A New Picture of Accretion Disk Boundary Layers

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Disk Boundary Layer

Accretion via a disk onto a WD, NS, protostar...

**Case 1:** accretion proceeds all the way to the surface of the central object.

**Case 2:** accretion is disrupted by magnetic field before the disk reaches the surface.
Boundary Layer in Cataclysmic Variables

$$L_{BL} \approx \frac{\dot{M} R_*^2}{2}(\Omega_K(R_*) - \Omega_*)^2$$ up to 50% of accretion energy released in BL.

$$L_{BL} \leq L_{disk}, \text{ if } \Omega_* \lesssim \Omega_K(R_*)$$

Patterson & Raymond (1985)

\[ M \equiv V_K / s_{BL} \sim 40 \]

Outburst, optically thick

\[ M \equiv V_K / s_{BL} \sim 1 \]

Quiescent, optically thin
Both variables have been vertically averaged along $z$. 

Resolution: $4096 \times 384 \times 128$
Both variables have been vertically averaged along $z$. 

Resolution: $4096 \times 384 \times 128$
Disk +BL + Star

Zoom-in of initial conditions around $R = R_*$

$\rho(R) = \begin{cases} 
\exp \left[ -\int \frac{dR'}{s^2} g(R') \right], & \text{star} \\
1, & \text{disk} 
\end{cases}$

fixed potential $\Phi(R) = -1/R$

isothermal EOS $P = \rho s^2$

$M \equiv \Omega_K(R_*) R_*/s \sim 10$
Three Wave Branches

**Upper**

\( R \nu_R \sqrt{\rho} \)

Wave branches characterized by:

- **Upper**: Sound wave in disk propagating in the direction of the flow.
- **Lower**: Gravitosonic wave in star propagating against direction of the flow.

**Middle**

\( R \nu_R \sqrt{\rho} \)

**Wave branches characterized by:**

- **Upper**: Sound wave in disk propagating in the direction of the flow.
- **Lower**: Gravitosonic wave in star propagating against direction of the flow.

\[
\frac{k_{R,disk}(R_*)}{k_{\phi}} \approx \frac{k_{R,star}}{k_{\phi}}
\]
$V_R$ 2D hydro

$R^2 \sum \delta V_R^2 \approx \text{const}$ in the absence of dissipation for waves.
$V_R$ 2D hydro

$R^2 \Sigma \delta V_R^2 \approx \text{const}$ in the absence of dissipation for waves.
The Role of Mach Number

• KH regime (\(M \ll 1\)) 2 modes:
  \[ \frac{\omega}{k_y} = \pm i \frac{\Delta V}{2} \]

• Acoustic mode regime (\(M \gg 1\)) 3 modes:
  \[ \frac{\omega}{k_y} = \pm (M - 1) s, \quad \frac{\omega}{k_y} = 0 \]

Finite width of shear layer very important

Shear instability: fastest growing mode

\[ \text{Im}[\omega_{max}] \sim \Delta V / \delta, \quad k_y \delta \sim 1 \]

True of both KH and acoustic regime

Acoustic regime

\[ \text{Im}[\omega] = 0, \quad k_y \delta \gg 1 \]
Comparison of Theory to Simulations

![Graph showing comparison of theoretical and simulated $\Omega_P$ values with $m$ values ranging from 5 to 25.](image)

**Upper**

<table>
<thead>
<tr>
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<th>time</th>
<th>$m$</th>
<th>$\Omega_P$ measured</th>
<th>$\Omega_P$ predicted</th>
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**Middle**

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<td>.57</td>
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Standard BL Picture

azimuthal momentum equation  
\[ \dot{M} \Omega R^2 + 2\pi R^3 \nu \Sigma \frac{d\Omega}{dR} = \dot{J}, \]

constant accretion rate  
\[ \dot{M} = -2\pi R v_R \Sigma, \]

alpha viscosity  
\[ \nu = \alpha \Sigma \min(H, h_s) \]

radial momentum equation  
\[ v_R \frac{dv_R}{dR} = (\Omega^2 - \Omega_K^2) R - \frac{1}{\Sigma} \frac{dP}{dR}, \]

disk scale height & star pressure scale height  
\[ H \equiv \frac{s}{\Omega_K R}, \quad h_s \sim \left( \frac{s}{\Omega_K R} \right)^2 R \]

**dynamical boundary layer width**  
\[ \delta_{BL} = n_{HS} h_s = n_{HS} M_*^{-2} R_* \]

**Mach number**  
\[ M \equiv \frac{\Omega_K R}{s} \sim 40 \]

What is the mechanism of ang. mom. transport in the boundary layer? Not MRI turbulence since BL is (linearly) stable to MRI \((d\Omega/dR > 0)\).
At $t \approx 75$ density gap opened in inner disk. Simultaneously, accretion onto the star.

Stress angular momentum current

$$C_S \equiv 2\pi R^2 \Sigma \langle \delta v_\phi v_R \rangle$$

Mass accretion rate

$$\dot{M} = \left( \frac{dl}{dR} \right)^{-1} \frac{\partial C_S}{\partial R}$$

Angular momentum is radiated away from the BL into both the star and the disk.
Angular Momentum Transport: MHD

- A.M.T by waves in inner disk and star, MRI throughout disk.
- Gap formation and BL widening due to magnetosonic modes.
- Possible stochastic re-excitation of modes on viscous timescale
Comparison: waves vs. anomalous viscosity

Waves:

- Travel long distances before dissipating - Nonlocal heating.
- $C_S$ changes sign at the corotation radius of the mode.
- $C_S$ depends on amplitude, wavenumber, and wave branch of excited mode. Modes are potentially stochastically excited.

Anomalous Turbulent Viscosity: $\nu_{\text{turb}} \equiv \alpha s H$, $\alpha \approx \text{constant}$

- Local dissipation and heating: $Q_d = \frac{1}{2} \sum \nu_{\text{turb}} (Rd\Omega/dR)^2$
- $C_S$ changes sign where $d\Omega/dR = 0: C_S = -2\pi R^3 \sum \nu_{\text{turb}} d\Omega/dR$
- Turbulent viscosity is typically quasi steady state.
Conclusion

Waves Transport Angular Momentum in the Boundary Layer

Acoustic Instability is like the “MRI” of the boundary layer.
BL Model #1: Compressible Shear Layer

Velocity profile

\[ V_y(x) = \begin{cases} 
\Delta V / 2, & x > \delta_{BL} \\
\Delta V x / 2\delta_{BL}, & -\delta_{BL} \leq x \leq \delta_{BL} \\
-\Delta V / 2, & x < \delta_{BL} 
\end{cases} \]

Mach number

\[ M \equiv \Delta V / 2s \]
Shear Instability for Linear Velocity Profile

Plot of $\delta V_x$ for radiation mechanism

$$M = 5$$

Plot of $\delta V_x$ for KH instability

$$M = 0.2$$

growth rate estimate

$$\text{Im}[\omega] \approx \epsilon \frac{s}{\delta_{BL}} = \frac{\epsilon M}{n_{HS}} \Omega_K(R_*)$$

$$\delta_{BL} \approx n_{HS} \frac{P}{dP/dr} = n_{HS} M^{-2} R_*$$

$\epsilon \sim 0.2$, $n_{HS} \sim 6$, $M \sim 10 - 100$

$$\text{Im}[\omega] \sim 0.3 \Omega_K(R_*) - 3 \Omega_K(R_*)$$

$$\text{KE-y}$$

$$0 \quad 10^{-16}$$

$$200 \quad 400 \quad 600 \quad 800$$

$t$
**BL Model #2 (MHD): Disk + BL + Star**

Initial Field: $B = 0$ for $R > R_*$

$$B(R \geq R_*, z) = \begin{cases} \frac{B_0}{R} \hat{z}, & \text{NVF} \\ B_0 \hat{\phi}, & \text{NAF} \\ \frac{B_0}{R} \sin \left[ \frac{2\pi}{\lambda_R} (R - R_*) \right] \hat{z}, & \text{ZNF} \end{cases}$$

Stability: Disk is MRI unstable $\frac{d\Omega}{dR} < 0$. BL is MRI stable $\frac{d\Omega}{dR} > 0$.

Convergence for MRI in disk (NAF geometry):

$$\tan(\theta_B) \equiv \frac{B_R}{B_\phi} \approx 13^\circ \text{ in resolved MRI, equivalent to } \alpha \beta = 1/2$$
The Role of $\beta$

One expects acoustic modes to be modified by terms $\mathcal{O}(\beta^{-1})$ in the MHD case.

$$\beta \equiv \frac{\rho s^2}{B^2/2\mu}$$

$$v_{ms} = \sqrt{s^2 + v_A^2}$$

$$= s\sqrt{1 + \frac{2}{\gamma} \beta^{-1}}$$

Since $\beta^{-1} \lesssim 0.05$, acoustic modes are not significantly modified by introduction of B field.

$$\beta_{BL}^{-1}/\beta_{disk}^{-1} \sim 1 - 5$$
Magnetosonic Modes in the MHD Context

- Magnetosonic modes resemble acoustic modes due to $\beta^{-1} \lesssim 0.05$.
- Amplification of B field in BL partly due to accretion and frozen-in flux law.
Astrophysical Implications

Hidden boundary layer problem in CVs:

\[ \frac{L_{BL}}{L_{disk}} \sim 1 \text{ in quiescence } \dot{M} \sim 10^{-12} - 10^{-10} \quad \text{(Pandel et al. 2005, Sion et al. 2005)} \]

\[ \frac{L_{BL}}{L_{disk}} \ll 1 \text{ in outburst } \dot{M} \sim 10^{-9} - 10^{-8} \quad \text{(very little EUV or soft X-ray flux)} \]

Two nearby systems UGem and VW Hyi have BL observed in outburst, but temperatures that are too low by factors of 2-3 according to models.

Why low BL temperature? **Solution:** waves carry energy away from BL!

**DNOs/QPOs?**

Very simplified model, but can generate regular periodicity.

Even in black holes could transition from accretion disk to radiatively inefficient accretion flow share some properties with a BL?
Future Prospects

- Realistic E.O.S. + cooling or radiative transfer
  - Excited gravity modes.
  - Connection to DNOs/QPOs.
- Field Amplification (stratified MHD)
  - Does the field amplify to equipartition in the BL?
  - In global simulations is there some global dynamo process going on that amplifies fields? What limits field growth?
- Meridional Spreading Layer
  - Differential rotation in meridional direction leading to possible Kelvin-Helmholtz and baroclinic instabilities.
  - Possible pinch (Tayler-Spruit) or Parker magnetic instabilities.
In hydro and MHD simulations, of an isothermal BL, acoustic modes are always present. Dimensionality, azimuthal extent of the simulation domain, Mach number, magnetic field, and stratification only modify the details.
Instability: const. density, reflecting wall

\[ M = 5 \]

Dimensionless growth rate

Plot of \( \delta V_x \) from Athena simulation

\[ n \approx 7 \]
Accretion by Wave Excitation and Dissipation

Stress angular momentum current when upper mode is dominant. The wave is generated at corotation in the BL and dissipates in the disk after shocking.

Mass accretion rate

\[ \dot{M} = \left( \frac{dl}{dR} \right)^{-1} \frac{\partial CS}{\partial R} \]

Specific angular momentum

\[ l = \Omega R^2 \]

Shock density profiles at two different radii. The sound wave steepens and shocks as it travels into the disk.
Observing CV Boundary Layer

Photoionization of neutral hydrogen 912 Angstroms.

Patterson & Raymond (1985)