Testing varying speed of light cosmologies in future experiments.

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28th Texas Symposium on Relativistic Astrophysics, 17 December 2015
Based on:

- V. Salzano, A. Balcerzak, MPD - in progress
Varying speed of light $c$ (VSL) theories

**Attempts:** Einstein (1907), Dicke (1957), J.-P. Petit (1988) (Einstein eqs remain same due to fine-tuned change of $c$ and $G$), Moffat (1992).

**Albrecht & Magueijo model (1998)** (AM model) (Barrow 1999; Magueijo 2003):

Introduce a scalar field

$$c^4 = \psi(x^\mu)$$ (1)

and so the action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right]$$ (2)

AM model **breaks Lorentz invariance** (relativity principle and light principle) so that there is a preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity. The Riemann tensor is computed in such a frame for a constant $\psi = c^4$ and no additional terms $\partial_\mu \psi$ appear in this frame (though they do in other frames). **Einstein eqs remain the same except $c$ now varies.**
VSL and varying fine structure constant $\alpha$ theories

Due to the definition of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \quad \text{i.e.} \quad \alpha(t) = \frac{e^2}{\hbar c(t)}, \quad \frac{\Delta \alpha}{\alpha} = -\frac{\Delta c}{c}, \quad (3)$$

VSL theories can be related with varying fine structure constant $\alpha$ (or charge $e = e_0 \epsilon(x^\mu)$) theories (Webb et al. 1999, Sandvik 2002))

$$S = \int d^4 x \sqrt{-g} \left( R - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} + L_m \right) \quad (4)$$

with $\psi = \ln \epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$. Constraints:

Oklo natural nuclear reactor: $\Delta \alpha / \alpha = (0.15 \pm 1.05) \cdot 10^{-7}$ at $z = 0.14$

VLT/UVES quasars: $\Delta \alpha / \alpha = (0.15 \pm 0.43) \cdot 10^{-5}$ at $1.59 < z < 2.92$

SDSS quasars: $\Delta \alpha / \alpha = (1.2 \pm 0.7) \cdot 10^{-4}$ at $0.16 < z < 0.8$

Atomic clocks (Rosenband (2008)) at $z = 0$: $\left( \frac{\dot{\alpha}}{\alpha} \right)_0 = (-1.6 \pm 2.3) \times 10^{-17} \text{yr}^{-1}$. 

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More bounds on variation of $\alpha$.

By Webb et al. (PRL 107, 191101 (2011)) ($\alpha$-dipole $R.A.17.4 \pm 0.9h$, $\delta = -58 \pm 9$: Keck ($\Delta \alpha < 0$) and VLT) as well as other specific measurements of $\alpha$ given in the table below (in parts per million):

<table>
<thead>
<tr>
<th>Object</th>
<th>$z$</th>
<th>$\Delta \alpha/\alpha$</th>
<th>Spectrograph</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE0515–4414</td>
<td>1.15</td>
<td>$-0.1 \pm 1.8$</td>
<td>UVES</td>
<td>Molaro et al. (2008)</td>
</tr>
<tr>
<td>HE0515–4414</td>
<td>1.15</td>
<td>$0.5 \pm 2.4$</td>
<td>HARPS/UVES</td>
<td>Chand et al. (2006)</td>
</tr>
<tr>
<td>HE0001–2340</td>
<td>1.58</td>
<td>$-1.5 \pm 2.6$</td>
<td>UVES</td>
<td>Agafonowa et al. (2011)</td>
</tr>
<tr>
<td>HE2217–2818</td>
<td>1.69</td>
<td>$1.3 \pm 2.6$</td>
<td>UVES–LP</td>
<td>Molaro et al. (2013)</td>
</tr>
<tr>
<td>Q1101–264</td>
<td>1.84</td>
<td>$5.7 \pm 2.7$</td>
<td>UVES</td>
<td>Molaro et al. (2008)</td>
</tr>
</tbody>
</table>

UVES - Ultraviolet and Visual Echelle Telescope
HARPS - High Accuracy Radial velocity Planet Searcher
LP - Large Program measurement
VSL - simple generalization of the Einstein equations.

Einstein eqs. the same except $c$ now varies - ($\rho$ - mass density; $\varepsilon = \rho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

\[ \rho(t) = \frac{3}{8\pi G} \left( \frac{\ddot{a}^2}{a^2} + \frac{k c^2(t)}{a^2} \right), \quad (5) \]

\[ p(t) = -\frac{c^2(t)}{8\pi G} \left( 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k c^2(t)}{a^2} \right), \quad (6) \]

but the continuity eq. contains an extra term (obtained from (5) and (6))

\[ \dot{\rho}(t) + 3\frac{\dot{a}}{a} \left( \rho(t) + \frac{p(t)}{c^2(t)} \right) = 3\frac{k c(t)\dot{c}(t)}{4\pi G a^2}. \quad (7) \]

Benefits:

Can solve the horizon, flatness and singularity problems.

Can mimic dark energy (see later).
Tests in future experiments.

1. **Redshift drift** (Sandage 1962, Loeb 1998) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.

There is a relation between the times of emission of light by the source $\tau_e$ and $\tau_e + \Delta \tau_e$ and times of their observation at $\tau_o$ and $\tau_o + \Delta \tau_o$ (VSL adopted):

$$\int_{t_e}^{t_o} \frac{c(t)dt}{a(t)} = \int_{t_e + \Delta t_e}^{t_o + \Delta t_o} \frac{c(t)dt}{a(t)} ,$$

which for small $\Delta t_e$ and $\Delta t_o$ transforms into

$$\frac{c(t_e)\Delta t_e}{a(t_e)} = \frac{c(t_0)\Delta t_o}{a(t_o)} .$$

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Redshift drift - varying $c$

Assuming the ansatz for the variability of the speed of light

$$c(t) = c_0 a^n(t), \quad n = \text{const.},$$  \hspace{1cm} (9)

and bearing in mind definitions of $\Omega$’s, assuming flat $k = 0$ model we have

$$\frac{\Delta z}{\Delta t_0} = H_0 \left[ 1 + z - (1 + z)^n \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_\Lambda} \right]$$  \hspace{1cm} (10)

which can further be rewritten to define new Hubble function ($w_{eff} = w_i + \frac{2}{3}n$)

$$\tilde{H}(z) \equiv (1 + z)^n H(z) = H_0 \sqrt{\sum_{i=1}^{i=k} \Omega_{wi}(1 + z)^3(w_{eff}+1)}.$$  \hspace{1cm} (11)
Redshift drift test - varying $c$

The VSL redshift drift effect for 15 year period of observations (Balcerzak, MPD 2014).
Redshift drift test - varying $c$

- If $n < 0$ ($c$ decreases) then dust matter becomes little negative pressure matter and the cosmological constant became phantom. **Varying $c$ mimics dark energy.**

- If $n > 0$ then (growing $c(t)$) VSL model becomes more like Cold Dark Matter (CDM) model.

- Theoretical error bars are taken from Quercellini et al. 2012 and presumably show that for $|n| < 0.045$ – **one cannot distinguish between VSL models and $\Lambda$CDM models.**

- Future observations:
  - European Extremely Large Telescope (EELT) (with its spectrograph CODEX (COsmic Dynamics EXperiment)), Thirty Meter Telescope (TMT), the Giant Magelllan Telescope (GMT)
  - **gravitational wave interferometers** DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer). Detection even at $z \sim 0.2$. 

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2. Measuring \( c \) with baryon acoustic oscillations (BAO)

Speed of light \( c \) appears in many observational quantities. Among them in the angular diameter distance

\[
D_A = \frac{D_L}{(1 + z)^2} = \frac{a_0}{1 + z} \int_{t_1}^{t_2} \frac{c(t) dt}{a(t)}
\]

(12)

where \( D_L \) is the luminosity distance, \( a_0 \) present value of the scale factor (normalized to \( a_0 = 1 \) later), and we have taken the spatial curvature \( k = 0 \) (otherwise there would be \( \sin \) or \( \sinh \) in front of the integral). Using the definition of redshift and the dimensionless parameters \( \Omega_i \) we have

\[
D_A = \frac{1}{1 + z} \int_0^z \frac{c(z) dz}{H(z)},
\]

(13)

where

\[
H(z) = \sqrt{\Omega_{r0}(1 + z)^4 + \Omega_{m0}(1 + z)^3 + \Omega_{\Lambda}}.
\]

(14)
Angular diameter distance maximum.

Due to the expansion of the universe, there is a maximum of the distance at

\[ D_A(z_m) = \frac{c(z_m)}{H(z_m)}. \]  

(15)

which can be obtained by simple differentiating (13) with respect to \( z \):

\[ \frac{\partial D_A}{\partial z} = -\frac{1}{(1+z)^2} \int_0^z \frac{c(z)dz}{H(z)} + \frac{1}{1+z} \frac{c(z)}{H(z)} = 0 \]  

(16)

In a flat \( k = 0 \) cold dark matter CDM model

\[ z_m = 1.25 \quad \text{and} \quad D_A \approx 1230 \quad \text{Mpc} \]  

(17)

For standard \( \Lambda \)CDM model of our interest:

\[ 1.4 < z_m < 1.8. \]  

(18)


**$D_A$ versus $H(z)$**

**The point:** The product of $D_A$ and $H$ gives **exactly** the speed of light $c$ at maximum (the curves intersect at $z_m$):

$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \text{ kms}^{-1}$$ \hspace{1cm} (19)

if we believe it is constant! (defined officially www.bipm.org; a relative error $10^{-9}$ by Evenson et al. 1972)

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The method to measure $c$ - cosmic "rulers" and "chronometers".

In fact what we end in the relation (Salzano, MPD, Lazkoz 2015):

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}.$$  \hspace{1cm} (20)

in which $D_A$ with the dimension of length plays the role of a “cosmic ruler”, and $1/H$ giving the dimension of time plays the role of a “cosmic clock/chronometer”. The method is then:

- Measure independently $D_A(z)$ and $H(z)$.
- Calculate $z_m$.
- Calculate the product $D_A(z_m)H(z_m) = c(z_m)$.
- If $c(z_m)$ is not be equal to $c_0$, then one measures the deviation from $c_0$, i.e. $\Delta c = c(z_m) - c_0$. 

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Measuring $z_m$

Measuring $z_m$ problematic if one uses $D_A$ only (large plateau around $z_m$ makes it difficult to avoid errors from small sample of data – besides, one has binned data, observational errors, and intrinsic dispersion).

However, one can appeal to an independent measurement of $c_0/H(z)$ which is the radial (line-of-sight) mode of the baryon acoustic oscillations surveys (BAO) for which $D_A(z)$ is the tangential mode (e.g. Nesseris et al. 2006). In other words, we have both tangential and horizontal modes as

$$y_t = \frac{D_A}{r_s} \quad y_r = \frac{c}{Hr_s},$$

(21)

where

$$r_s = \int_{z_{dec}}^{\infty} \frac{cc_s(z)dz}{H(z)}$$

(22)

is the sound horizon size at decoupling and $c_s$ the speed of sound.
The scenarios.

Take background $\Lambda$CDM model with an ansatz (Magueijo 2003)

$$c(a) \propto c_0 \left(1 + \frac{a}{a_c}\right)^n$$

(23)

where $a_c$ is the scale factor at the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \rightarrow c_0$ (at late times to now).

Three scenarios (Salzano, MPD, Lazkoz 2015):

1) standard case $c = c_0$;

2) $a_c = 0.005$, $n = -0.01 \rightarrow \Delta c/c \approx 1\%$ at $z \propto 1.5$;

3) $a_c = 0.005$, $n = -0.001 \rightarrow \Delta c/c \approx 0.1\%$ at $z \propto 1.5$. 
Based on $10^3$ Euclid project (Laureijs et al. 0912.0914) mock data simulations (Font-Ribeira et al. 2014):

1) $z_m = 1.592^{+0.043}_{-0.039}$ (fiducial model input $z_m = 1.596$) and $c/c_0 = 1 \pm 0.009$

2) $z_m = 1.528^{+0.038}_{-0.036}$ (fiducial $z_m = 1.532$) and $c(z_m)/c_0 = 1.00925 \pm 0.00831$

and

$$< c(z_m)/c_0 - 1\sigma c(z_m)/c_0 > = 1.00094^{+0.00014}_{-0.00033}$$ (24)

so that **a detection by Euclid of 1\% variation at 1\sigma-level will be possible.**

3) $z_m = 1.584^{+0.042}_{-0.039}$ (fiducial $z_m = 1.589$) and $c(z_m)/c_0 = 1.00095 \pm 0.00852$

and

$$< c(z_m)/c_0 - 1\sigma c(z_m)/c_0 > = 0.99243^{+0.00016}_{-0.00013}$$ (25)

so that **a detection by Euclid of 0.1\% variation at 1\sigma-level will not be possible.**
The results - Matérn(9/2) function approach.

- Better estimation of errors (Seikel at al. arXiv: 1311.6678)

- To determine $H(z)$ the cosmic chronometers (passively-evolving early-type galaxies - Jimenez, Loeb Ap.J. 573, 37 (2002)) to directly measure $\Delta t$ and $\Delta z$ have been used i.e. $H(z) = -\Delta z/(1+z)\Delta t$

- Euclid will not detect at $1\sigma$ (errors larger than in GP approach)

- Possible measurement of 1% of variation of $c$ by Square Kilometer Array (SKA) even at $3\sigma$ - errors on $z_M$ about 30% smaller than from Euclid.

- Measurement of 0.1% variation is possible at $1\sigma$ level, but needs reduction of errors by a factor of 10.
### The results - Euclid vs SKA.

<table>
<thead>
<tr>
<th>$\Delta c/c_0$</th>
<th>$z_M$</th>
<th>$c(p_{&gt;1})$</th>
<th>$c_{1\sigma}(p_{&gt;1})$</th>
<th>$c_{2\sigma}(p_{&gt;1})$</th>
<th>$c_{3\sigma}(p_{&gt;1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euclid</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>1.559$^{+0.054}_{-0.051}$</td>
<td>1.00872$^{+0.00003}_{-0.00003}$ (1)</td>
<td>0.99993$^{+0.00013}_{-0.00024}$ (0.32)</td>
<td>0.99436$^{+0.00023}_{-0.00041}$ (0)</td>
<td>0.98879$^{+0.00032}_{-0.00056}$ (0)</td>
</tr>
<tr>
<td>0.1%</td>
<td>1.587$^{+0.058}_{-0.052}$</td>
<td>1.000880$^{+0.000006}_{-0.000006}$ (0.98)</td>
<td>0.99199$^{+0.00014}_{-0.00024}$ (0.001)</td>
<td>0.98636$^{+0.00024}_{-0.00038}$ (0)</td>
<td>0.98072$^{+0.00034}_{-0.00053}$ (0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Delta c/c_0$</th>
<th>$z_M$</th>
<th>$c(p_{&gt;1})$</th>
<th>$c_{1\sigma}(p_{&gt;1})$</th>
<th>$c_{2\sigma}(p_{&gt;1})$</th>
<th>$c_{3\sigma}(p_{&gt;1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SKA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>1.593$^{+0.018}_{-0.017}$</td>
<td>$1.3\cdot10^{-7}$</td>
<td>$4\cdot10^{-7}$</td>
<td>0.99708$^{+0.00003}_{-0.00004}$</td>
<td>0.99524$^{+0.00006}_{-0.00007}$</td>
</tr>
<tr>
<td>1%</td>
<td>1.561$^{+0.017}_{-0.017}$</td>
<td>1.00873$^{+0.00001}_{-0.00001}$ (1)</td>
<td>1.00585$^{+0.00003}_{-0.00003}$ (1)</td>
<td>1.004036$^{+0.00005}_{-0.00005}$ (1)</td>
<td>1.00221$^{+0.00008}_{-0.00009}$ (1)</td>
</tr>
<tr>
<td>0.1%</td>
<td>1.590$^{+0.018}_{-0.017}$</td>
<td>1.000880$^{+0.000001}_{-0.000001}$ (1)</td>
<td>0.99797$^{+0.00003}_{-0.00003}$ (0)</td>
<td>0.99612$^{+0.00006}_{-0.00006}$ (0)</td>
<td>0.99428$^{+0.00008}_{-0.00008}$ (0)</td>
</tr>
<tr>
<td>0.1% (err/3)</td>
<td>1.590$^{+0.006}_{-0.006}$</td>
<td>1.000880$^{+0.000001}_{-0.000001}$ (1)</td>
<td>0.999834$^{+0.000009}_{-0.000009}$ (0)</td>
<td>0.99917$^{+0.00001}_{-0.00001}$ (0)</td>
<td>0.998510$^{+0.00002}_{-0.00002}$ (0)</td>
</tr>
<tr>
<td>0.1% (err/10)</td>
<td>1.590$^{+0.003}_{-0.003}$</td>
<td>1.000880$^{+0.000003}_{-0.000002}$ (1)</td>
<td>1.00032$^{+0.00014}_{-0.00018}$ (0.94)</td>
<td>0.99996$^{+0.00023}_{-0.00029}$ (0.44)</td>
<td>0.99961$^{+0.00032}_{-0.00040}$ (0.10)</td>
</tr>
</tbody>
</table>

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Observations.

- **Current:**
  - Euclid will have \( \frac{1}{10} \) of the errors of the current missions like WiggleZ Dark Energy Survey (e.g. Blake et al. 2011, 2012).

- **Future:**
  - Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 1308.0847)
  - Square Kilometer Array (SKA) (Bull et al. 1405.1452)
  - Wide-Field Infrared Survey Telescope (WFIRST) (Spergel et al. 1305.5425) (esp. having largest sensitivity at potential \( z_m \) region i.e. \( 1.5 < z < 1.6 \)).
Summary:

- **Redshift drift test** which give clear prediction for redshift drift effect which can potentially be measured by future telescopes (E-ELT, TMT, GMT, DECIGO/BBO).

- Baryon acoustic oscillations test to independently measure the radial $D_A$ and tangential mode $c/H$ of the volume distance at the angular diameter distance maximum $z_m$.

- In simple terms we have a “cosmic” measurement of the speed of light $c$ with $D_A$ giving the dimension of length being a “cosmic ruler” and $1/H$ giving the dimension of time being a “cosmic clock/chronometer” i.e.

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}.$$  \hspace{1cm} (26)