Aspects of infrared non-local modifications of General Relativity

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Based on:

- Jaccard, Maggiore, Mitsou, PRD 2013, 1305.3034
- Maggiore, PRD 2014, 1307.3898
- Foffa, Maggiore, Mitsou, PLB 2014, 1311.3421, & IJMPA 2014, 1311.3435
- Maggiore and Mancarella, PRD 2014, 1402.0448
- Dirian, Foffa, Khosravi, Kunz, Maggiore, JCAP 2014, 1403.6068
- Dirian, Mitsou, JCAP 2014, 1408.5058
- Dirian, Foffa, Kunz, Maggiore, Pettorino, JCAP 2015, 1411.7692
- GC, Fumagalli, Maggiore, JHEP 1409, 1407.5580
- Maggiore, 1506.06217
- GC, Foffa, Maggiore, Mancarella, 1512.????

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Outline

- general idea/inspiration behind IR non-local modifications of GR
- how to interpret (time) non-localities
- localization methods – how to count physical dofs
- subtleties in the stability analysis
- possible connections with fundamental local quantum theories

Illustrative purposes: action-based model

$$S = EH + m^2 R\Box^{-2}R$$

$$G_{\mu\nu} - m^2\left(g_{\mu\nu}\Box^{-1}R\right)^T = 0$$ Maggiore et al. 2014

$$S = EH + m_1^2 R\Box^{-2}R + m_2^2 C_{\mu\nu\alpha\beta}\Box^{-2}C^{\mu\nu\alpha\beta} + m_3^2 R_{\mu\nu}\Box^{-2}R^{\mu\nu}$$ GC et al. coming out

General ideas behind IR non-local modifications of GR

Inspiration: non-local formulation Proca-theory

$$\left(1-\frac{m^2}{\Box}\right)\partial_\mu F^{\mu\nu}=j^\nu \quad \rightarrow \quad \text{filtered source} \quad \left(1-\frac{m^2}{\Box}\right)^{-1} j^\nu$$

- $k \gg m \rightarrow$ filter ~ 1 $-$ $k \ll m \rightarrow$ filter $\ll 1$
- filter $\equiv$ Yukawa suppression e^{-mr}/r^2

General ideas behind IR non-local modifications of GR

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Is it possible to do the same with (massive) gravity? $\rightsquigarrow$ degravitation! Dvali et al. 2007

$$\left(1-\frac{m^2}{\Box}\right) G_{\mu\nu}=8\pi G\, T^{\mu\nu} \quad \rightarrow \quad \text{filtered source} \quad \left(1-\frac{m^2}{\Box}\right)^{-1} T^{\mu\nu}$$

- observed Λ: small because "filtered"
- m^2 small ($\sim \mathcal{H}_0$) but technically natural

Our illustrative model

Maggiore-Mancarella model

Maggiore, Mancarella [1402.0448]

$$S = \frac{1}{16\pi G}\int d^{d+1}x\,\sqrt{-g}\left[R - \frac{m^2}{2}R\frac{1}{\Box^2}R\right],$$

where $\Box^{-1}$ is a formal inverse of $\Box$ in the scalar representation

$$\left(\Box^{-1}R\right)(x) = \cancel{hom} + \int d^{d+1}y\,\sqrt{-g(y)}\mathcal{G}(x,y)R(y)\,, \quad \Box_x\mathcal{G}(x,y) = \frac{\delta^{(d+1)}(x-y)}{\sqrt{-g(x)}}\,.$$

Further prescriptions (theory data, part of the model definition)

(1) $\mathcal{G}(x,y) = 0\,,$ unless y is in the past light cone of x

(2) $\mathcal{G}(x,y)_{x^0=t^0} = 0\,, \quad \partial_0\mathcal{G}(x,y)_{x^0=t^0} = 0\,.$

Our illustrative model: prescriptions on the Green function

(1) to ensure causality: $\mathcal{G}(x,y) \to \mathcal{G}_{ret}(x,y)$

- causality implemented by hand on *eom*: $G_{\mu\nu} + K_{\mu\nu}|_{ret} = 8\pi G T_{\mu\nu}$

$$\text{e.g.} \quad \frac{\delta}{\delta\phi(x)} \int d^4x' \, \phi(x') \left(\Box^{-1}\phi\right)(x') = \int d^4x' \left[\mathcal{G}(x,x') + \mathcal{G}(x',x)\right] \phi(x')$$

- diff. invariant action $\to \nabla_\mu T^{\mu\nu} = 0$ (not spoiled by causality prescription)

(2) initial conditions: $\mathcal{G}(x,y)_{x^0=t^0} = 0\,, \quad \partial_0 \mathcal{G}(x,y)_{x^0=t^0} = 0\,,$

- EFT valid below given energy scale ($\leftrightarrow$ after some t_0)
- for models $\Box^{-1}R$ cosmology independent on t_0 in radiation

$$\text{e.g. FRW} \quad (\Box^{-1}R)(t) = -\int_{t_0}^{t} dt' \, \frac{1}{a^d(t')} \int_{t_0}^{t'} dt'' \, a^d(t'') R(t'')$$

Good properties of the model

This illustrative model has some (very) appealing features

- simple: same number of parameters of ΛCDM (+ some prescriptions)
- predictive: once m^2 chosen to reproduce $\Omega_{DE}(0)$, all the rest is fixed
- phenomenologically viable
  - absence vDVZ discontinuity: GR recovered on small scales
  - stable linear perturbations — Dirian et al. [1403.6068], [1411.7692]
  - comparison with Planck 2013: statistically equivalent to ΛCDM

Localizing fields introduced making use of Lagrange multipliers Maggiore et al.[1402.0448]

$$S_{loc} = \frac{1}{16\pi G}\int d^{d+1}x\,\sqrt{-g}\left[R\left(1-\frac{m^2}{2}S\right)-\xi_1\left(\Box U+R\right)-\xi_2\left(\Box S+U\right)\right],$$

$$U = -\Box^{-1}R = -\int d^{d+1}y\,\sqrt{-g(y)}\,\mathcal{G}(x,y)R(y) + \cancel{U_{hom}}\,,$$

$$S = -\Box^{-1}U = \Box^{-2}R = -\int d^{d+1}y\,\sqrt{-g(y)}\,\mathcal{G}(x,y)U(y) + \cancel{S_{hom}}\,,$$

with initial conditions inredited from the prescriptions on the Green function

$$U(t_0) = \dot{U}(t_0) = S(t_0) = \dot{S}(t_0) = 0\,.$$

An alternative localization

Introducing a new scalar field Φ

$$S_{loc} = \int d^{d+1}x\, \sqrt{-g} \left[M \Phi R + \frac{1}{2m^2} \left(\Box \Phi \right)^2 \right] ,$$

$$\Phi(t_0) = M\,, \quad \dot{\Phi}(t_0) = \ddot{\Phi}(t_0) = \dddot{\Phi}(t_0) = 0 \quad \Leftrightarrow \quad \Phi = M \left(1 - \frac{m^2}{\Box^2} R \right)$$

Einstein frame, field redefinitions (...)

$$S_{loc} = \int d^{d+1}x\, \sqrt{-\tilde{g}} \left[M^2 \tilde{R} - \frac{1}{2} \tilde{\nabla}_\mu \phi \tilde{\nabla}^\mu \phi + \frac{1}{2} \tilde{\nabla}_\mu \psi \tilde{\nabla}^\mu \psi - \frac{1}{2} m^2 \psi^2\, e^{-\frac{2(\phi+\psi)}{M}} \right] ,$$

$$\phi(t_0) = \dot{\phi}(t_0) = \psi(t_0) = \dot{\psi}(t_0) = 0\,.$$

ϕ and ψ are dynamical fields but with fixed initial conditions $\rightsquigarrow$ no new dofs!!

Spurious dofs and stability analysis

- Kinetic term of ψ has wrong sign $\rightsquigarrow$ is classical stability screwed up?

- Localizing fields have to be handled in special way in stability analysis

- In other terms: "standard" stability criterium . . .

  Chosen a background, perturbations are stable if under a small change of initial conditions, the solutions stay "close" to the original ones

  . . . can not be applied in this case

"spurious" ghost is not a priory dangerous for the classical stability:
direct investigation of perturbations needed!

Stability on Minkowski & cosmological background

Minkowski:

- ψ perturbation $\delta\psi \sim \exp\sqrt{(m^2 - k^2)t}$
- time scale instability $\Delta t \sim m^{-1} \sim \mathcal{H}_0^{-1}$
- cosmologically relevant scales $k \gg m \sim \mathcal{H}_0$ stable (see absence vDVZ)

FLRW:

- the instability of "spurious ghost" is power low (sets in at z_{eq} for $\Box^{-1}R$)
- matter/metric perturbations very close to ΛCDM ones Dirian et al. [1403.6068]
- why? background evolves to $w_{DE} < -1$: Hubble friction stabilizes pert.

Where could non-localities come from?

Model comes from an action ($+$ prescriptions): class. limit of quantum theory?

(1) Difficulties of promoting the model to a quantum theory

(2) Non-localities: quantum correction to gravity effective action?

(3) Non-localities could come from RG-flow couplings in UV-completed GR

(1) Can the (linearized) action be considered a quantum action?

how to embed prescriptions on the Green functions in quantum context?

$$\langle h_{\mu\nu}(k) h_{\alpha\beta}(k)\rangle_S = \text{massless graviton} - \frac{\eta_{\mu\nu}\eta_{\rho\sigma}}{d(d-1)}\left(\frac{i}{k^2 - i\epsilon} + \frac{i}{-k^2 + m^2 - i\epsilon}\right)$$

where $i\epsilon$ prescription chosen in such a way to have convergent path integral

$$\langle h_{\mu\nu} h_{\alpha\beta}\rangle_S = \frac{\int Dh \ldots e^{iS}}{\int Dh \, e^{iS}}$$

but. . . path integral does not know about interpretation of extra poles. . .

Where could non-localities come from?

(2) Quantum correction to gravity effective action Γ?

non-local terms from matter and gravity loops have complex structure ...

e.g. anomaly induced

$$S_{an} = -\frac{1}{8}\int d^4x\sqrt{-g}\left(E-\frac{2}{3}\Box R\right)\Delta_4^{-1}\left[b\left(E-\frac{2}{3}\Box R\right)-2bC^2\right]$$

where

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}\left(\nabla^\mu R\right)\nabla_\mu$$

(3) Non-localities could come from RG-flow couplings in UV-completed GR

e.g. Stelle theory $\sim M^2 R + aR^2 + bC^2$ Maggiore [1506.06217]

- if a UV asymptotically free, QCD-like situation: IR generation mass scale
- non-perturbative structure IR QCD propagator: mass for gluon
- IR pole $a(\Box) \propto \Box^{-2}$: dynamical generation of a mass for conformal mode

e.g. strong IR effects $EH+$anomaly induced

Summarizing

(e.g. $EH + m^2 R\Box^{-2}R\,,\; G_{\mu\nu} - m^2\left(g_{\mu\nu}\Box^{-1}R\right)^T = 0$)

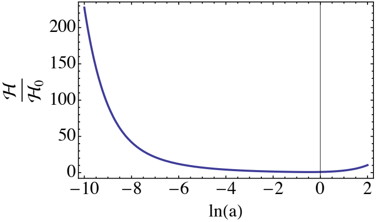
Recently proposed class of non-local models with appealing features:

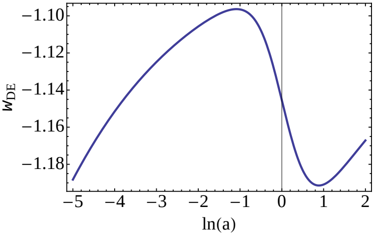
- same number of free parameters as ΛCDM
- developed techniques to deal with non-localities
- cosmological perturbations stable
- models statistically equivalent to ΛCDM

Open questions:

- low energy classical EFT...
- which is the underlying fundamental quantum field theory?
- which is the quantum origin of non-localities?

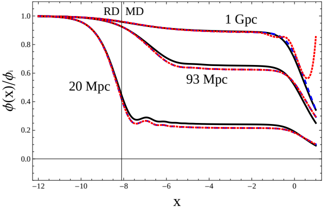
Thank you!





$EH + R(\Box^{-2})R$

$EH + R(\Box^{-2})R + C_{\mu\nu\alpha\beta}(\Box^{-2})C^{\mu\nu\alpha\beta}$



GC et al. [1512.????]