Critical Phenomena in the Aspherical Gravitational Collapse of Radiation Fluids

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TWB & P. J. Montero, PRD, in press

Critical Phenomena

- Consider family of initial data, parametrized by p
- assume $p > p^\star$ leads to black-hole formation
- for p close to $p^\star$ can then observe *critical phenomena*:
  - for $p > p^\star$, mass of black hole scales with

    $$M \propto (p - p^\star)^\gamma$$

    where γ depends on matter model only
  - in strong-field region prior to black-hole formation, spacetime approaches *self-similar* critical solution
  - depending on matter model, self-similarity may be *discrete* or *continuous*

[Choptuik, 1993; see review by Gundlach & Martín-García, 1999]

Brief History

- Original discovery: scalar fields
  [Choptuik 1993]
- Other matter models:
  - vacuum (gravitational waves) [Abrahams & Evans, 1993]
  - radiation fluids [Evans & Coleman, 1994]
  - etc...
- Analytical results [Koike *et.al.*, 1995; Maison, 1995; etc...]
- non-spherical perturbations of critical solution [Martín-García & Gundlach, 1999]

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But,

- despite wealth of interesting phenomena in absence of spherical symmetry, and
- despite enormous progress in 3D numerical relativity,

> "*there has been less progress in going beyond spherical symmetry than we anticipated.*"

[Gundlach & Martín-García, 2007]

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- Cartesian coordinates not well suited for study of critical collapse

Outline

- Consider critical collapse of radiation fluid

$$P = (\Gamma - 1)\rho$$

  with $\Gamma = 4/3$
  $\Longrightarrow$ critical solution displays *continuous self-similarity*
  [Evans & Coleman, 1994]

- Use new numerical-relativity code written in spherical polar coordinates
  [Baumgarte $et.al.$, 2013, 2015]
  $\Longrightarrow$ well-suited to study critical collapse

$\Longrightarrow$ Study deviations from spherical symmetry

Computational Setup

- Solve Einstein's equations using BSSN formulation
- Adopt implementation in spherical polar coordinates [Baumgarte $et.al.$, 2013, 2015]
  - reference-metric formulation of BSSN [Bonazzola $et.al.$, 2004; Brown, 2009]
  - geometric rescaling of all tensor components
  - "partially-implicit Runge-Kutta" time integrator [Montero & Cordero-Carrión, 2012]
- Impose
  - axisymmetry ($N_\varphi = 1$)
  - equatorial symmetry
- Evolve using moving-puncture coordinates
  - "1+log" slicing
  - Gamma-driver shift condition
- logarithmic grid in radial direction

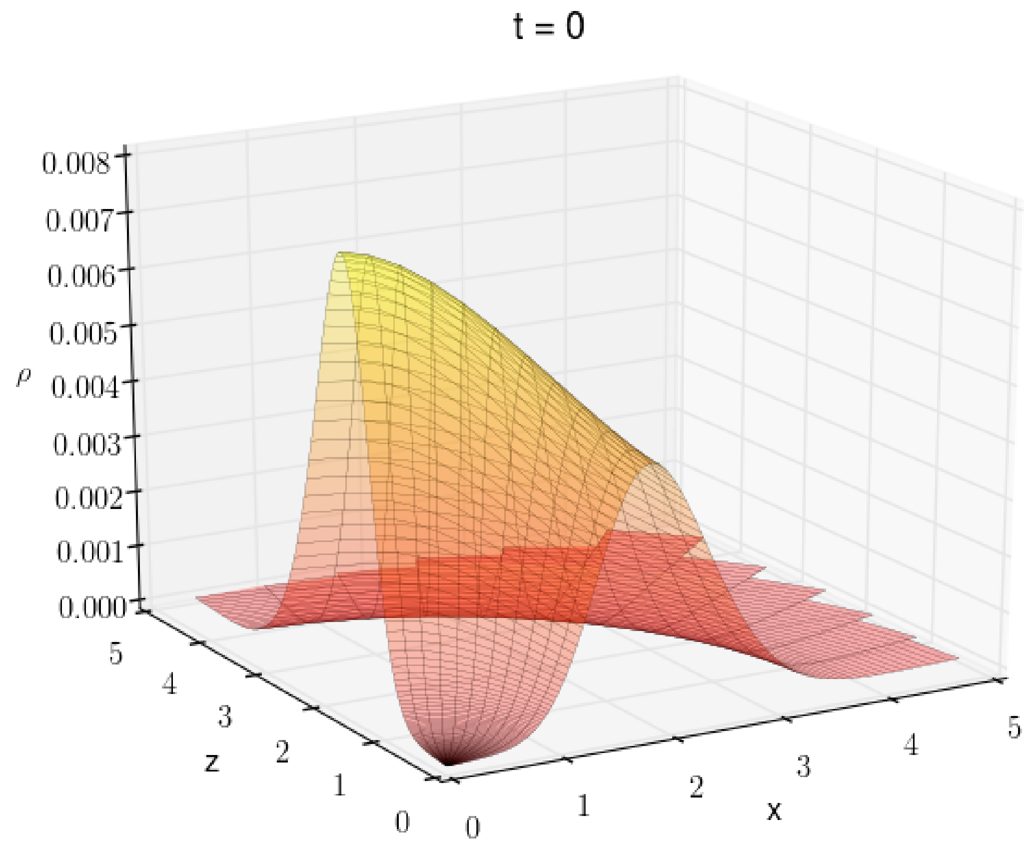
Initial Data

- Gaussian density distribution ρ, parameterized by:
  - amplitude η
  - centered on R_c
  - quadrupole moment parameter ϵ

- conformally flat: $\bar{\gamma}_{ij} = \eta_{ij}$

- time-symmetry: $K_{ij} = 0$

$\Longrightarrow$ solve Hamiltonian constraint



Consider three cases:

- spherical and centered: $\epsilon = 0$, $R_c = 0$: Evans & Coleman [1994]
- spherical and off-centered: $\epsilon = 0$, $R_c > 0$
- aspherical and off-centered: $\epsilon > 0$, $R_c > 0$

Spherical Symmetry

Compare with results of Evans & Coleman [1994] for spherical ($\epsilon = 0$) and centered ($R_c = 0$) data

- vary η to find onset of black-hole formation

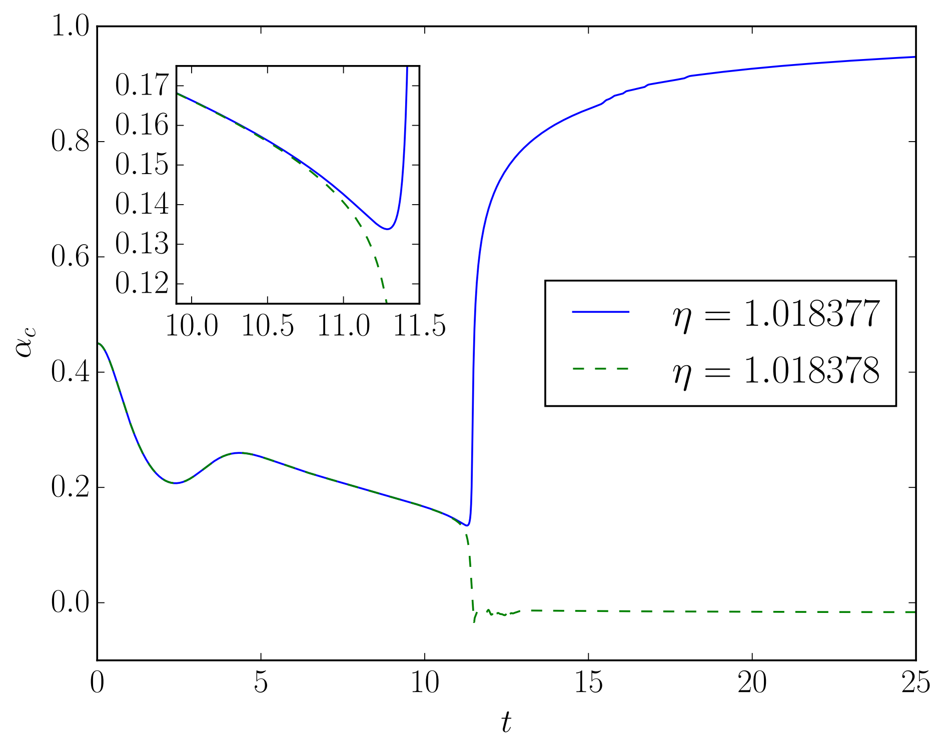
$$\eta_c \approx 1.018377$$

- Evans & Coleman:

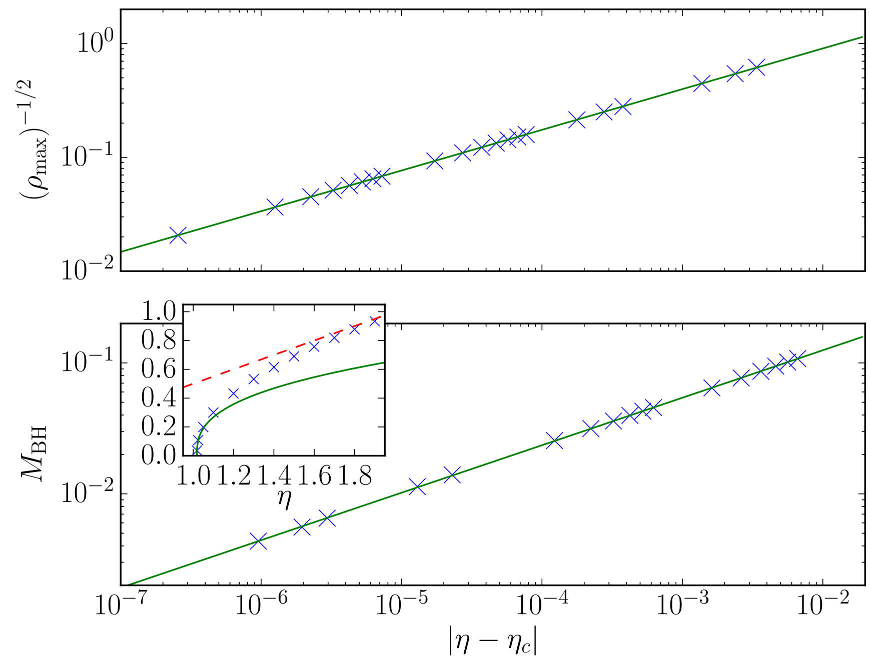
$$\eta_c \approx 1.0188$$

(completely different code, completely different gauge conditions)

$\Longrightarrow$ excellent agreement



Critical Scaling



- for *supercritical* data, measure black-hole mass from area of apparent horizons $\Longrightarrow$ fit to

$$M_{\rm BH} = C_{\rm super}(\eta - \eta_c)^\gamma$$

- for *subcritical data*, maximum space-time curvature, hence density, satisfies similar scaling [Garfinkle & Duncan, 1998] $\Longrightarrow$ fit *subcritical* data to

$$\rho_{\max}^{-1/2} = C_{\rm sub}(\eta_c - \eta)^\gamma$$

- find $\gamma \approx 0.36$ in both cases
- consistent with both analytical and previous numerical results

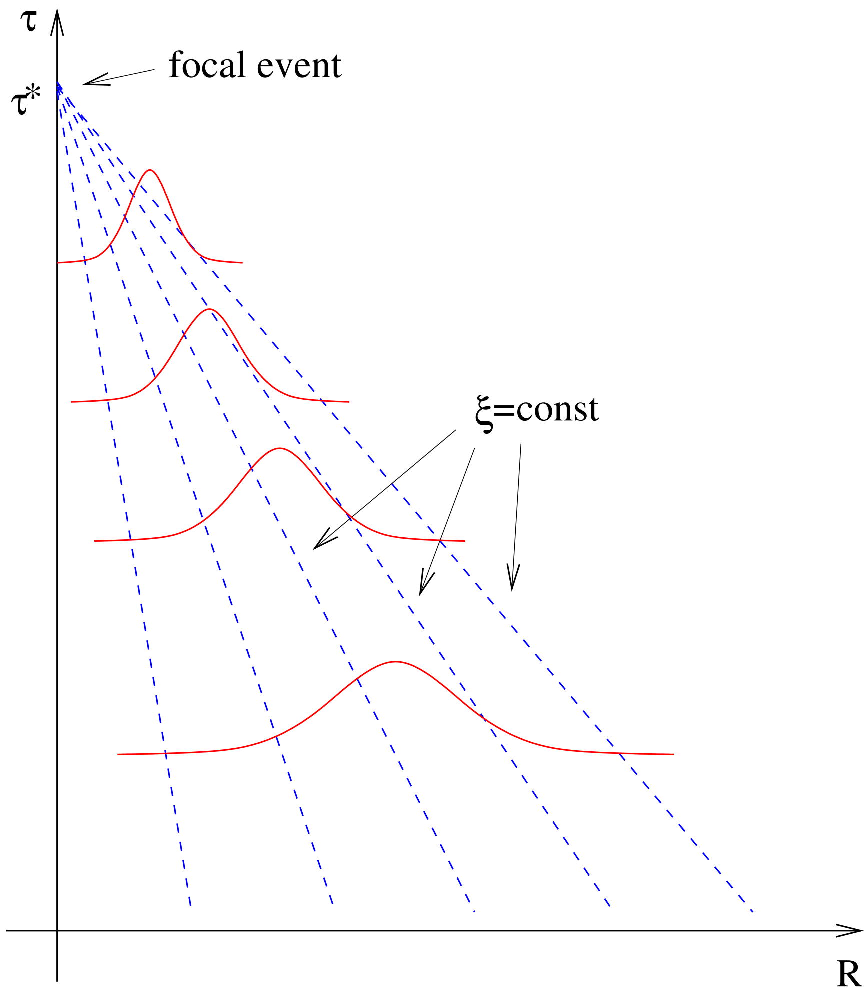
Self-Similarity of Critical Solution

In strong-field region of spacetime, close to criticality and prior to black-hole formation, solution contracts to "focal event" in self-similar fashion

$\Longrightarrow$ introduce self-similar coordinate

$$\xi \equiv \frac{R}{\tau^* - \tau}$$

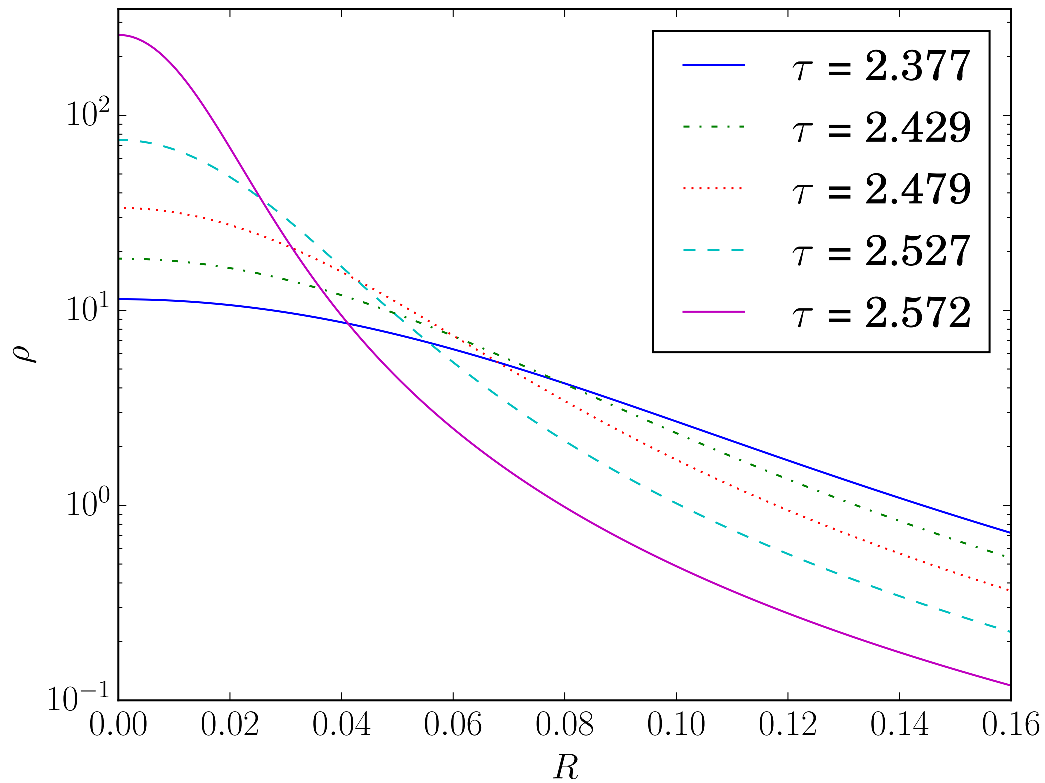
$\Longrightarrow$ self-similar solution can be expressed as function of ξ



(adapted from [Neilsen & Choptuik, 2000]

Self-similarity

- Not every variable displays self-similar behavior



$$\eta = 1.0183772$$

Self-similarity

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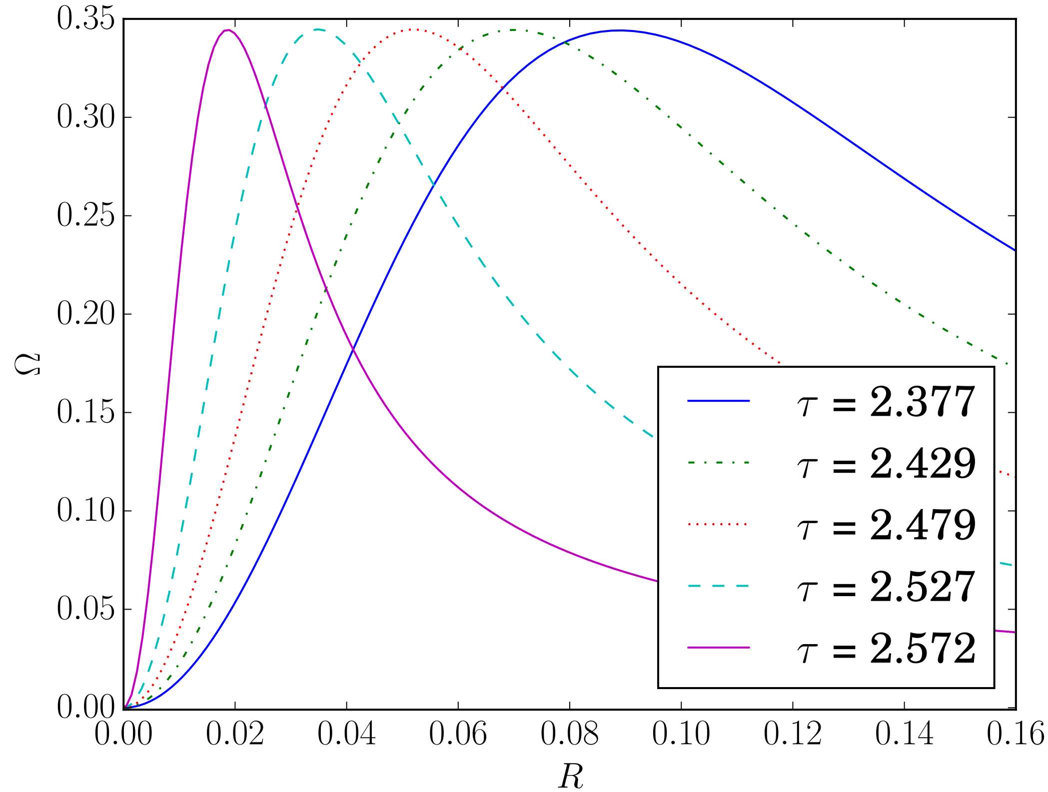
- Instead of ρ, consider

$$\Omega \equiv 4\pi R^2 \rho$$

[Evans & Coleman, 1994]



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Self-similarity

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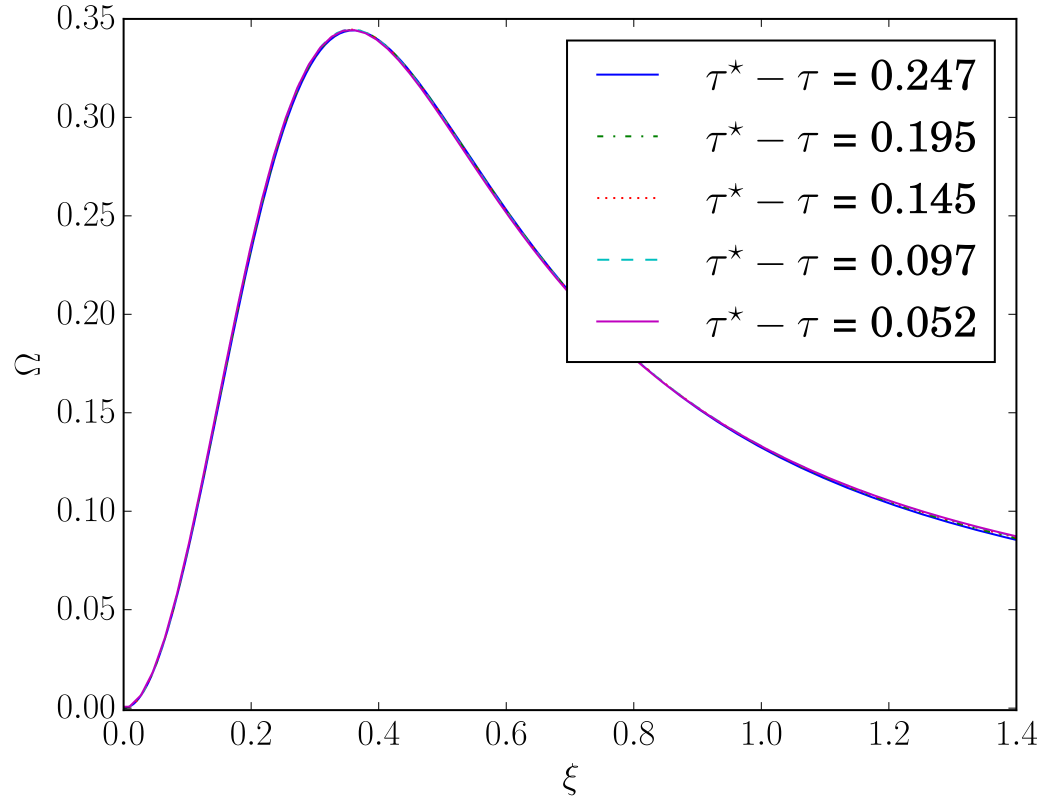
- Plot as function of

$$\xi \equiv \frac{R}{\tau^* - \tau}$$

with $\tau^* = 2.624$

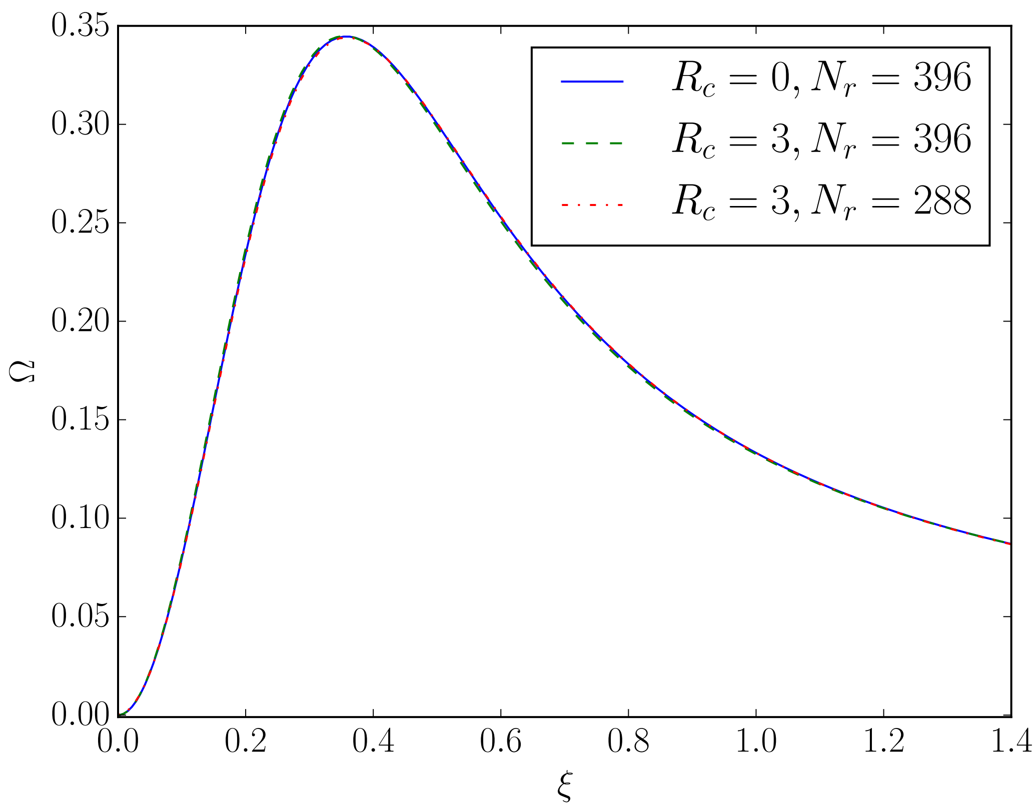
$\Longrightarrow$ self-similarity evident



$\eta = 1.0183772$

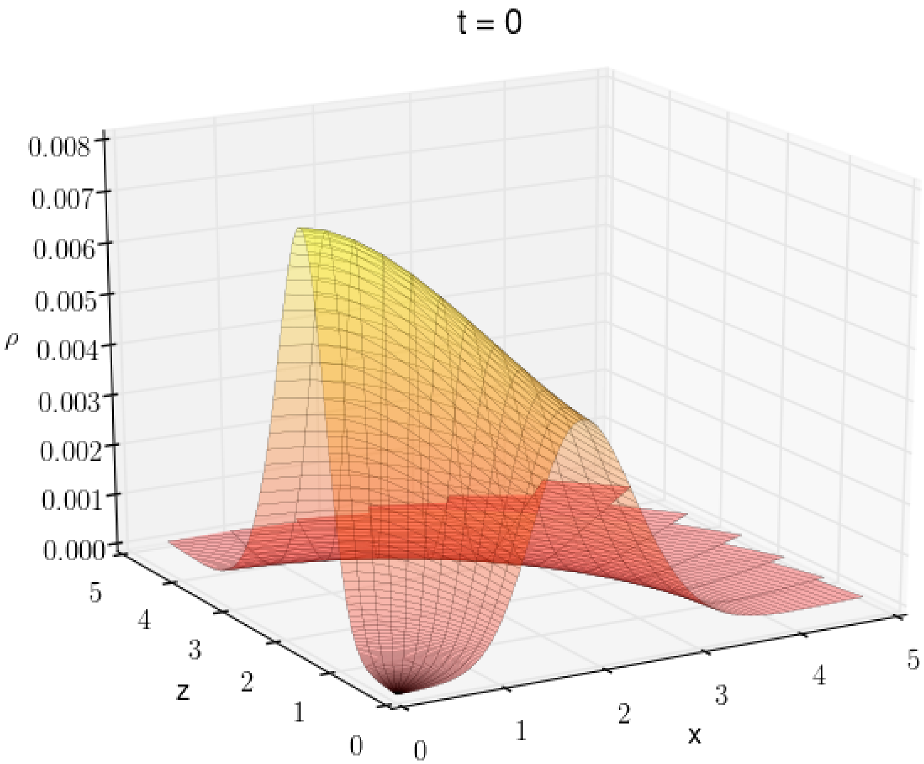
Off-centered Data

- Results for spherical off-centered ($R_c = 3$) data very similar
- Critical parameter: $\eta_c \approx 0.12409$
- Self-similar solution $\Omega(\xi)$ nearly indistinguishable from centered data



$\Longrightarrow$ Evidence for *universality* of critical solution

Aspherical Data



- Aspherical data with $\epsilon = 0.5$
- colored surface: $\eta = 0.12443$
- wireframe: $\eta = 0.12442$

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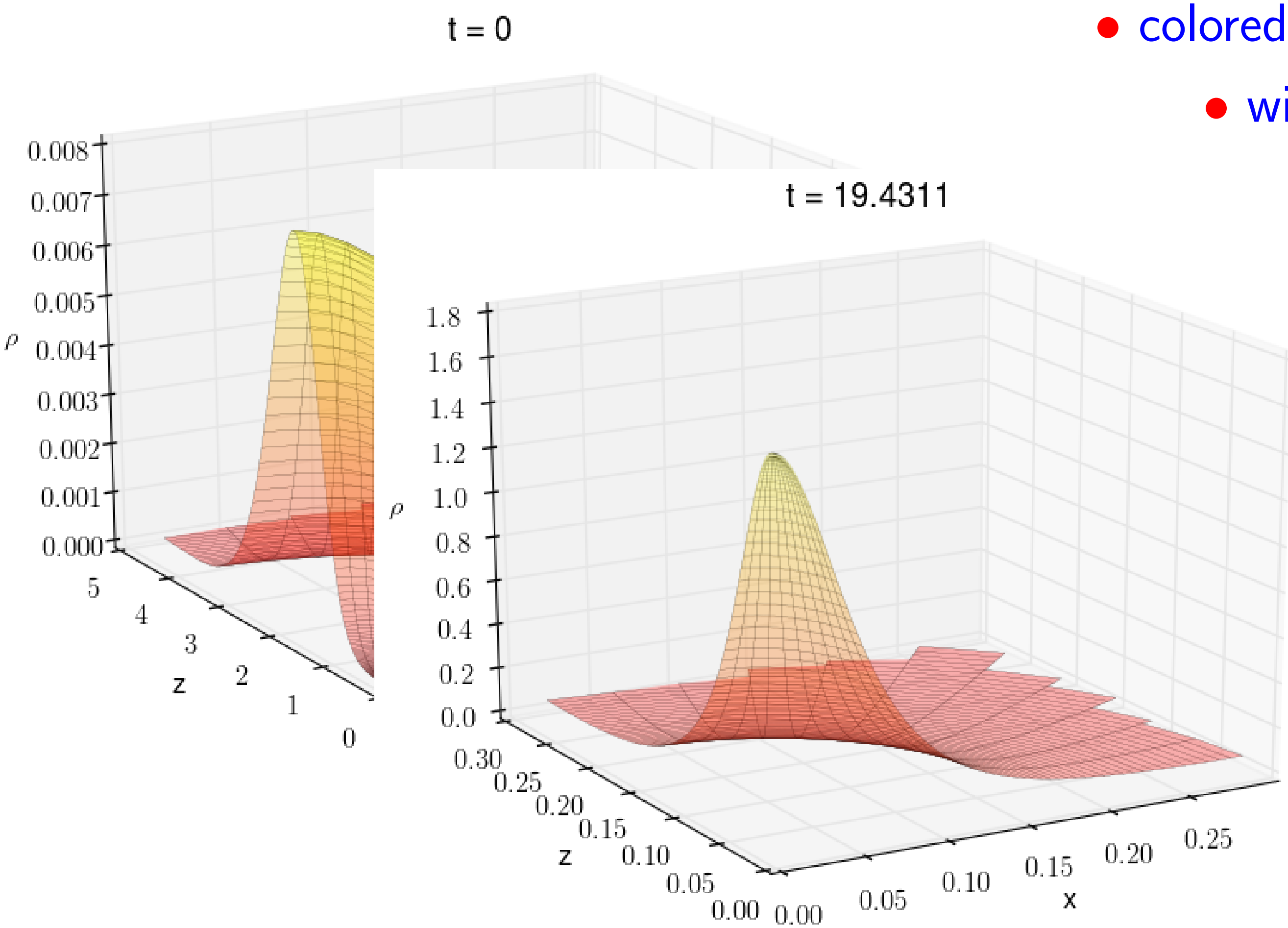

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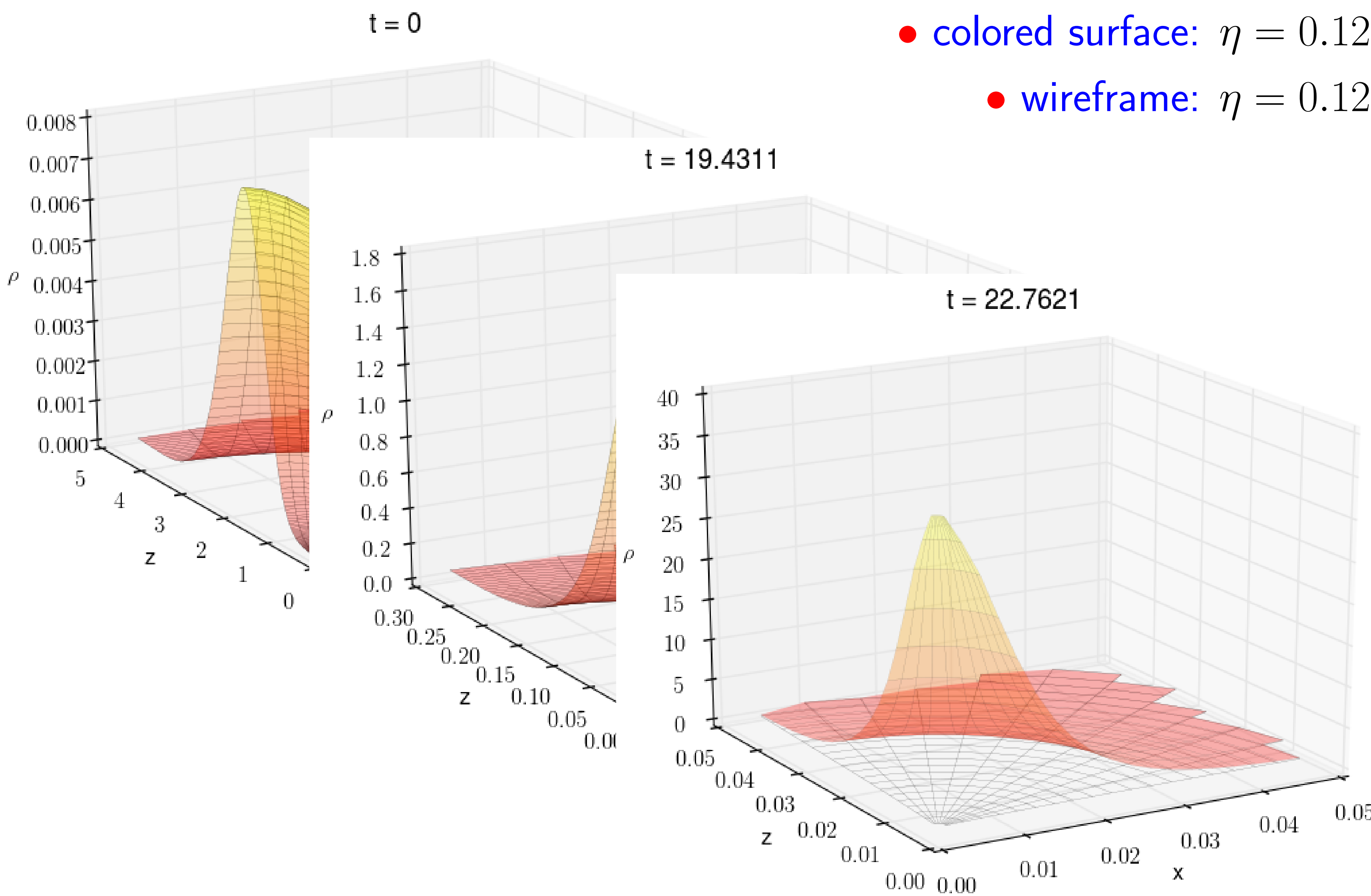
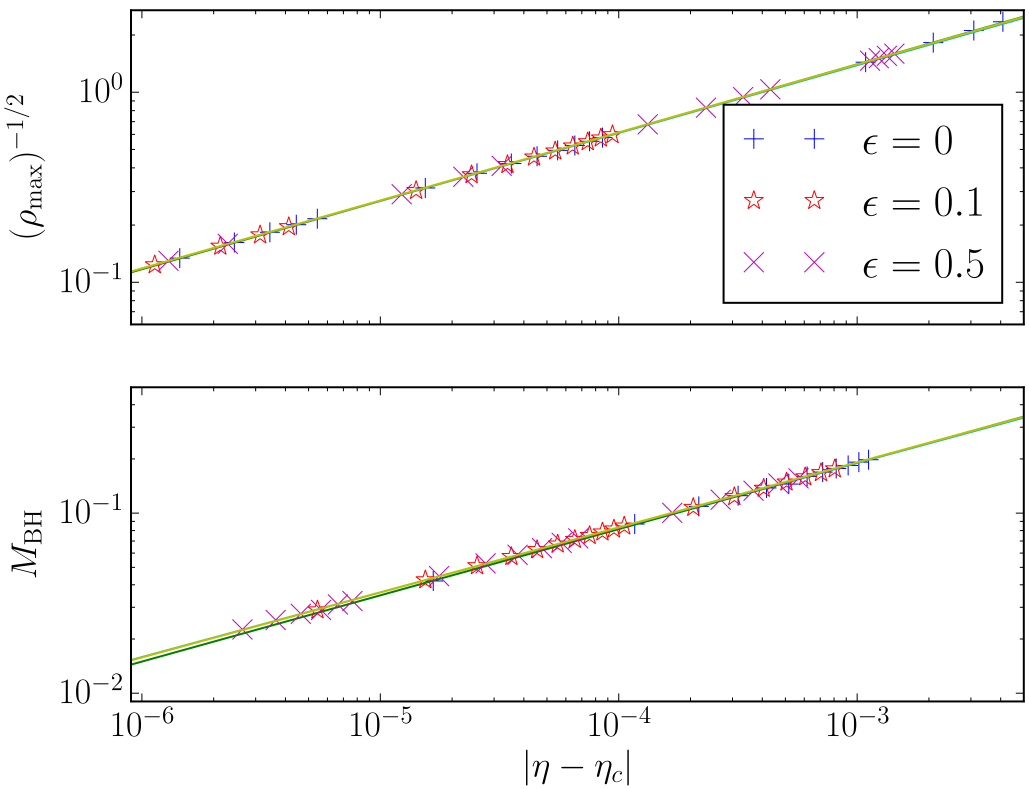


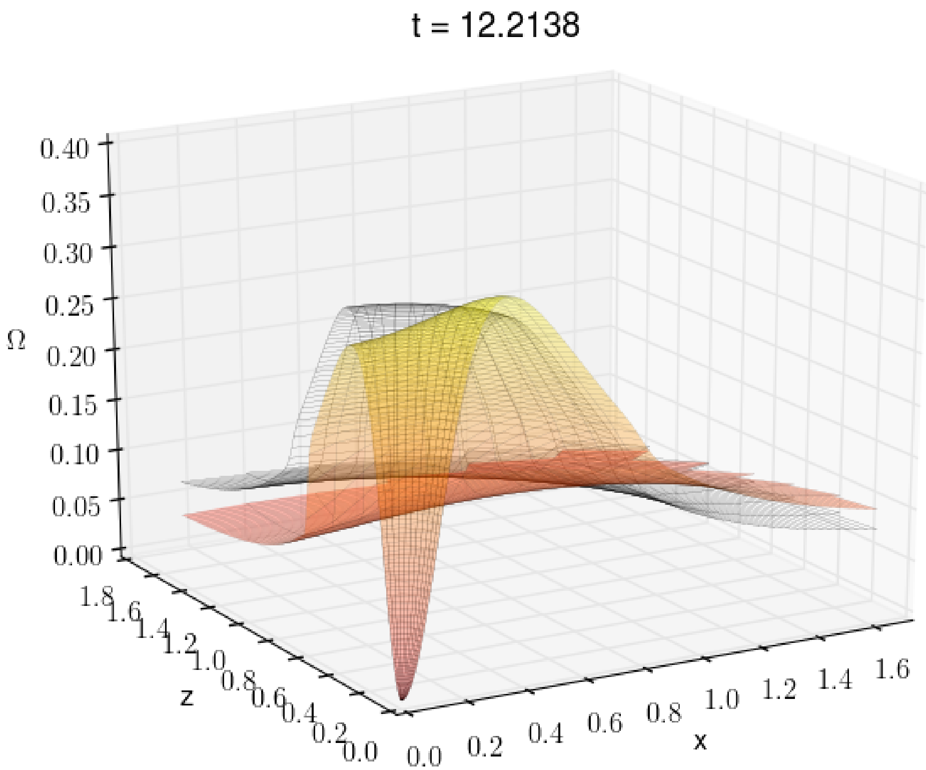
Critical Scaling

- Consider critical scaling for both sub- and supercritical data



$\Longrightarrow$ deviations from spherical symmetry do not appear to affect critical scaling

Approach to Self-Similarity



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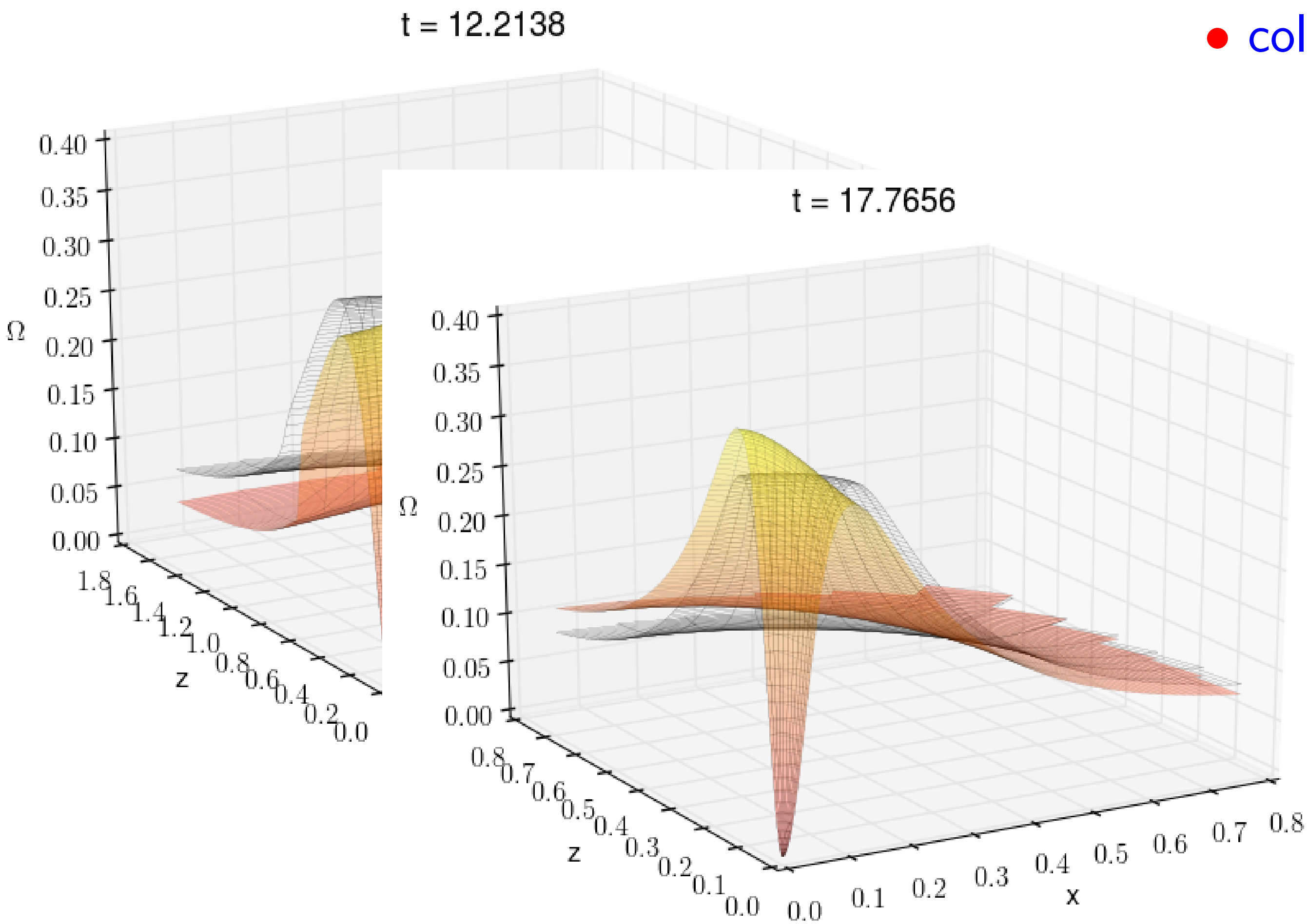

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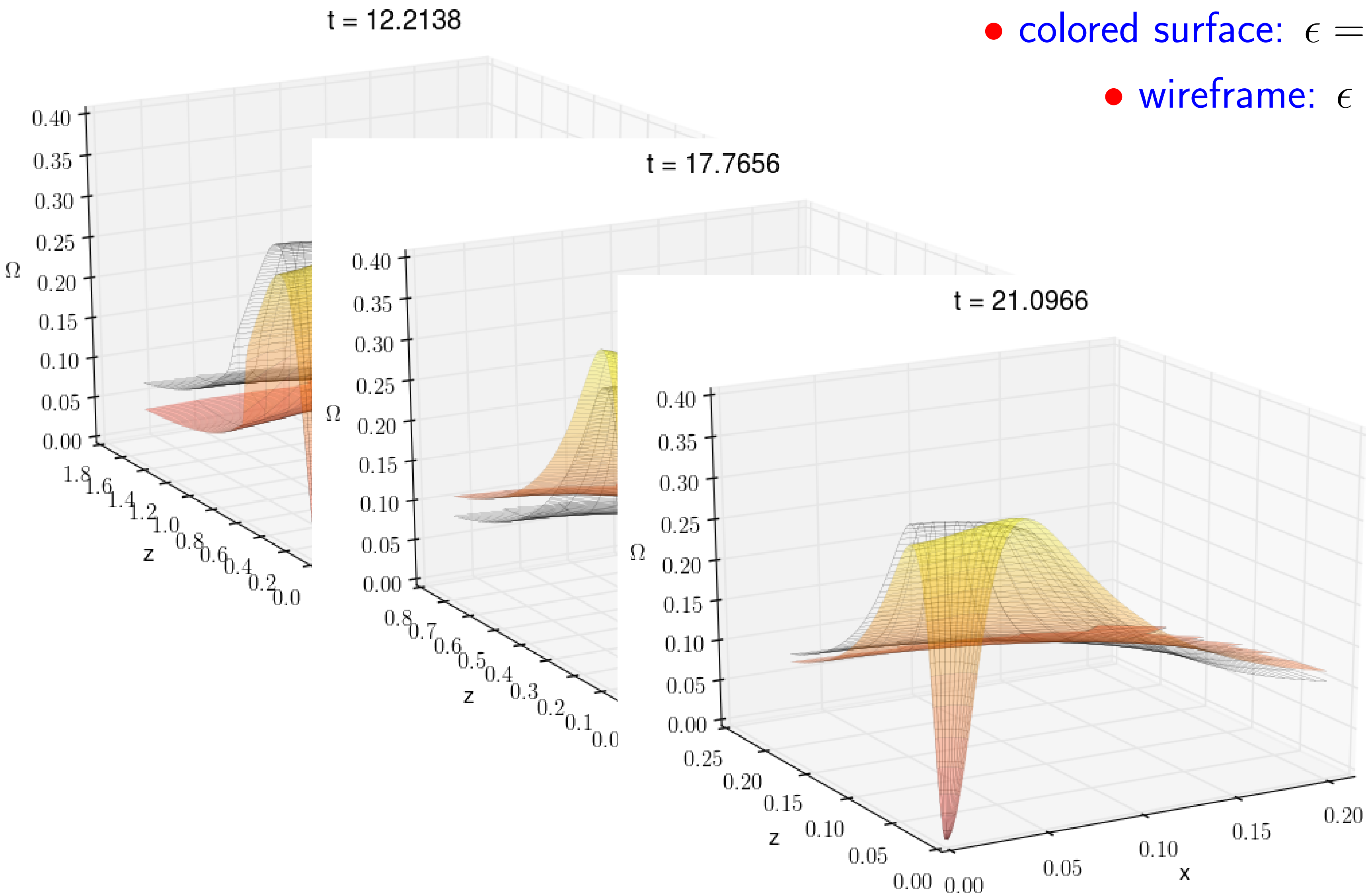

- colored surface: $\epsilon = 0.5$
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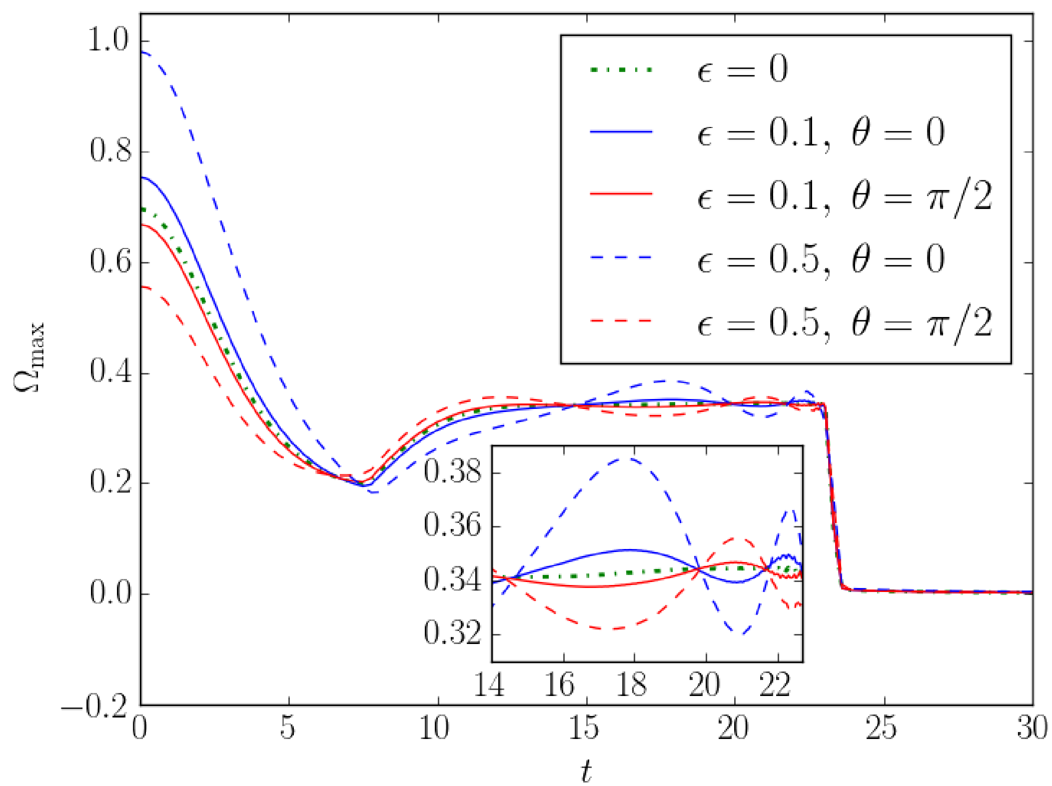
Approach to Self-Similarity

⟹ In aspherical collapse, solution close to criticality oscillates around spherical critical solution

⟹ plot maxima of Ω along axis ($\theta = 0$) and in equatorial plane ($\theta = \pi/2$)



⟹ Oscillations are *damped* with *decreasing* periods

Approach to Self-Similarity

- plot deviations from spherical data
- plot as function of

$$T \equiv -\ln(\tau^* - \tau)$$

(stretches τ^* to ∞)

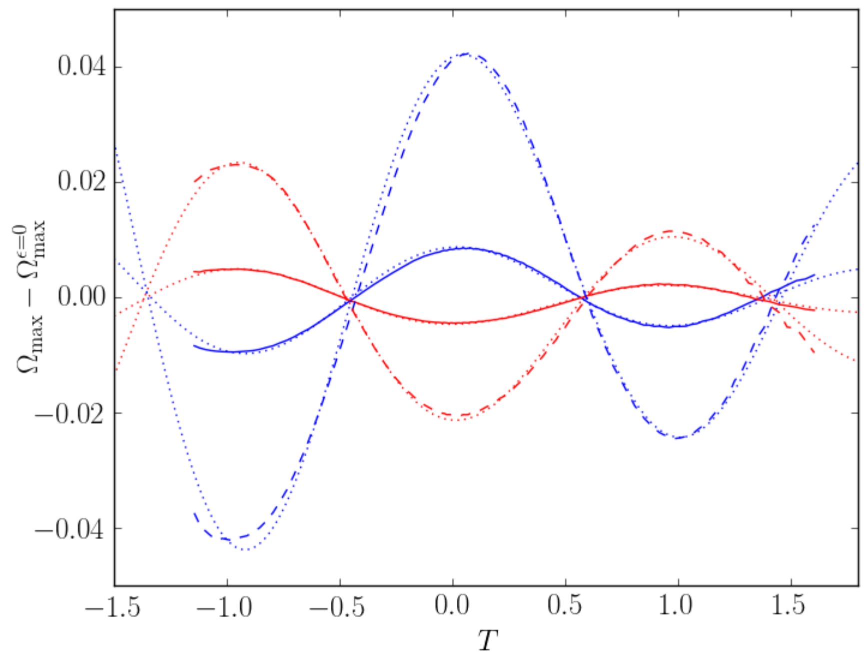
- fit to damped oscillations

$$u(T) = Ae^{-\kappa T}\cos(\omega T + \phi)$$

$\Longrightarrow$ good agreement for

$$\kappa \approx 0.35$$

$$\omega \approx 3.33$$



$\Longrightarrow$ Solution approaches spherically symmetric critical solution as $T \to \infty$, and hence $\tau \to \tau^*$

Comparison with perturbative calculations

Gundlach [1999, 2002] studied aspherical perturbations of critical solution

- perturbations obey

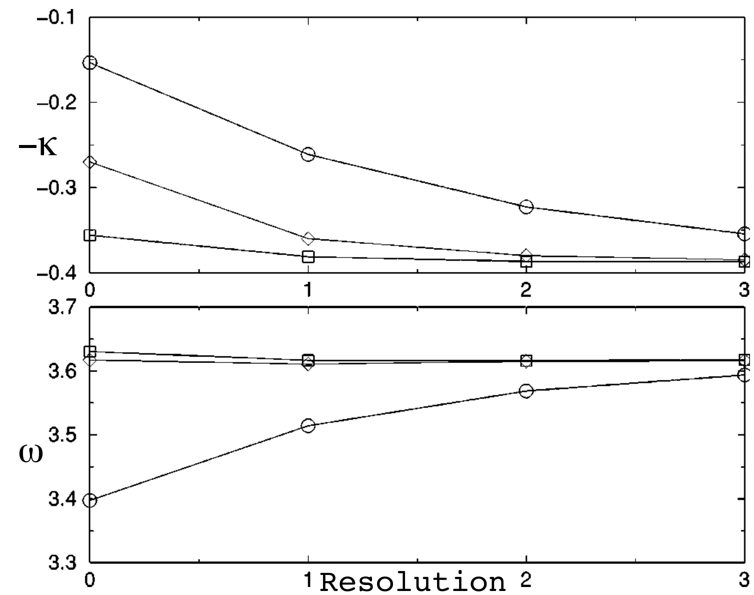
$$u(T) = Ae^{-\kappa T}\cos(\omega T + \phi)$$

- κ and ω determined numerically
- approximate values

$$\kappa \approx 0.38 \qquad (0.35)$$

$$\omega \approx 3.62 \qquad (3.33)$$

$\Longrightarrow$ to within 10% of our values



[Gundlach, 2002]

Summary

- Critical phenomena in gravitational collapse of radiation fluid
- Generalized results of Evans & Coleman [1994] for aspherical data
- Found that...
  - critical scaling not affected by deviations from spherical symmetry
  - close to criticality, evolution performs damped oscillations around critical solution
  - period and damping in good agreement with perturbative results [Gundlach 2002]
  - evolution approaches spherically symmetric critical solution
- Demonstrated that BSSN with moving-puncture coordinates suitable for study of critical collapse
  (compare [Akbarian & Choptuik, 2015])