Anna Chashkina<sup>1,2</sup>, Pavel Abolmasov<sup>1,2</sup> and Juri Poutanen<sup>1,3</sup>

<sup>1</sup>Tuorla Observatory, Department of Physics and Astronomy, University of Turku, Väisäläntie 20, FI-21500 Piikkiö, Finland

<sup>2</sup>Sternberg Astronomical Institute, Universitetskiy pr., 13, 119992 Moscow, Russia <sup>3</sup>Nordita, Roslagstullsbacken 17, SE-10691 Stockholm, Sweden

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#### Abstract

An ultraluminous X-ray source M82 X-2 was recently identified as an X-ray pulsar accreting at a super-Eddington rate. In this object, the accretion disc outside the magnetosphere probably still remains below the local Eddington limit but its structure may be affected by the radiation of the central source (accretion column) that together with magnetic torques shifts the centrifugal balance in the inner parts of the accretion disc thus increasing its surface density and thickness. Magnetospheric radius is also affected by the structure of the disc and can be calculated self-consistently in the framework of our model. We consider the structure of such a disc and corrections to the magnetospheric radius. For large magnetic moments (surface magnetic field  $B > 10^{13}$ G), the structure of the accretion disc is very close to the standard accretion disc model [1], and the magnetospheric radius is proportional to the classical Alfvén radius with a constant coefficient. A small magnetic field, on the other hand, allows the disc to penetrate further inside the magnetosphere, but the radius of the magnetosphere becomes relatively larger with respect to the classical Alfvén radius. The inner disc parts in this case show sub-Keplerian rotation (slower by a factor of about 0.75).

# 1 Introduction

Usually, magnetospheric accretion upon neutron stars in binary systems is proposed to run through a thin accretion disc surrounding a nearly-dipole magnetosphere. At low mass accretion rates, this approximation may be violated, and the disc should be replaced by a quasi-spherical envelope [2]. At the high mass accretion rates close to or exceeding the Eddington limit, the disc again should become thick. Until recently, the discs in all the X-ray pulsars were well understood as thin, gas-pressure dominated, and nearly Keplerian, and trimmed from the inside by the magnetospheric radius.

In the recent work [3], pulsations with a period  $P \simeq 1.37$  s were found from the ultraluminous X-ray source M82 X-2, that clearly identifies this object as a neutron star rather than a black hole. The high luminosity of this source ( $\sim 10^{40} \text{ ergs}^{-1}$ ) is well in excess of the Eddington limit for a neutron star that clearly points to importance of radiation pressure, as well as disc thickness, in this source. Most of the radiation is released close to the surface of the neutron star, in the accretion column, that can in principle emit much more than the Eddington luminosity due to geometric reasons and transparency of a strongly magnetized plasma [4]. The accretion disc, at the same time, becomes illuminated by a radiation flux exceeding the local Eddington limit and thus its structure as well as the position of the disc-magnetosphere interface should be affected significantly.

Both radiation from the central source (accretion column) and the magnetic field of the neutron star create in the disc a radial pressure gradient, violating one of the basic assumptions of the standard disc theory. In this work we discuss mainly the boundary of the disc  $R_{\rm in}$ . The disc boundary is often

 $\mathbf{2}$ 

described by Alfvénic radius  $R_A$ , the radius where the ram pressure is equal to the magnetic pressure. In reality there is a correction factor of  $\xi \sim 1$  that depends on the disc geometry:

$$R_{\rm in} = \xi R_{\rm A} = \xi \left(\frac{\mu^2}{2\dot{M}\sqrt{2GM}}\right)^{2/7},$$
 (1)

where  $\mu$  is the magnetic moment of the neutron star with mass M,  $\dot{M}$  is the accretion rate. Analytical models predict  $\xi$  from 0.5 [5] to > 1 [6], simulations argue for  $\xi \sim 0.5$ .

One of the most important ingredients of the model, neutron star magnetic field, is still unknown. There is a very wide range of the magnetic fields proposed in the literature: from very small up to magnetar magnetic field  $B = 10^{14}$ G [7]. The latter has the advantage of explaining the highly super-Eddington luminosity of the object.

Besides the magnetic field value, it is very important to understand the interaction of the magnetic field with the accretion disc. The models considering interaction of the magnetosphere with the disc can be divided into two main classes: magnetically threaded discs (see [5] and [8]) and the models with open magnetic field lines (see [9]). It was shown recently [10] that the stellar magnetic field cannot thread the disc but is rather pushed out very quickly. In our model we consider a disc with only a small area at the boundary where the matter interacts with the magnetic field and flows to the neutron star along the magnetic field lines.

## 2 The model

We start from the equations of hydrodynamics plus viscosity prescription from [1]. The Euler equation in the vector form takes the form:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla\varphi.$$
(2)

In steady-state, the radial component of this equation in cylindrical coordinate system can be rewritten as:

$$v_R \frac{\partial v_R}{\partial R} - \frac{v_{\phi}^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial R} - \frac{GM}{R^2}.$$
(3)

Here p is the pressure,  $v_{\phi} = \Omega R$  is the rotation velocity,  $v_R$  is the radial velocity, and  $\Omega$  is the disc angular velocity. Radiation pressure of the central source shifts the radial force equilibrium of the Keplerian disc ( $\Omega^2 R = GM/R^2$ ) and the radial pressure gradient cannot be neglected in the inner parts of the disc. The disc angular velocity becomes non-Keplerian, and the ratio of the disc angular velocity to the Keplerian value  $\omega = \Omega/\Omega_K$  should be a function of radius. Because of the strong radiation pressure at the boundary, the inner parts of the disc are additionally decelerated in the radial direction and their radial velocity is smaller than that in the standard disc, so we neglect the first term, quadratic in  $v_R$ , on the left hand side of eq. (3).

Integrating eq. (3) over the vertical coordinate z yields:

$$\Omega^2 R = \frac{1}{\Sigma} \frac{\partial \Pi}{\partial R} + \frac{GM}{R^2}.$$
(4)

Here  $\Pi = \int_{-H}^{H} pdz$  is the vertically integrated pressure at the inner face of the disc,  $\Sigma = \int_{-H}^{H} \rho dz$  is the surface density, and H is the disc thickness. At the boundary internal disc pressure  $(p_{\rm rad} + p_{\rm gas})$ is balanced with the external pressure of the radiation source  $p_{\rm rad} = L/4\pi R^2 c$  and the magnetic field pressure  $p_{\rm mag} = \mu^2/8\pi R^6$ . Note that the radiation pressure can be even higher by a factor up to two due to reflection of photons, depending on the scattering albedo of the disc. Thus vertically integrated pressure at  $R_{\rm in}$  is  $\Pi = (p_{\rm rad} + p_{\rm mag})H$ .

The vertical disk structure (see [11] in more detail) gives the relation between the central pressure  $p_c$  and the central density  $\rho_c$  as:

$$p_c = \rho_c \frac{GM}{R^3} \frac{H^2}{4}.$$
(5)

Eq. (5) provides us with the thickness of the disc:

$$H = \sqrt{\frac{5\Pi}{\Sigma} \frac{R^3}{GM}}.$$
(6)

Following the standard accretion disc theory we assume alpha-prescription:

$$W_{r\phi} = \alpha \Pi, \tag{7}$$

where  $W_{r\phi}$  is the  $r\phi$ -component of the vertically integrated viscous stress tensor. Under this assumption the angular momentum conservation equation is:

$$\dot{M}\frac{d(\Omega R^2)}{dR} = \frac{d}{dR}(2\pi R^2 W_{r\phi}).$$
(8)

As in [1], we assume that the energy released in the accretion disc is radiated locally from its surface. In this case, the local energy release will be coupled to the radiation emitted from the disc surface, on one side, and to the velocity gradient in the disc, on the other, allowing to link the rotation law of the disc  $\omega(R)$  with the vertically-integrated pressure. The pressure is itself linked to the energy release rate by the vertical radiation diffusion equation

$$F = -D\nabla\epsilon,\tag{9}$$

where F is the vertical energy flux,  $\epsilon = aT^4$  is the radiation energy density and D is the diffusion coefficient. Local energy dissipation rate:

$$\frac{dF}{dz} = \alpha p R \frac{d\Omega}{dR},\tag{10}$$

where p is the total (gas+radiation) pressure:

$$p = p_{\text{gas}} + p_{\text{rad}} = 2nkT + \frac{aT^4}{3}.$$
 (11)

Solving equations (9) and (10) one obtains the following relation between the surface temperature  $T_s$  and the central temperature  $T_c$ :

$$T_s^4 = T_c^4 - \frac{73}{120} \frac{1}{a} \frac{\alpha \kappa \rho_c p_c R H^2}{c},$$
(12)

here  $\kappa$  is the opacity. Assuming that the surface temperature  $T_s$  equals to the effective temperature,

$$2\sigma_{\rm SB}T_{\rm eff}^4 = \alpha \Pi R \frac{d\Omega}{dR},\tag{13}$$

we get an expression for the central disc temperature:

$$T_c^4 = R \frac{d\Omega}{dR} \frac{W_{r\phi}}{\sigma_{\rm SB}} \left[ \frac{219}{512} \kappa \Sigma + 1 \right]. \tag{14}$$

This expression differs from a similar equation for the standard disc (see eq. 2.24 in [1]) because we account for the vertical structure of the disc.

The central pressure expressed as a sum of radiation and gas pressure together with the alpha prescription gives us:

$$\frac{15}{16}\frac{W_{r\phi}}{\alpha H} = \frac{aT_c^4}{3} + \frac{3}{2}\frac{\Sigma kT_c}{Hm_p}.$$
(15)

Close to the disc boundary, the infalling matter has the angular velocity that differs from the angular velocity of the neutron star and the magnetosphere thus should lose a certain amount of excess angular

momentum. We assume that the excess angular momentum is removed by the magnetic and radiation stresses at the magnetospheric boundary:

$$\dot{M}(\Omega_{\rm ns} - \Omega_{\rm in})R_{\rm in}^2 = k_t \frac{\mu^2 H_{\rm in}}{R_{\rm in}^4} - L \frac{\Omega_{\rm in}}{c^2} H_{\rm in} R_{\rm in},$$
(16)

where  $k_t = B_{\phi}/B_z$  is the ratio of magnetic field components. The second boundary condition is for the pressure at the inner boundary of the disc that should be balanced by the magnetic and radiation pressure:

$$W_{r\phi}^{\rm in} = 2\alpha \left( \frac{\mu^2 H_{\rm in}}{8\pi R_{\rm in}^6} + \frac{LH_{\rm in}}{4\pi R_{\rm in}^2 c} \right).$$
(17)

# **3** Results and discussion

We are interested in the disc boundary  $R_{\rm in}$  or in other words in parameter  $\xi = R_{\rm in}/R_{\rm A}$ . In each simulation, we used the following input parameters aimed to reproduce the conditions in M82 X-2: neutron star rotation period P = 1.37 s, luminosity  $L = 10^{40} \,\mathrm{erg s^{-1}}$  with the efficiency  $\eta = 0.1$  and the neutron star magnetic moment between  $10^{29}$  and  $10^{32} \mathrm{G \, cm^3}$ . For determining the disc boundary we used the shooting method: first we assume some  $R_{\rm in}$ , using it we calculate  $\Omega_{\rm in}$  with the boundary condition (16) and then calculate the whole structure of the disc from the outside. We then vary  $R_{\rm in}$  until the angular velocity at the boundary is equal to  $\Omega_{\rm in}$  calculated from the boundary condition.

Our results are given in Fig. 1. The dependence of the magnetospheric radius on the magnetic moment is shown by circles. For high magnetic fields ( $\mu > 5 \times 10^{30}$  G cm<sup>3</sup>), our results are in agreement with the other simulations of the standard disc, but for a lower magnetic field, the disc boundary approaches the Alfvénic radius. The dependence of the non-Keplerianity  $\Omega/\Omega_{\rm K}$  on the magnetic fields have nearly Keplerian discs  $\Omega(R_{\rm in}) > 0.95$  while decreasing magnetic field leads to a significant non-Keplerianity.

In Fig. 2, the thickness of the disc is given as a function of the radial coordinate. The standard disc theory predictions (with the zero torque at the inner boundary) are shown by the blue lines, our results are shown by the red lines. Different plots correspond to different magnetic field strengths. Here we neglect the gas pressure inside the disc because the luminosity is close to the Eddington limit. Gas pressure may become important due to matter accumulation in the disc. It is also important at smaller mass accretion rates and larger magnetic fields, when the disc is cooler. The effect of the gas pressure is discussed further in [11]. Our model conforms with the standard disc far from the boundary, but near the boundary our solution corresponds to a thicker disc. For the strong magnetic field, the disc is still thin, but when the magnetic field decreases it becomes thicker. The solution presented in the last two plots, where the disc is thick, i.e.  $H/R \sim 1$ , should be taken with the grain of salt, as the standard disc theory is not applicable when the flux locally exceeds the Eddington limit. In this case, we should take into account outflows and other effects such as radial advection [12]. To reproduce the properties of M82 X-2, the magnetic field of the neutron star in M82 X-2 should be  $B > 10^{13}$ G or higher to remain below the local Eddington limit outside the magnetors boundary. This conclusion is consistent with the previous fundings by [4, 7] that this source hosts a magnetar.

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Figure 1: Magnetospheric radius (circles) and non-Keplerianity (triangles) as functions of the magnetic moment.



Figure 2: The thicknesses of the disc are shown as functions of radial coordinate. The standard disc theory predictions are shown by the blue lines, our results are shown by the red lines.

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