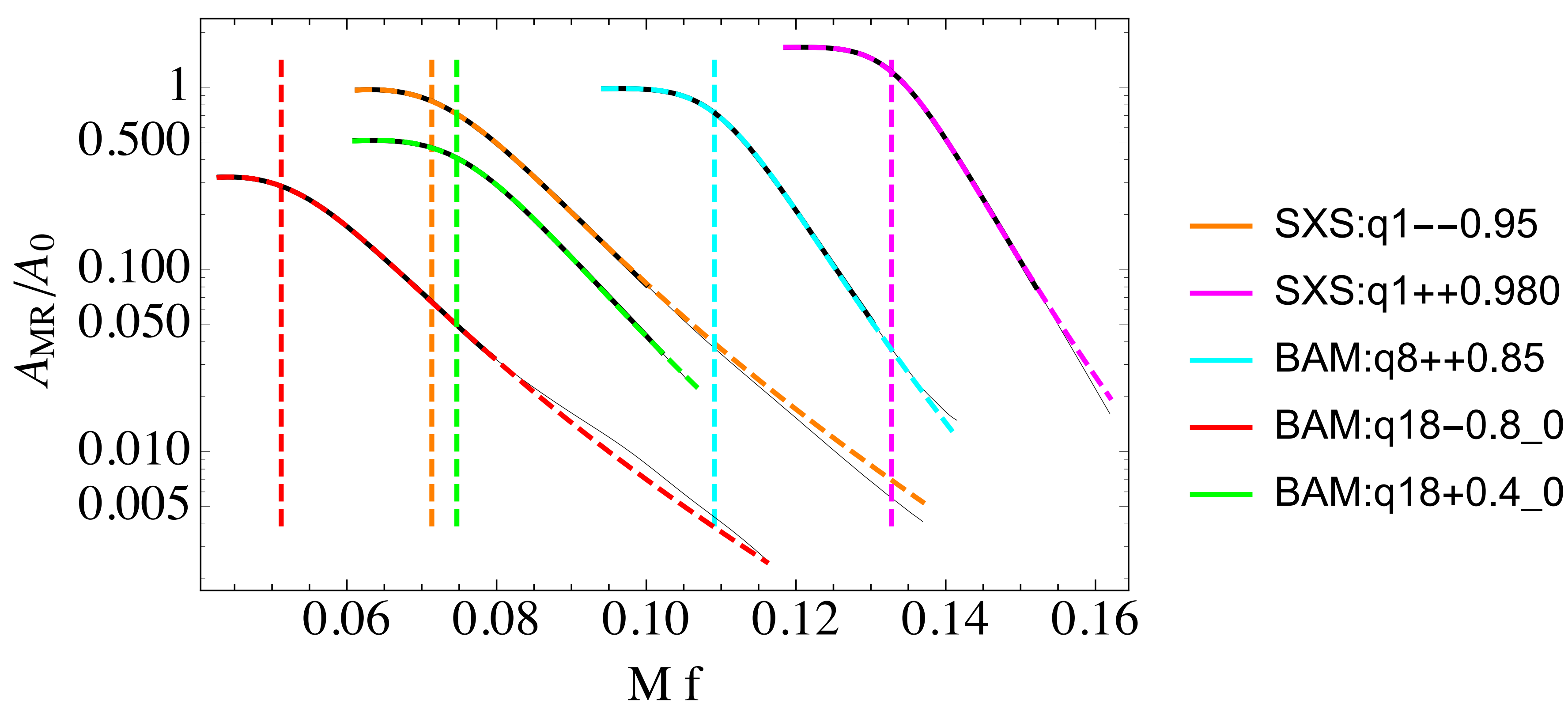
Complete amplitude model

- Deal with smooth functions -> high frequency falloff faster than polynomial.
- Previous Phenom ringdown based on Lorentzian, now multiply with exponential.

$$h_{\mathrm{RD}} = \frac{a\, e^{-\lambda(f - f_{\mathrm{ring}})}}{(f - f_{\mathrm{ring}})^{\,2} + \sigma^2}$$

- 0.018 < f < (local maximum of ringdown): rational function connected C^1 to inspiral and ringdown with 1(2) further parameters, or polynomial.



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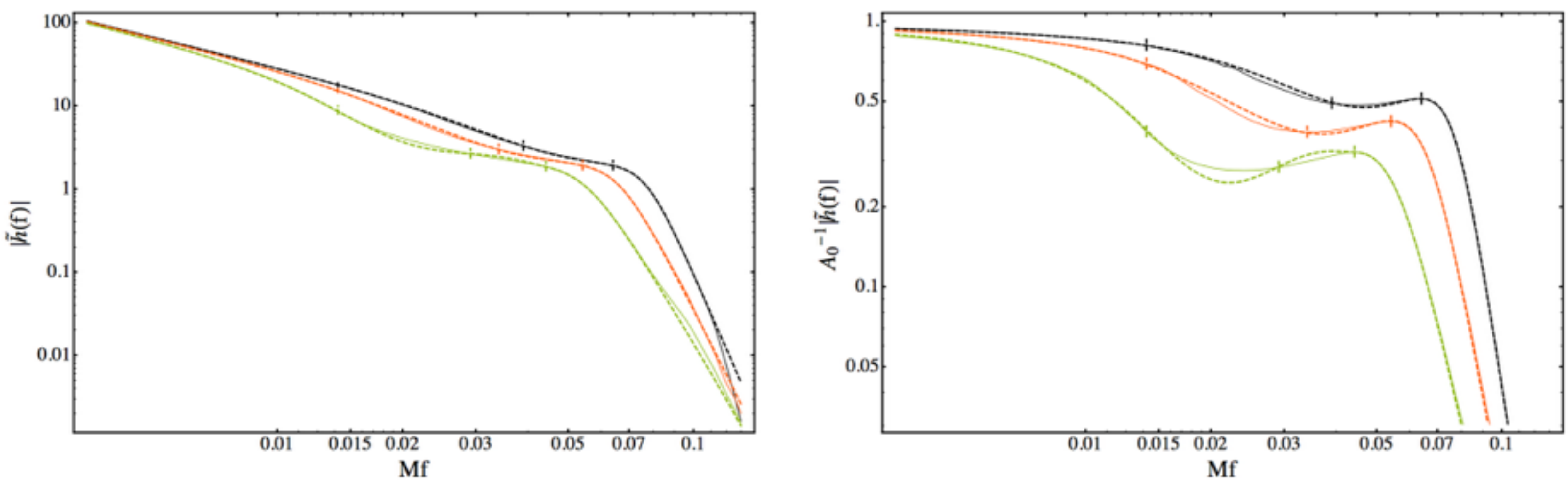
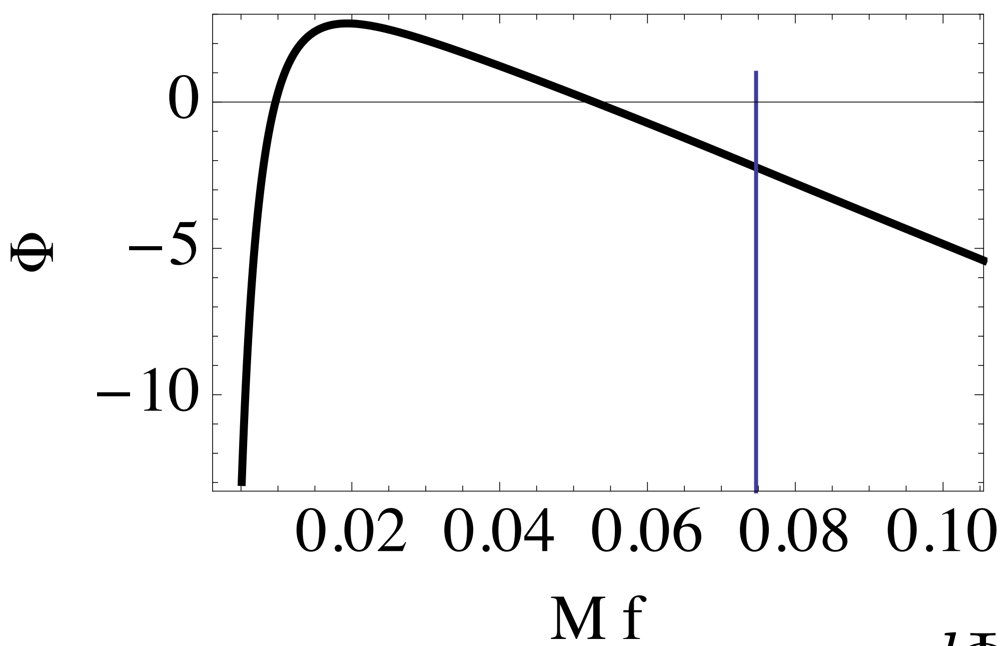
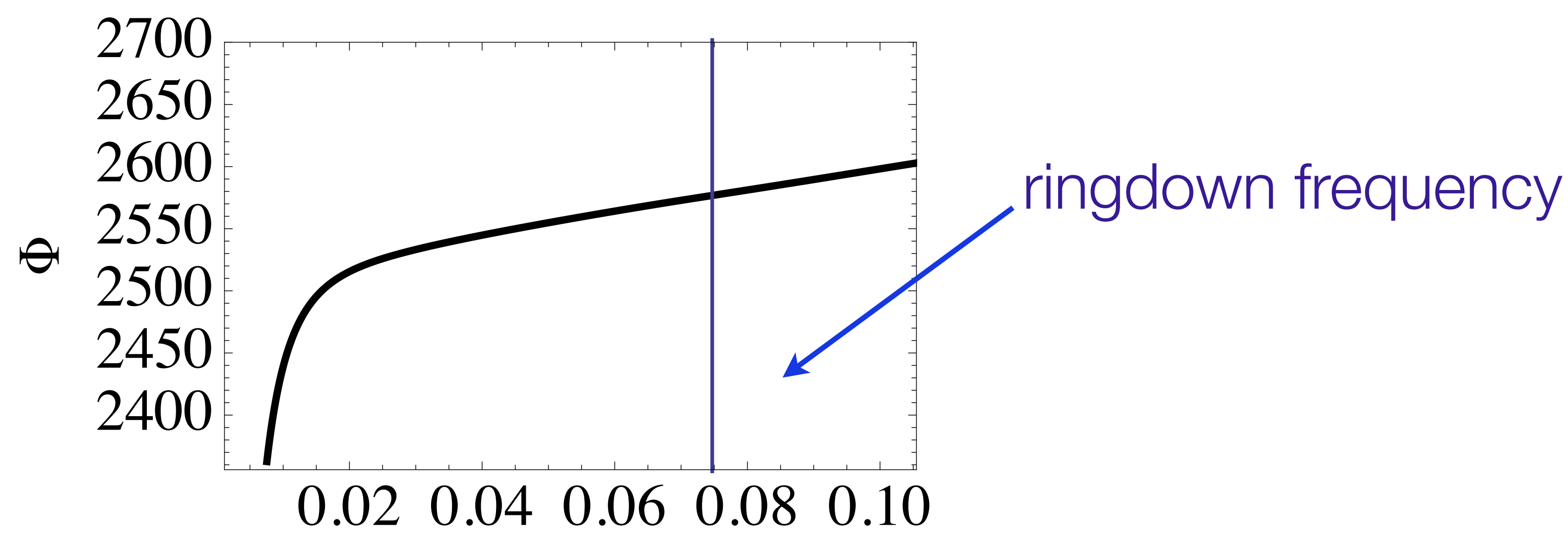


FIG. 10: The same quantities as in Fig. 9, but now for three $q = 18$ configurations, $\chi_1 = 0.4, \chi_2 = 0$, $\chi_1 = \chi_2 = 0$ and $\chi_1 = -0.8, \chi_2 = 0$.

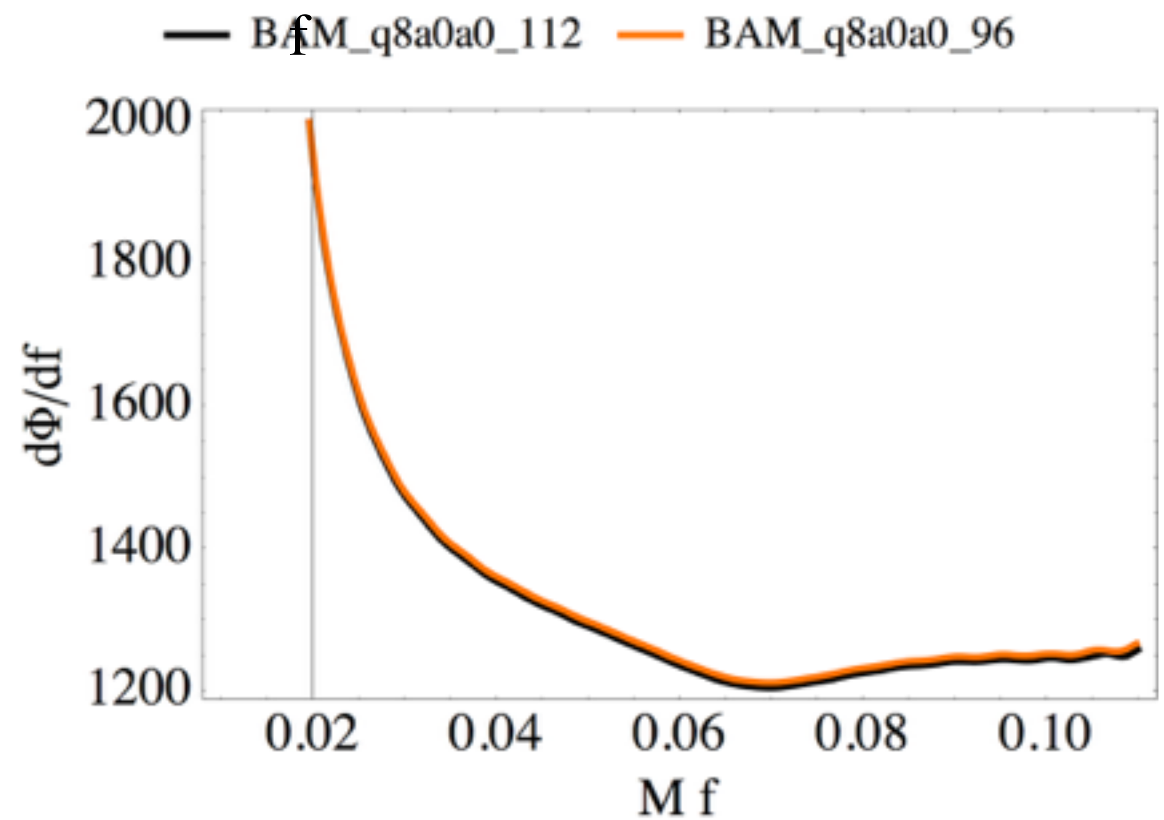
Modelling the Fourier domain phase

- Bad news: Freedom in initial phase & time shift: $\Phi(f) \to \Phi(f) + \Phi_0 + 2\pi t$





- Look at first derivative: $\dfrac{d\Phi(f)}{df}$
  - 2nd derivative often too noisy.
- Can you spot the ringdown frequency?
- MRD-Ansatz:



$$\phi'_{\rm MR} = \alpha_1 + \sum_{i=2}^{n} \alpha_n f^{-p_n} + \frac{a}{f_{damp}^2 + (f - f_{RD})^2}$$

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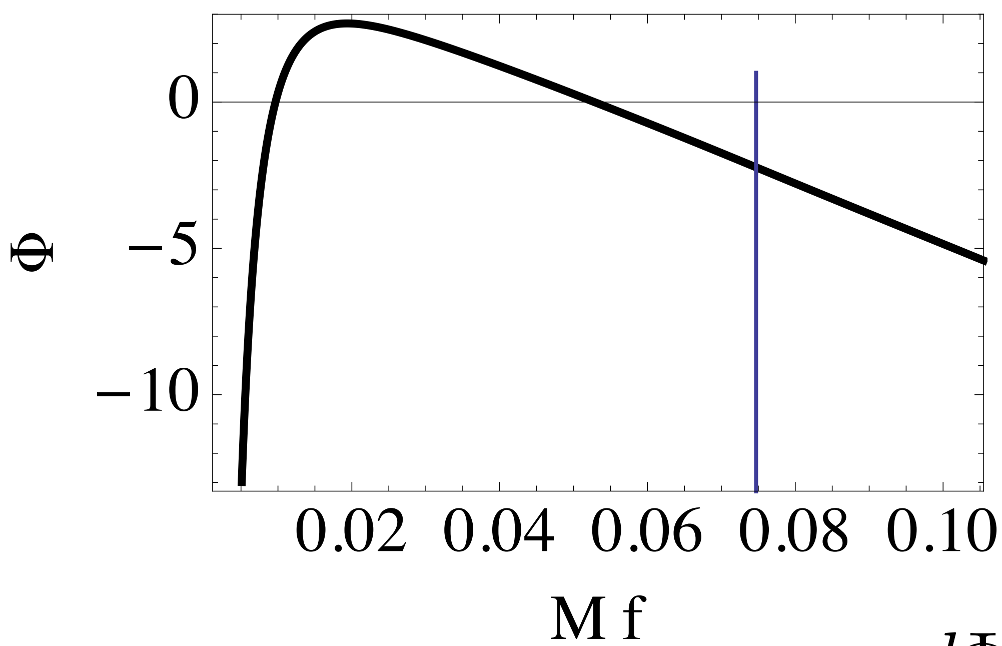

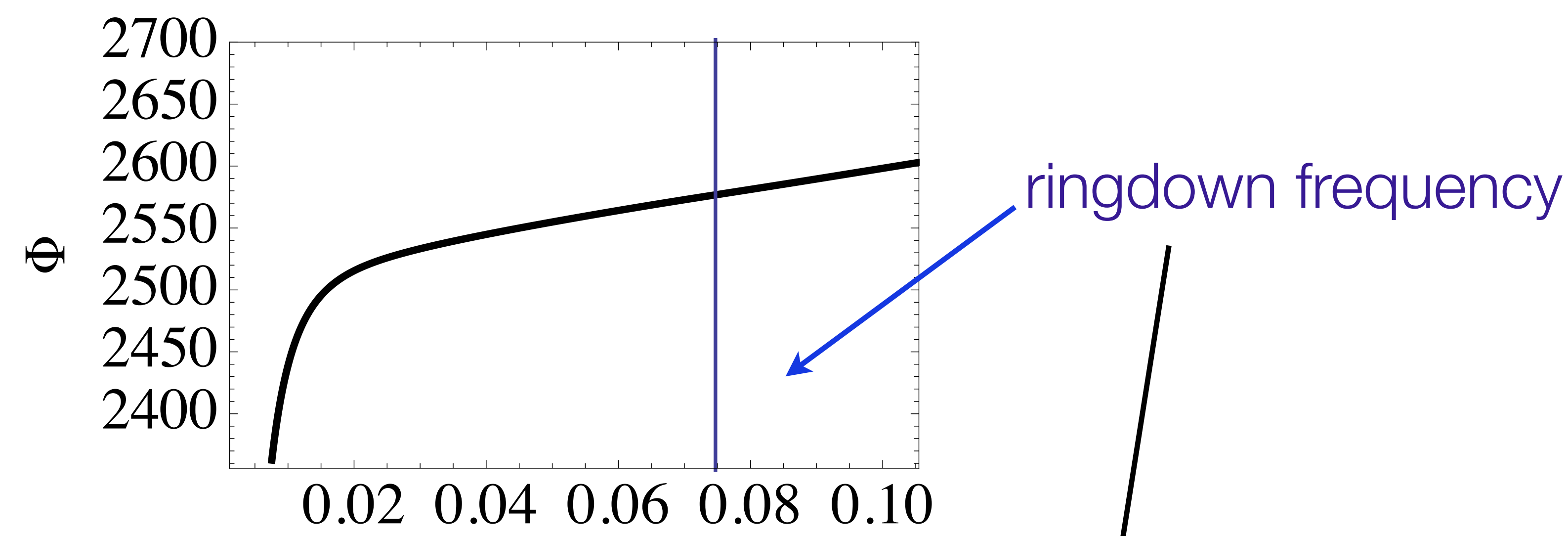

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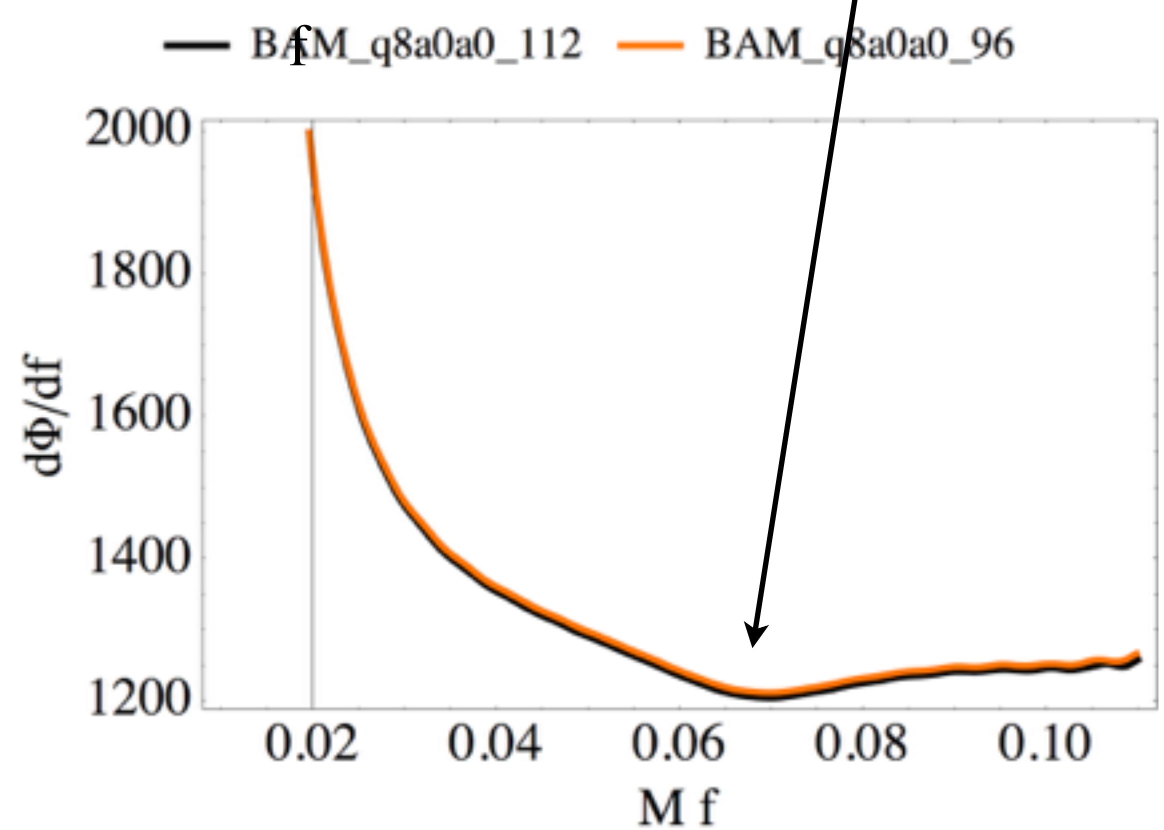

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Modelling the Fourier domain phase

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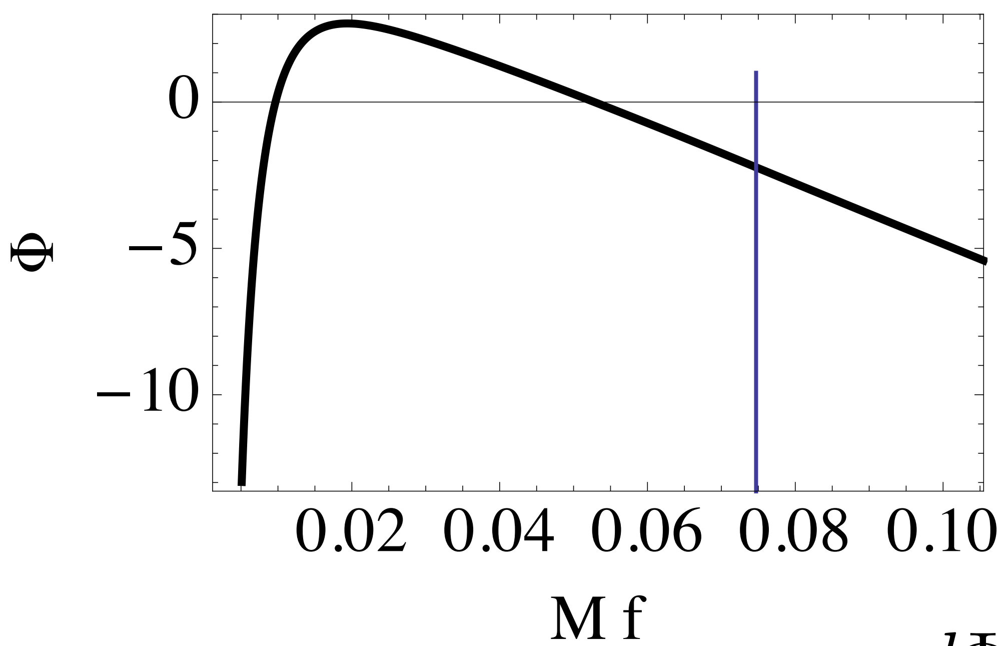

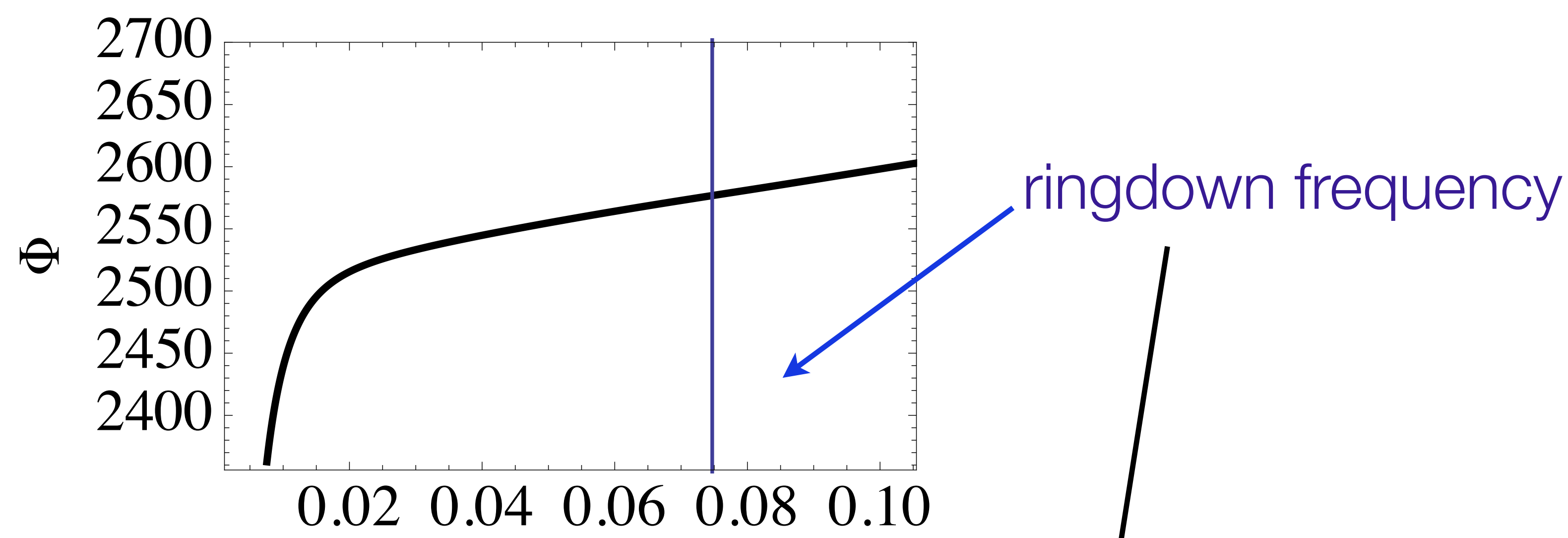

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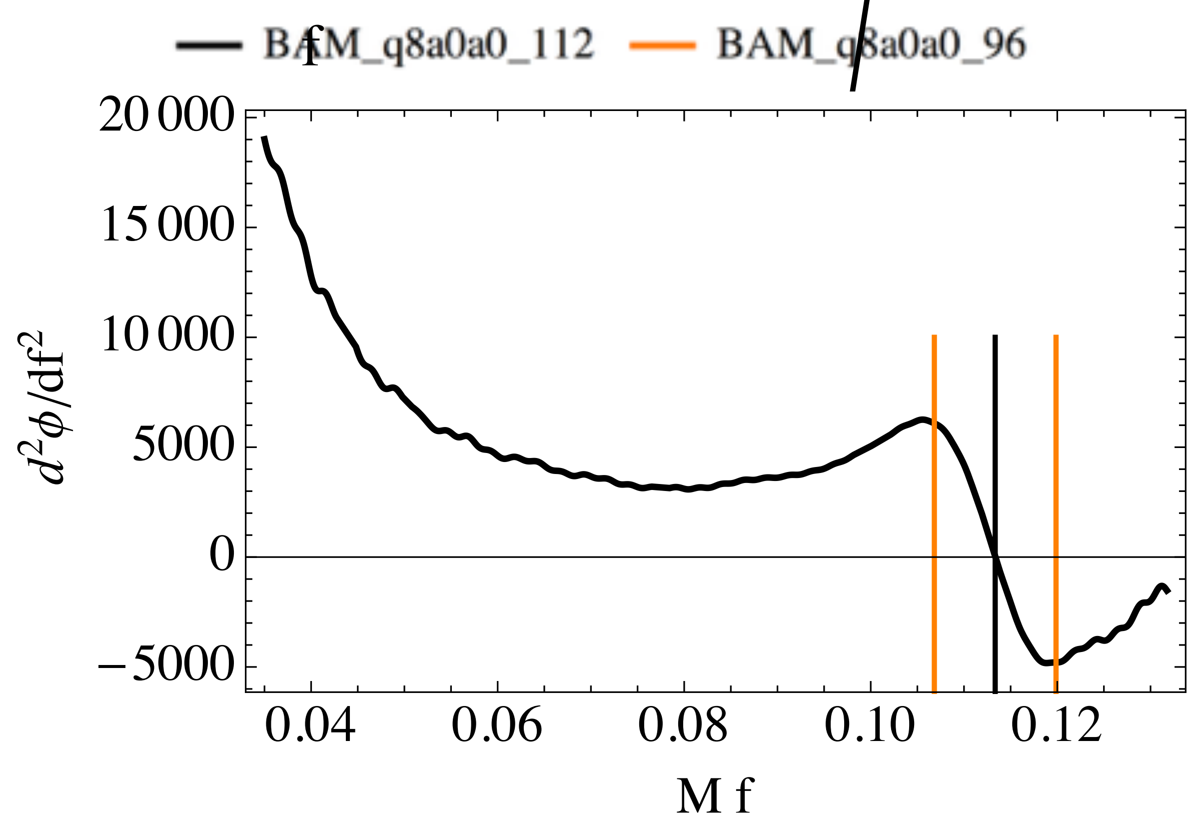

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$$\phi'_{\mathrm{MR}} = \alpha_1 + \sum_{i=2}^{n} \alpha_n f^{-p_n} + \frac{a}{f_{damp}^2 + (f - f_{RD})^2}$$

Phase model

- Inspiral: as for amplitude, PN + 3 higher order terms + Φ_0
- Ringdown: $\Phi'_{\mathrm{MR}} = \alpha_1 + \alpha_2 f^{-2} + \alpha_3 f^{-1/4} + \dfrac{\alpha_4 f_{\mathrm{damp}}}{f_{\mathrm{damp}}^2 + (f - \alpha_5 f_{\mathrm{RD}})^2}$
- Intermediate: $\Phi'_{\mathrm{Int}} = \beta_1 + \beta_2 f^{-1} + \beta_3 f^{-4}$

Phase & residuals
example:
intermediate freq.

Phase model

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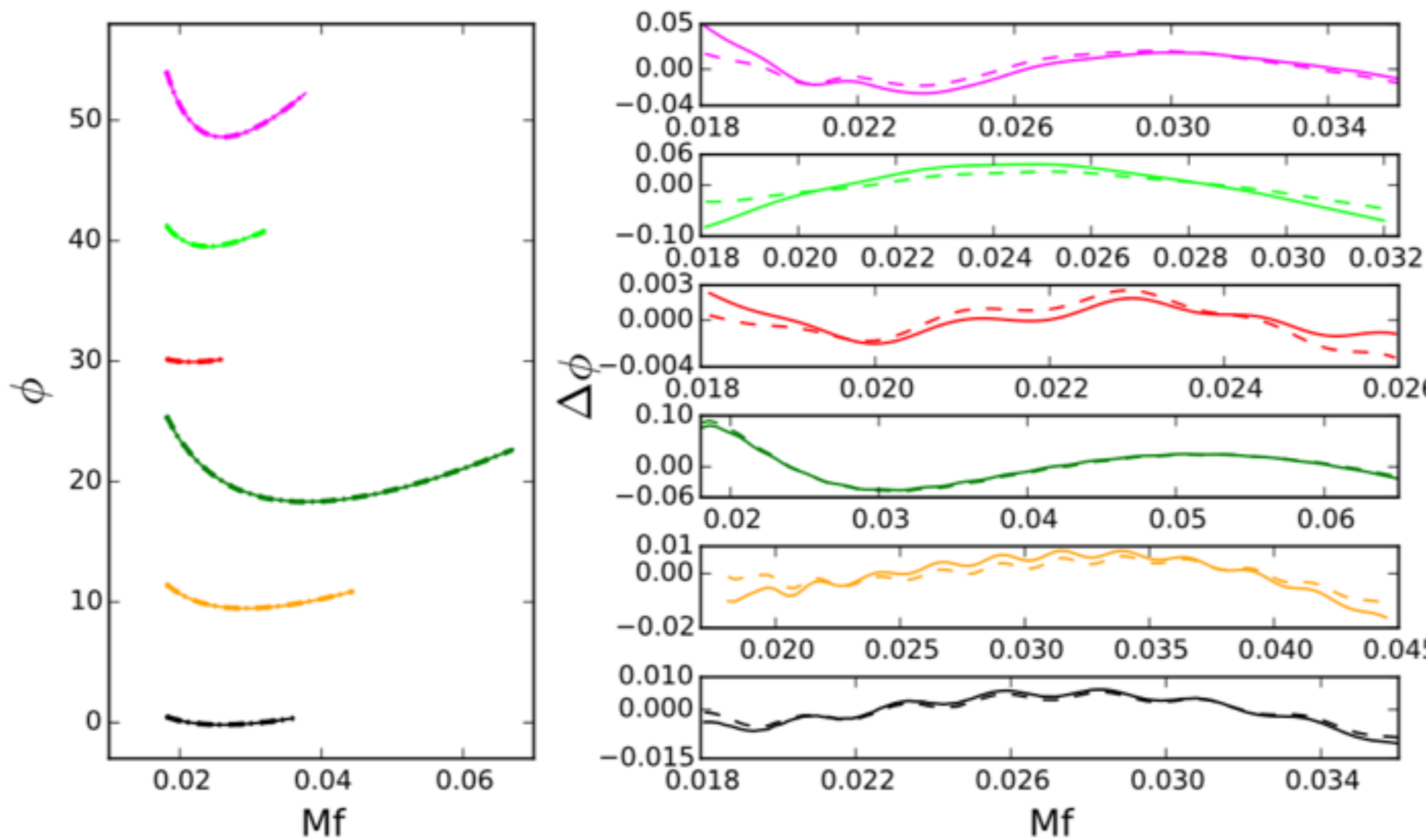
Phase & residuals example: intermediate freq.



Phase coefficients as functions of $\eta, \hat{\chi}$

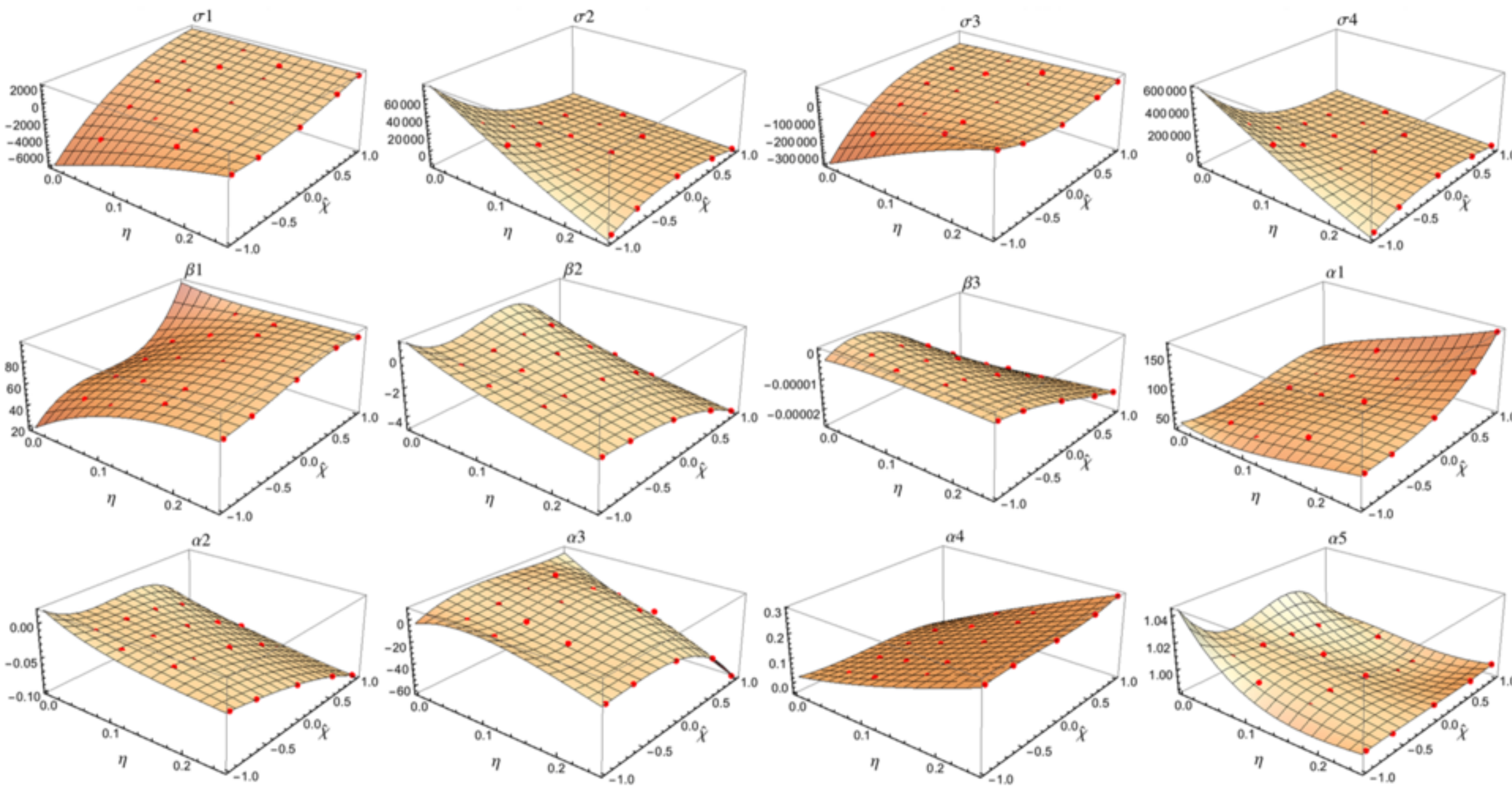
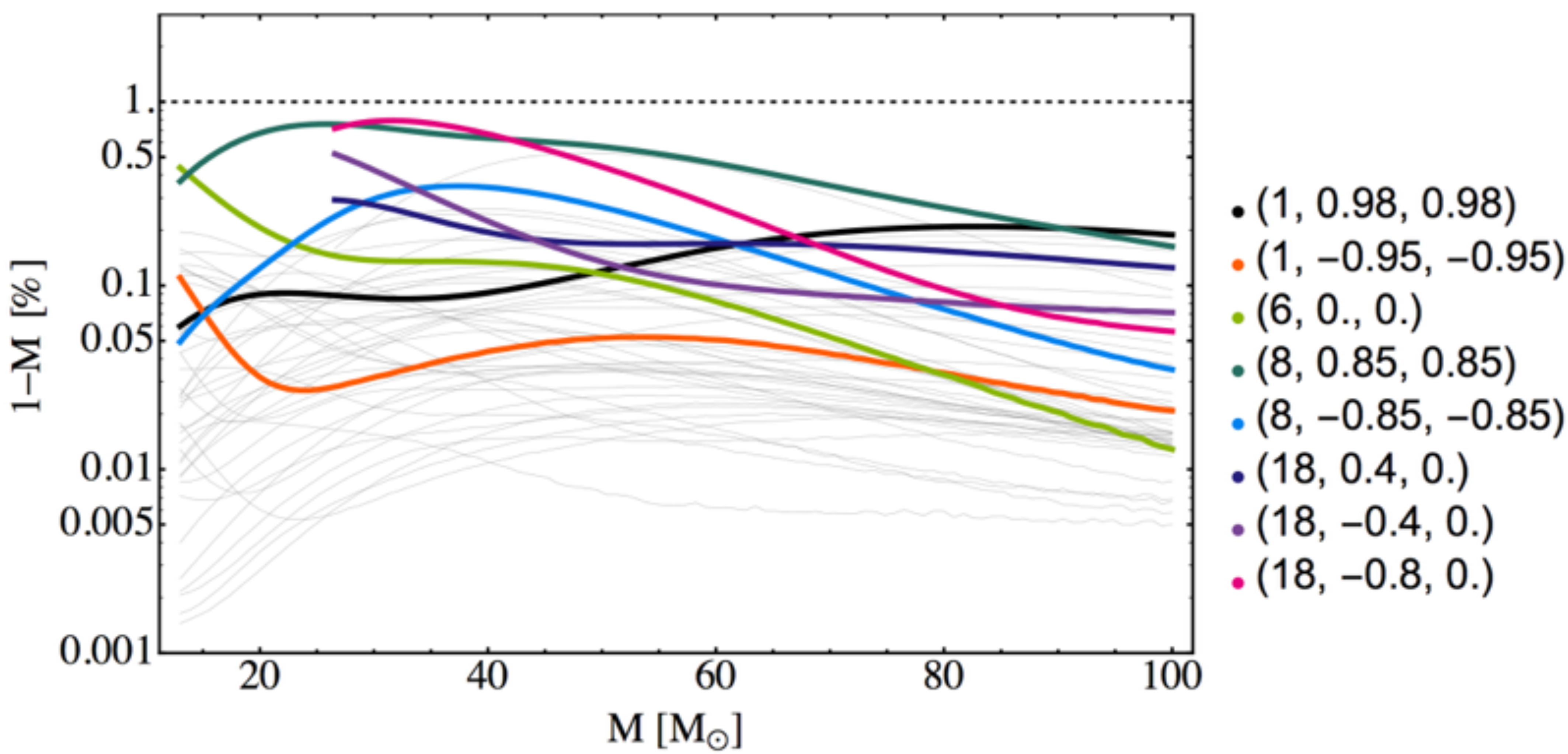


FIG. 12: Phase coefficients for region I and II. The calibration points and the model, extrapolated to the boundary of the physical parameter space are shown.

PhenomD mismatches against all 48 hybrids

early aLIGO noise curve, low freq. cutoff @ 30 Hz



Mismatches between models: SEOBNRv2 vs PenD

PhenD/SEOBNRv2_ROM PhenD+EOB insp. PhenD+TaylorF2

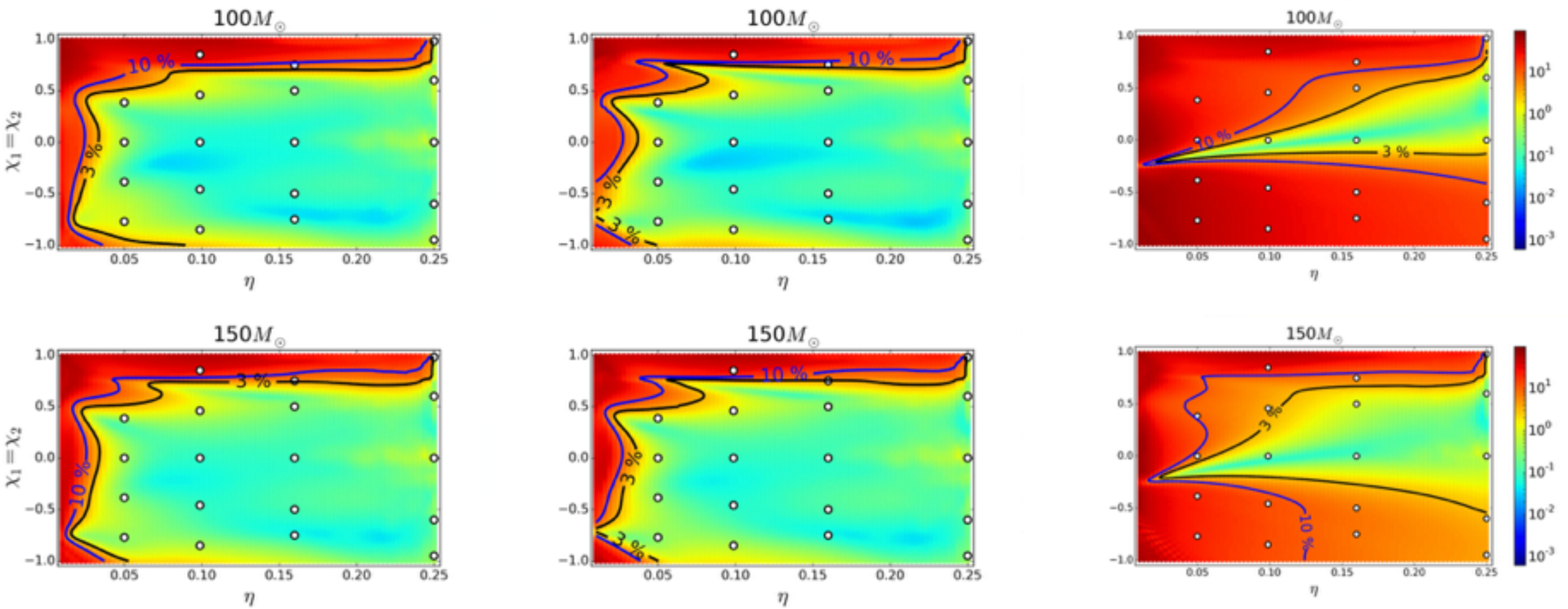
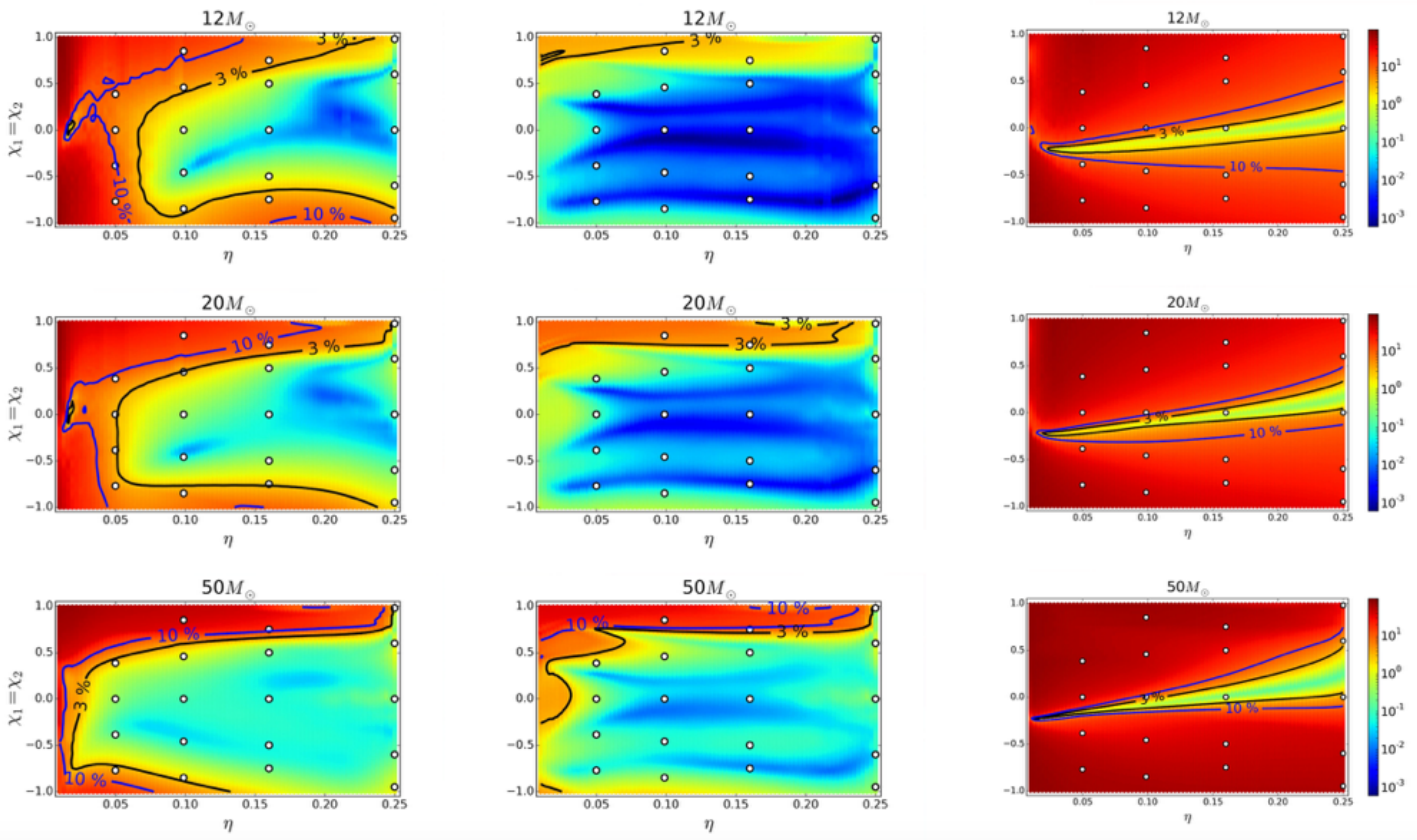
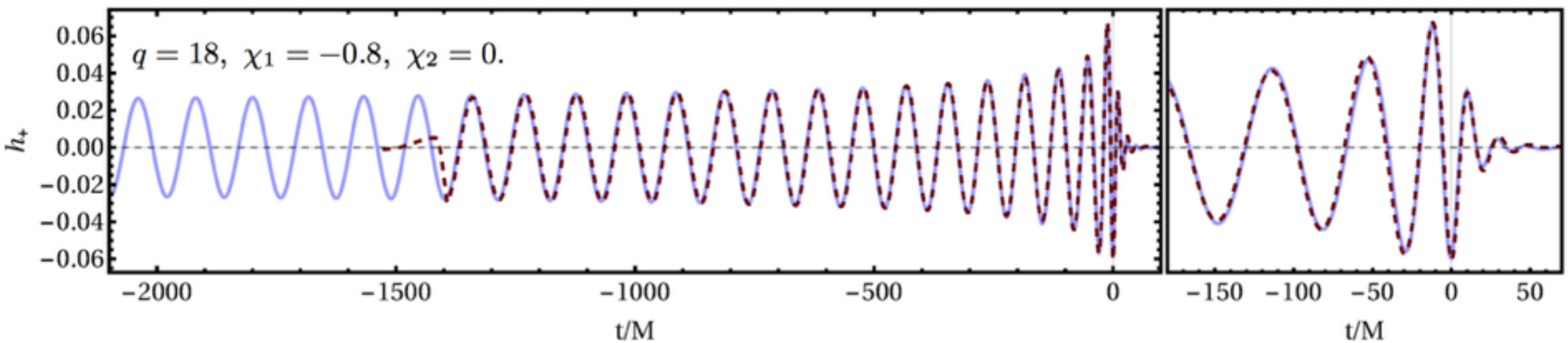


FIG. 18: Mismatch comparisons between the SEOBNRv2_ROM model, and three versions of PhenomD. Left: the final PhenomD model. Middle: SEOBNRv2_ROM is used for the inspiral part of PhenomD, i.e., up to $Mf = 0.018$. Right: TaylorF2 is used for the inspiral part of PhenomD. See text for discussion.

Matches (Faithfulness) vs. hybrids & between models



Time domain waveforms



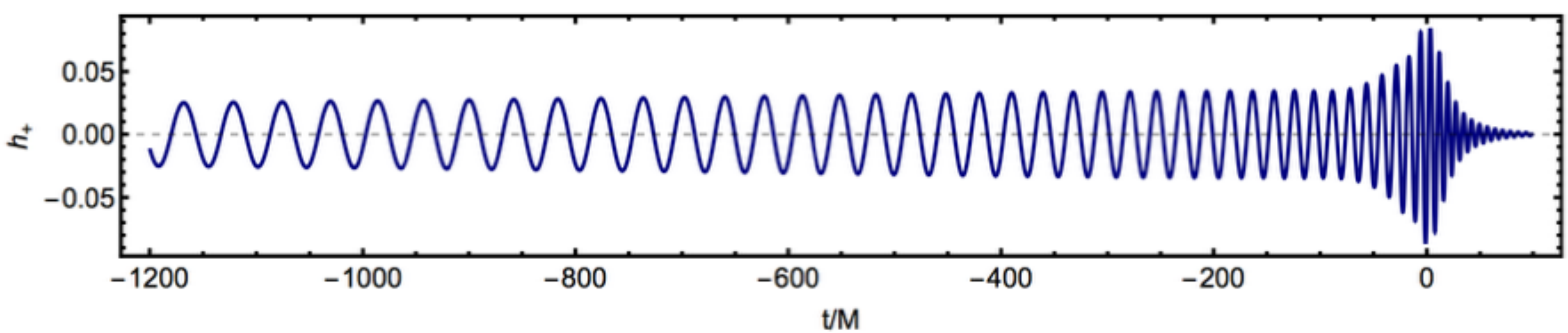


FIG. 21: Time-domain representation of the PhenomD model outside its calibration region, here for mass ratio 50 and spin parameters of $\chi_1 = \chi_2 = 0.99$.

Summary

- PhenomD: very accurate WFs in time & frequency domain.
  - Open source C implementation (LAL); Mathematica on request.
  - Builds upon EOB inspiral description & detailed study of WF anatomy.
- Phenom* & SEOBNR agree extremely well in their calibration regions.
  - Need more NR simulations for large spins || orbital ang. momentum.
- PhenomD is modular, e.g. inspiral and MRD can be tuned from different waveform sets, variations of Phen* models easy to generate.
  - Application to precession -> Mark's talk
  - Similar modifications may be possible for modGR, eccentricity …