Impact of other scalar fields on oscillons after hilltop inflation

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Outline

1. Preheating in single-field hilltop inflation
2. Oscillon formation
3. Parametric resonance of $\chi$ sourced by inhomogeneous $\phi$
4. Impact of $\chi$ on oscillons
5. Conclusions
\[ V(\phi) = V_0 \left( 1 - \frac{\phi^6}{v^6} \right)^2, \quad \text{with } v \ll m_{Pl} \]

\[ N_e \text{ e–folds} \]

\[ \eta(\phi_{\text{end}}) = -1 \]

\[ V_0 \]

\[ \phi_* \quad \phi_{\text{end}} \quad v \]

\text{tachyonic preheating}

\[ \Rightarrow \phi \text{ fluctuations grow for } k^2 + V_{\phi\phi} < 0 \]

- leads to formation of IR dominated spectrum
- ends when \( \phi = v \) for the first time
- for \( v \lesssim 10^{-5} m_{Pl} \), \( \langle \delta \phi^2 \rangle \sim v^2 \) during this phase
- for \( v \gtrsim 10^{-5} m_{Pl} \), tachyonic preheating is followed by coherent oscillations of the inflaton

\[ P_{\phi(k)}/v^2 \]

\[ k/(aH_i) \]
\[ V(\phi) = V_0 \left( 1 - \frac{\phi^6}{v^6} \right)^2 , \quad \text{with } v \ll m_{Pl} \]

\[ N_\ast \text{ e–folds} \]

before \( \phi_{\text{end}} \)

\[ \eta(\phi_{\text{end}}) = -1 \]

\[ V_0 \]

\[ \phi_* \]

\[ \phi_{\text{end}} \]

\[ \nu \]

tachyonic oscillations

⇒ \( \phi \) fluctuations grow around \( k_p/a \sim 200 \sqrt{V_0/3} \)

• leads to spectrum peaked at \( k_p \)

• for \( v \lesssim 10^{-1} m_{pl} \), \( \langle \delta\phi^2 \rangle \sim v^2 \) during this phase

⇒ for \( 10^{-5} \lesssim v/m_{pl} \lesssim 10^{-1} \), growth of fluctuations at \( k_p \) leads to formation of ”hill-crossing” oscillons, separated by \( \xi \sim 2\pi/k_p \)

• oscillons eventually settle around \( \phi = v \)
$V(\phi) = V_0 \left(1 - \frac{\phi^6 v}{v^6}\right)^2$, with $v = 10^{-2} m_{Pl}$.

Movie of $\rho$ from 2D simulation with $1024^2$ points.
Oscillons:

Spatially localized oscillating configurations of a real scalar field

- form when scalar field oscillates in potential that is shallower than quadratic away from the minimum
- they have a surprisingly long lifetime: stable for large number of oscillations, can survive for many $e$-folds
- arise in many inflationary models favoured by observations

Interesting and potentially observable consequences:

- generate gravitational waves when they form and when they decay
- affect the expansion history of the universe possibly leading to signatures in the CMB
- ...
\[ V(\phi, \chi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2} \phi^2 \chi^2 (\phi^2 + \chi^2), \quad \text{with } v = 10^{-2} m_{Pl} \]

\[ N_* \text{ e–folds before } \phi_{end} \]

\[ \eta(\phi_{end}) = -1 \]

parametric resonance of $\chi$ sourced by inhomogeneous $\phi$

$\Rightarrow$ $\chi$ fluctuations grow after $\delta \phi \ll \sqrt{\langle \delta \phi^2 \rangle}$
- exponential growth only if $\lambda$ is inside resonance band
\[ V(\phi, \chi) = V_0 \left( 1 - \frac{\phi^6}{v^6} \right)^2 + \frac{\chi^2}{2} \phi^2 \chi^2 (\phi^2 + \chi^2), \quad \text{with } v = 10^{-2} m_{Pl} \]

\[ N_* \text{ e-folds before } \phi_{\text{end}} \]
\[ \eta(\phi_{\text{end}}) = -1 \]

parametric resonance of \( \chi \) sourced by inhomogeneous \( \phi \)

\( \Rightarrow \) \( \chi \) fluctuations grow after \( \delta \phi \ll \sqrt{\langle \delta \phi^2 \rangle} \)

- exponential growth only if \( \lambda \) is inside resonance band
- \( \chi \) develops spectrum peaked at \( k \sim k_p \)

\( \Rightarrow \) what is the impact of this resonance on oscillons?
\[ V(\phi, \chi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2} \phi^2 \chi^2 (\phi^2 + \chi^2), \]

with \( v = 10^{-2} m_{Pl} \)

Consider three cases:

**I** no resonance

\[ \Rightarrow \lambda \text{ outside resonance band} \]

\( \langle \delta \chi^2 \rangle \) not amplified and its vacuum fluctuations redshift

**II** ”slow” resonance

\[ \Rightarrow \lambda \text{ inside resonance band} \]

\( \langle \delta \chi^2 \rangle \) amplified \( \sim 1 \) e-fold after \( \phi \) has formed oscillons

**III** ”fast” resonance

\[ \Rightarrow \lambda \text{ inside resonance band} \]

\( \langle \delta \chi^2 \rangle \) amplified during initial phase of oscillons formation
\[ V(\phi, \chi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 + \frac{\chi^2}{2} \phi^2 \chi^2 (\phi^2 + \chi^2), \quad \text{with } v = 10^{-2} m_{\text{Pl}} \]

In what follows: results of lattice simulations of cases I, II and III
⇒ solve system of equations:

\[
\begin{align*}
\ddot{\phi}(t, \bar{x}) + 3H \dot{\phi}(t, \bar{x}) - \frac{1}{a^2} \bar{\nabla}^2 \phi(t, \bar{x}) + \frac{\partial V}{\partial \phi} &= 0 \\
\ddot{\chi}(t, \bar{x}) + 3H \dot{\chi}(t, \bar{x}) - \frac{1}{a^2} \bar{\nabla}^2 \chi(t, \bar{x}) + \frac{\partial V}{\partial \chi} &= 0
\end{align*}
\]

\[ H^2 \equiv \frac{\langle \rho \rangle}{3m_{\text{Pl}}^2} = \frac{1}{3m_{\text{Pl}}^2} \left\langle V + \sum_f \left( \frac{1}{2} f^2 + \frac{1}{2a^2} |\bar{\nabla} f|^2 \right) \right\rangle \]

on discretized lattice with parameters:

<table>
<thead>
<tr>
<th>( v/m_{\text{Pl}} )</th>
<th>( \langle \phi \rangle_i/v )</th>
<th>( \langle \dot{\phi} \rangle_i/v^2 )</th>
<th>( \langle \chi \rangle_i/v )</th>
<th>( \langle \dot{\chi} \rangle_i/v^2 )</th>
<th>( H_i/m_{\text{Pl}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-2} )</td>
<td>0.08</td>
<td>2.49 \times 10^{-9}</td>
<td>0</td>
<td>0</td>
<td>1.9 \times 10^{-10}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D</th>
<th>N</th>
<th>( k_{uv} )</th>
<th>( k_{ir} )</th>
<th>( \delta x )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1024</td>
<td>6.2 \times 10^4 H_i</td>
<td>60.5 H_i</td>
<td>10^{-4}/H_i</td>
<td>0.1/H_i</td>
</tr>
</tbody>
</table>
case I: no parametric resonance

\[ \lambda = 1.5 \times 10^{-3}/m_{\text{pl}}, \rho \text{ slice at } a = 10.0 \]

\[ \lambda = 1.5 \times 10^{-3}/m_{\text{pl}}, \rho \text{ slice at } a = 22.0 \]

- at \( a = 10 \): many oscillons still present
- at \( a = 22 \): less oscillons

\[ \Rightarrow \] they start decaying before the end of the simulation

without parametric resonance of \( \chi \) the evolution of oscillons is equal to single-field case
case II: ”slow” parametric resonance

\[ \lambda = 1.37 \times 10^{-3}/m_{pl}, \rho \text{ slice at } a=10.0 \]

\[ \lambda = 1.37 \times 10^{-3}/m_{pl}, \rho \text{ slice at } a=22.0 \]

- at \( a = 10 \): many oscillons still present, as in case I
- at \( a = 22 \): more oscillons, more pronounced

\[ \Rightarrow \] ”slow” parametric resonance of \( \chi \) enhances the oscillons formed by \( \phi \)
**case II:** "slow" parametric resonance

\[ \lambda = 1.37 \times 10^{-3}/m_{pl}, \ \phi \ \text{slice at} \ a = 22.0 \]

\[ \lambda = 1.37 \times 10^{-3}/m_{pl}, \ \chi \ \text{slice at} \ a = 22.0 \]

- at \( a = 10 \): many oscillons still present, as in case I
- at \( a = 22 \): more oscillons, more pronounced
  
  \( \Rightarrow \) \( \phi \) and \( \chi \) are correlated: the oscillons get imprinted in the \( \chi \) field

\( \Rightarrow \) "slow" parametric resonance of \( \chi \) enhances the oscillons formed by \( \phi \)

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case **III**: "fast" parametric resonance

\[ \lambda = 10^{-3}/m_{pl}, \rho \text{ slice at } a = 10.0 \]

\[ \lambda = 10^{-3}/m_{pl}, \rho \text{ slice at } a = 22.0 \]

- at \( a = 10 \): no oscillons
- at \( a = 22 \): no oscillons

\[ \Rightarrow \] "fast" parametric resonance of \( \chi \) suppresses the oscillons formed by \( \phi \)
case I: no parametric resonance

\[ \lambda = 1.5 \times 10^{-3}/m_{pl}, \quad \rho \text{ slice at } a=10.0 \]

\[ \lambda = 1.5 \times 10^{-3}/m_{pl}, \quad \rho \text{ slice at } a=22.0 \]

\[ \rho \text{ tail distribution for } \lambda = 1.5 \times 10^{-3}/m_{pl} \]

**tail distribution**: number of lattice points with energy density \( \rho \)

in grey: tail distribution of energy \( \rho_g = m_\phi^2 g_1^2/2 + m_\chi^2 g_2^2/2 \),

where \( g_1 \) and \( g_2 \) are discrete Gaussian fields with variances \( \langle g_1^2 \rangle = \langle \phi^2 \rangle \) and \( \langle g_2^2 \rangle = \langle \chi^2 \rangle \)
case I: no parametric resonance

- at $a = 10$: tail distribution stretches to $\rho \sim 300\langle \rho \rangle$, far away from Gaussian tail
  ⇒ expected when oscillons are present
- at $a = 22$: tail distribution shrunk, but still more spread than Gaussian
case II: ”slow” parametric resonance

- at $a = 10$: tail distribution stretches to $\rho \sim 700 \langle \rho \rangle$, longer than case I tail
  $\Rightarrow$ more energetic oscillons are present
- at $a = 22$: tail distribution spread to $\rho \sim 3000 \langle \rho \rangle$ but lower around $\rho \sim 100 \langle \rho \rangle$
case III: ”fast” parametric resonance

- **at $a = 10$:** tail distribution close to Gaussian tail
- **at $a = 22$:** tail distribution close to Gaussian tail

$\Rightarrow$ no oscillons are formed and field fluctuations close to Gaussian
The evolution of $\rho$ in cases I, II and III is shown in the graph. The line colors and markers correspond to:

- Blue line: $\lambda = 1.5 \times 10^{-3}/m_{pl}$
- Green line: $\lambda = 1.37 \times 10^{-3}/m_{pl}$
- Red line: $\lambda = 10^{-3}/m_{pl}$
- Black dashed line: $\lambda = 0$
- Gray dashed line: Gaussian

$\rho_{0.1}$ for which 10% of the energy is stored in regions with $\rho > \rho_{0.1}$

$\Rightarrow$ larger $\rho_{0.1}$ indicates that more energy is in regions with larger $\rho$

$\Rightarrow$ more oscillons

I no resonance case as single field case (dashed black):

$\rho_{0.1}$ grows until $a = 10$, then decreases

II "slow" resonance leads to more energy stored in oscillons

III "fast" resonance suppresses the formation of oscillons, statistics close to Gaussian

$\Rightarrow$ the timing of the resonance of $\chi$ determines whether the oscillons are enhanced or suppressed
Summary:

- $\phi$ oscillons form after hilltop inflation, during the phase of tachyonic oscillations.
- When another scalar field $\chi$ is present, a parametric resonance of $\chi$ sourced by the inhomogeneous $\phi$ can happen.
- Depending on the occurrence and timing of the resonance, $\chi$ can have different impacts on the oscillons:
  
  I. If no resonance, $\chi$ has no effect on oscillons $\Rightarrow$ as single field case.
  
  II. “Slow” resonance: $\chi$ amplified $\sim 1$ e-fold after $\phi$ oscillons have formed.
    $\Rightarrow$ oscillons are enhanced, their lifetime extended.
  
  III. “Fast” resonance: $\chi$ amplified during formation of oscillons.
    $\Rightarrow$ oscillons are suppressed, and the fields have statistics close to Gaussian.
Summary:

- \( \phi \) oscillons form after hilltop inflation, during the phase of tachyonic oscillations
- when another scalar field \( \chi \) is present, a parametric resonance of \( \chi \) sourced by the inhomogeneous \( \phi \) can happen
- depending on the occurrence and timing of the resonance, \( \chi \) can have different impacts on the oscillons:
  - I if no resonance, \( \chi \) has no effect on oscillons \( \Rightarrow \) as single field case
  - II "slow" resonance: \( \chi \) amplified \( \sim 1 \) e-fold after \( \phi \) oscillons have formed
    \( \Rightarrow \) oscillons are enhanced, their lifetime extended
  - III "fast" resonance: \( \chi \) amplified during formation of oscillons
    \( \Rightarrow \) oscillons are suppressed, and the fields have statistics close to Gaussian

Outlook:

- oscillons affect the expansion history of the universe: effect on equation of state, delay thermalization
- their evolution depends on \( \chi \), light degree of freedom during inflation
  \( \Rightarrow \) possible effect on CMB scales via dependence on \((\phi_*, \chi_*)\)
- production of gravitational waves during formation and decay of oscillons
- ...