

General Relativity and Cosmic Structure Formation

based on Phys. Rev. **D88** 103527 (arXiv:1308.6524),
Class. Quant. Grav. **31** 234006 (arXiv:1408.3352), **arXiv:1509.01699**,
and work in progress with D. Daverio, R. Durrer, and M. Kunz

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Newtonian N-body simulations have a long tradition in LSS modelling

- conceptually simple
- “good enough” given the limited precision of past LSS surveys

Models beyond Λ CDM may have relativistic sources of stress-energy perturbations

- Newtonian limit not always a good approximation

Increasing data quality imposes new challenge to take into account relativistic effects (e.g. in modelling RSD, WL. . .)

- advantage of conceptual simplicity is lost

Let's go relativistic!

Strategy

- choose ansatz for the metric (perturbed FLRW)

$$ds^2 = a^2(\tau) \left[-(1 + 2\Psi) d\tau^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j + h_{ij} dx^i dx^j - 2B_i dx^i d\tau \right]$$

- metric components are evolved with Einstein's equations

$$G_{\nu}^{\mu} = 8\pi G T_{\nu}^{\mu}$$

- stress-energy tensor is determined by solving the EOM's of all sources of stress-energy

$$T_{\text{m}}^{\mu\nu} = \sum_n m_{(n)} \frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}_{(n)})}{\sqrt{-g}} \left(-g_{\alpha\beta} \frac{dx_{(n)}^{\alpha}}{d\tau} \frac{dx_{(n)}^{\beta}}{d\tau} \right)^{-\frac{1}{2}} \frac{dx_{(n)}^{\mu}}{d\tau} \frac{dx_{(n)}^{\nu}}{d\tau}$$

- instead of peculiar velocities, use canonical momentum

$$\mathbf{v} = \frac{\mathbf{q}}{\sqrt{\mathbf{q}^2 + m^2 a^2}} \left(1 + \Psi + \frac{\mathbf{q}^2 \Phi + 2m^2 a^2 \Phi + \frac{1}{2} \mathbf{q} \cdot \mathbf{h} \cdot \mathbf{q}}{\mathbf{q}^2 + m^2 a^2} \right) + \mathbf{B} - \frac{\mathbf{h} \cdot \mathbf{q}}{\sqrt{\mathbf{q}^2 + m^2 a^2}}$$

gives “simple” equations valid for arbitrary \mathbf{q} :

- stress-energy tensor

$$T_0^0 = -\frac{1}{a^4} \sum_n \delta^{(3)}(\mathbf{x} - \mathbf{x}_{(n)}) \left[\sqrt{\mathbf{q}_{(n)}^2 + m_{(n)}^2 a^2} \left(1 + 3\Phi + \frac{\mathbf{q}_{(n)}^2}{\mathbf{q}_{(n)}^2 + m_{(n)}^2 a^2} \Phi \right) + \mathbf{B} \cdot \mathbf{q}_{(n)} - \frac{\mathbf{q}_{(n)} \cdot \mathbf{h} \cdot \mathbf{q}_{(n)}}{2\sqrt{\mathbf{q}_{(n)}^2 + m_{(n)}^2 a^2}} \right]$$

$$\Pi_{ij} \doteq \delta_{k(i} T_{j)}^k - \frac{1}{3} \delta_{ij} T_k^k = \dots$$

- geodesic equation

$$\mathbf{q}' = -\sqrt{\mathbf{q}^2 + m^2 a^2} \nabla \left(\Psi + \frac{\mathbf{q}^2}{\mathbf{q}^2 + m^2 a^2} \Phi + \frac{\mathbf{B} \cdot \mathbf{q}}{\sqrt{\mathbf{q}^2 + m^2 a^2}} - \frac{1}{2} \frac{\mathbf{q} \cdot \mathbf{h} \cdot \mathbf{q}}{\mathbf{q}^2 + m^2 a^2} \right)$$

Einstein's equations at leading order

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = -4\pi G a^2 \delta T_0^0$$

$$\left(\frac{\partial^2}{\partial x^i \partial x^j} - \frac{1}{3} \delta_{ij} \Delta \right) (\Phi - \Psi) = 8\pi G a^2 \Pi_{ij}^{(S)}$$

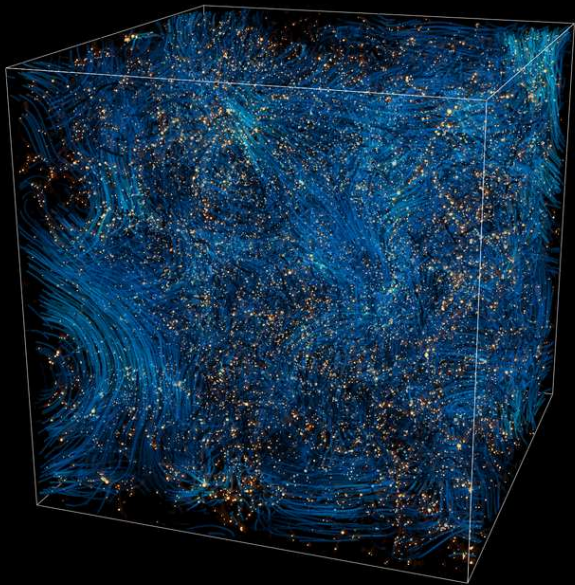
$$B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} = 8\pi G a^2 \Pi_{ij}^{(V)}$$

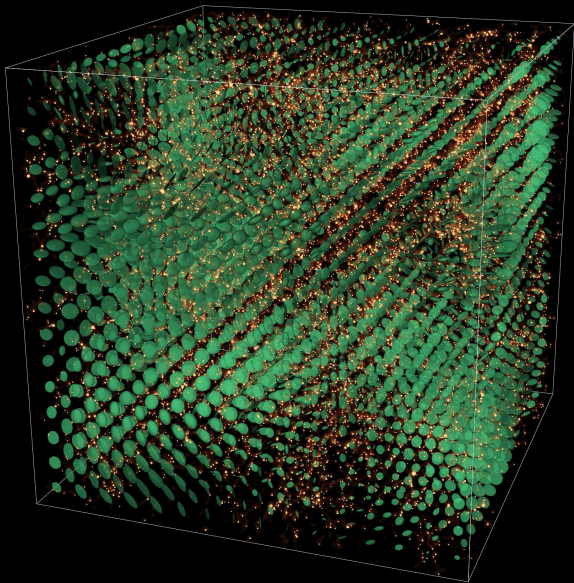
$$\frac{1}{2} h''_{ij} + \mathcal{H} h'_{ij} - \frac{1}{2} \Delta h_{ij} = 8\pi G a^2 \Pi_{ij}^{(T)}$$

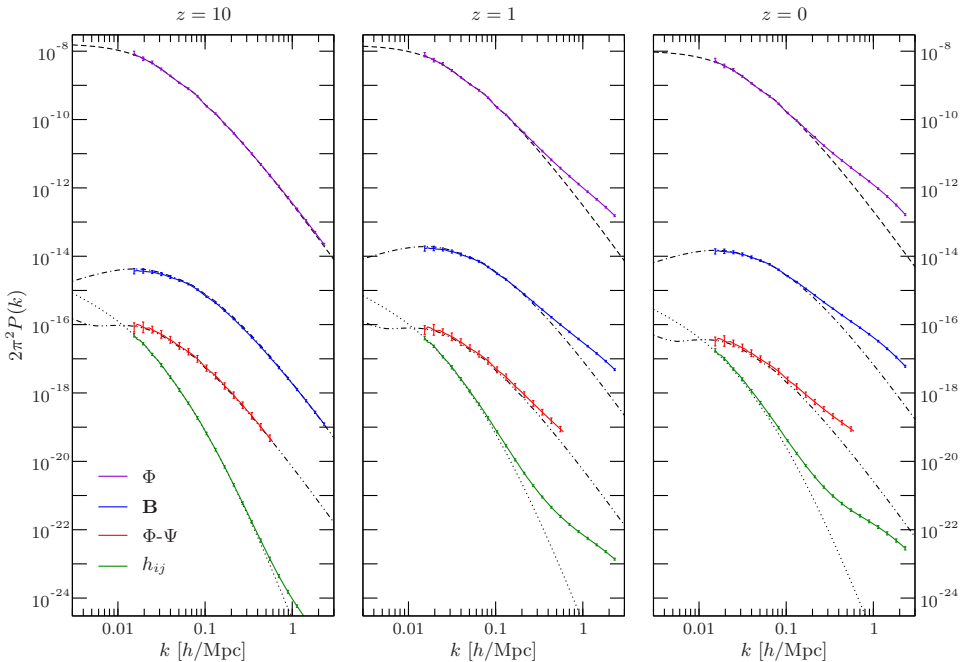
Including “shortwave corrections” ...

$$(1 + 4\Phi) \Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi + \frac{3}{2} (\nabla\Phi)^2 = -4\pi Ga^2\delta T_0^0$$

$$\begin{aligned} & \left(\frac{\partial^2}{\partial x^i \partial x^j} - \frac{1}{3} \delta_{ij} \Delta \right) (\Phi - \Psi) + \\ & \quad B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} + \\ & \quad \frac{1}{2} h''_{ij} + \mathcal{H}h'_{ij} - \frac{1}{2} \Delta h_{ij} + \\ & 2(\Phi + \Psi) \Phi_{,ij} - \frac{2}{3} \delta_{ij} (\Phi + \Psi) \Delta\Phi + \\ & 2\Phi_{,i} \Phi_{,j} - \frac{2}{3} \delta_{ij} \delta^{kl} \Phi_{,k} \Phi_{,l} = 8\pi Ga^2 \Pi_{ij} \end{aligned}$$







Summary

- N-body simulations within a GR framework are feasible
- unified relativistic treatment is a clear, logical and transparent way to address the most general observables with minimal restrictions on the cosmological model
- technology should be useful for simulations with relativistic sources (dynamical DE, neutrinos, ...)
- our code, `gevolution`, will soon be available on a public Git repository:

<https://github.com/gevolution-code/gevolution-1.0.git>