Beginning inflation in an inhomogeneous universe

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work with Matthew Kleban, Andrei Linde, and Leonardo Senatore
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Inflation proposes to explain large scale homogeneity and flatness of the visible universe. However, most treatments of inflation assume homogeneity to begin with. Lore: inflation can only begin if you initially assume homogeneity on Hubble scales. Arguably no initial conditions problem if $V_{\text{Inflation}} \sim M_{\text{Pl}}$, however, models with $V_{\text{Inflation}} \ll M_{\text{Pl}}$ popular, especially with new results from observations.

Question to address: can we begin from very inhomogeneous conditions and eventually transition to exponential expansion?
Motivation

- Inflation is an attractor — need to consider large/nonlinear perturbations
- Overdensities may form black holes — strong field gravity is important
- Study in 3D (c.f. earlier work in spherical symmetry (Goldwirth & Piran, 1990))
- Use full numerical GR simulations to probe start of inflation beginning from initial conditions where gradient/kinetic energy dominates
Einstein equations: \[ R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} \]
Coupled to inflaton: \[ \Box \phi = V'(\phi) \]
Periodic boundary conditions: either numerical convenience or universe with toroidal topology
“Flat on average"
Initial conditions

\[ \phi(t = 0, x) = \phi_0 + \delta \phi \left[ \sum_{1 \leq |kL/2\pi|^2 \leq N} \cos(k \cdot x + \theta_k) \right] \]

- Start with gradient dominated energy conditions
  \[ \langle \rho \rangle \approx \langle \rho_{\text{grad}} \rangle \]
- Inflationary energy scale is initially much smaller:
  \[ V_{\text{Inflation}} = 10^{-3} \langle \rho \rangle \]
- Start with constant \( K (= -3H_0) \) set by average energy density
- Note that \( H_0^2 \propto \rho(t = 0) \propto k^2 \delta \phi^2 \) (Hamiltonian constraint)
- Hence different amplitudes corresponding to different values of \( H_0/k \)
Consider different values of \( k/H_0 \)

- When \( k/H_0 \gg 1 \) expect you should be able to average over inhomogeneities
- When \( k \sim H_0 \) expect black hole formation by hoop conjecture: mass in over density \( \sim \) size of over density (in geometric units)
- Hence becomes a strong-field gravity problem where gravitational collapse may spoil expansion
- When \( k/H_0 \ll 1 \) have superhorizon modes which act like curvature (not considering).
Inflaton values are entirely contained on the flat part of the potential — inflationary “plateau”:

\[ V \approx \Lambda \]
Volume-averaged energy density and expansion

\[ \langle \rho \rangle / \rho \Lambda \propto a^{-4} \]

\[ k/H_0 = 1 \quad k/H_0 = 8 \]
\[ k/H_0 = 2 \quad k/H_0 = 4 \times \]
\[ k/H_0 = 4 \times (1^{1/2}, \ldots, 5^{1/2}) \]

\[ \exp(\langle N \rangle) \]

\[ \langle H \rangle / H_\Lambda \propto a^{-2} \]
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Distance between black holes

\[ d_{BH}/L \propto a^{k/H_0} \]

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Second case

Consider cases when average field value gives slow-roll inflation, but variations can exceed inflationary plateau.

Step and notch-like potentials (e.g. cosmological attractors from Kallosh & Linde, 2013)
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Leaving the inflationary plateau

\[ \exp(\langle N \rangle) \]

\[ \langle H \rangle / H_\Lambda \propto a^{-2} \]

\[ \phi_0 / \delta \phi = 0.2 \text{ step} \]
\[ \phi_0 / \delta \phi = 0.5 \text{ step} \]
\[ \phi_0 / \delta \phi = 1.0 \text{ step} \]
\[ \phi_0 / \delta \phi = 0.2 \text{ notch} \]
\[ \phi_0 / \delta \phi = 0.5 \text{ notch} \]
\[ \phi_0 / \delta \phi = 1.0 \text{ notch} \]
When \( \delta \phi > \phi_0 \), field “feels” bottom of potential which rapidly pulls it to lower energy/non-inflationary values. For step-like potential:

- Average “force” on scalar field is proportional to
  \[
  \langle V' \rangle = V / \langle \phi \rangle
  \]
- Slow roll condition \( M_p V' / V = \epsilon < 1 \)
- \[
  \langle V'(\phi) \rangle \sim V(\langle \phi \rangle) / \langle \phi \rangle = \left( \frac{M_p}{\langle \phi \rangle} \right) \left( \frac{V'(\langle \phi \rangle)}{\epsilon} \right) > V'(\langle \phi \rangle)
  \]
  for \( \langle \phi \rangle < M_p / \epsilon \)
- Similar nonlinear effects cause potential energy decrease for notch-like potential
Conclusion

- Inflation can proceed from a large class of initial inhomogeneous initial conditions — no need to initially assume Hubble-sized homogeneous patches
- Collapsing regions with black holes form, but exponentially expanding regions eventually dominate the volume
- Holds except when distance to end of inflationary plateau \( \lesssim \delta \phi \)

Future work:

- Consider wider class of initial conditions/potentials
- Fate of crunching regions potentially produced during inflation by metastability of the Higgs

Many interesting applications of tools from numerical GR to addressing inhomogeneous/strong-field cosmology questions (see e.g. Eloisa Bentivegna’s talk from Monday).