A loophole in the electromagnetic cascade theory : application to the cosmological lithium problem

Vivian Poulin

LAPTh, Université Savoie Mont Blanc & CNRS, BP 110,
F-74941 Annecy-le-Vieux Cedex, France

Institute for Theoretical Particle Physics and Cosmology (TTK),
RWTH Aachen University, D-52056 Aachen, Germany.

March 1, 2016

Abstract

We discuss a loophole in the standard electromagnetic cascade theory that can have a large effect in the early universe, notably in altering primordial nucleosynthesis bounds on electromagnetically decaying relic particles. We show how this may greatly simplify the possibility to address the long-standing "lithium problem" in terms of new physics models.

1 Introduction

Electromagnetic cascades, namely the evolution of γ, $e^{\pm}$ particle numbers and energy distribution following the injection of a energetic γ or e in a medium filled with radiation, magnetic fields and matter, is one of the physical processes most frequently encountered in astroparticle physics, in domains as disparate as high-energy gamma-ray astrophysics, ultra-high-energy cosmic ray propagation, or the physics of the early universe. In particular, the elementary theory of such a cascade onto a photon background has been well known since decades, and can be shown via a textbook derivation (see Chapter VIII in [1], for instance) to lead to a universal "meta-stable" spectrum—attained on timescales much shorter than the thermodynamical equilibration scale—of the form:

$$\frac{dN_\gamma}{dE_\gamma} = \begin{cases} K_0 \left(\frac{E_\gamma}{\epsilon_X}\right)^{-3/2} \text{for } E_\gamma < \epsilon_X \,, \\ K_0 \left(\frac{E_\gamma}{\epsilon_X}\right)^{-2} \quad \text{for } \epsilon_X \leq E_\gamma \leq \epsilon_c \,, \\ 0 \qquad\qquad\quad \text{for } E > \epsilon_c \,. \end{cases} \tag{1}$$

In the above expression, $K_0 = E_0 \epsilon_X^{-2} [2 + \ln(\epsilon_c/\epsilon_X)]^{-1}$ is a normalization constant enforcing that the total energy is equal to the injected electromagnetic energy, E_0; the characteristic energy $\epsilon_c = m_e^2/\epsilon_\gamma^{\max}$ denotes the effective threshold for pair-production ($\epsilon_\gamma^{\max}$ being the highest energy of the photon background onto which pairs can be effectively created); $\epsilon_X \leq \epsilon_c/3$ is the maximum energy of up-scattered inverse Compton (IC) photons (See e.g. [2, 3, 4] for monte carlo studies leading to further justification of these parameters).

A notable application of this formalism concerns the possibility of a non-thermal nucleosynthesis phase in the early universe (for recent review on this and other aspects of primordial nucleosynthesis, or BBN, see [5, 6]). The determination of the baryon energy density of the universe Ω_b inferred from the CMB acoustic peaks measurements can be used in fact to turn the standard BBN into a parameter-free theory. The resulting predictions for the deuterium abundance (or ^{2}H, the most sensitive nuclide to Ω_b) are in remarkable agreement with observations, providing a tight consistency check for the standard cosmological scenario. The ^{4}He and ^{3}He yields too are, broadly speaking, consistent with this value, although affected by larger uncertainties and hence, are used to put severe constraints on the abundance and lifetime of unstable early universe relics, decaying electromagnetically. The ^{7}Li prediction, however, is a factor ~ 3 above its determination in the atmosphere of metal-poor halo stars. If this is interpreted as reflecting a cosmological value—as opposed to a post-primordial astrophysical reprocessing, a question which is far from settled [7, 8]—it requires a non-standard BBN mechanism, for which a number of possibilities have been explored [5, 6].

In particular, cosmological solutions based on electromagnetic cascades have been proposed in the last decade, see for instance [9]. However, typically they do not appear to be viable [6], as confirmed also in recent investigations (see for instance Fig. 4 in [10], dealing with massive "paraphotons") due to the fact that whenever the cascade is efficient in destroying enough ^{7}Li, the destruction of ^{2}H is too extreme, and spoils the agreement with the CMB observations mentioned above. Actually, this tension also affects some non-e.m. non-thermal BBN models, see for instance [11].

This difficulty can be evaded if one exploits the property that ^{7}Be (from which most of ^{7}Li come from for the currently preferred value of Ω_b, via late electron capture decays) has the lowest photodissociation threshold among light nuclei, of about 1.59 MeV vs. 2.22 MeV for next to most fragile, ^{2}H. Hence, to avoid any constraint from ^{2}H while being still able to photo-disintegrate some ^{7}Be, it is sufficient to inject photons with energy $1.6 < E_\gamma/\mathrm{MeV} < 2.2$, with a "fine-tuned" solution (see e.g. the remark in [6] or the discussion in [12]). Nonetheless, it turns out to be hard or impossible to produce a sizable reduction of the final ^{7}Li yield, while respecting other cosmological bounds, such as those coming from extra relativistic degrees of freedom (N_{eff}) or spectral distortions of the CMB. A recent concrete example of these difficulties has been illustrated in [13], which tried such a fine-tuned solution by studying the effects of O(10) MeV sterile neutrino decays.

However, in Refs. [14] and [15], we have recently argued that when the energy of a photon injected in the primordial plasma falls below the pair-production threshold, the universality of the non-thermal photon spectrum from the standard theory of electromagnetic cascades onto a photon background breaks down. This can have major consequences : first of all, the meta-stable spectrum for such models beeing non universal, the constraints in the abundance vs. lifetime plane for unstable particles decaying electromagnetically in the early universe, derived from the ^{2}H, ^{4}He and ^{3}He measurements also are non universal. In fact, they are often more stringent than commonly thought, up to an order of magnitude. But it also yields an unexpected gift. Indeed, we have illustrated in Ref. [14] how this may reopen the possibility of purely electromagnetic new physics solution to the lithium problem, and hence the possibility of solving this lond-standing issue with very simple extension to the standard model of particle physics.

2 E.m. cascades and breakdown of universal non-thermal spectrum

In general, in order to compute the non-thermal photon spectrum which can photo-disintegrate nuclei, one has to follow the coupled equations of photons and electron-positron populations. For the problem at hand, however, it is a good first approximation to ignore the non-thermal electrons (except if they were to constitute the first generation of injected particles, of course!): while the injected photons will in general Compton scatter and produce them, a further process, typically inverse Compton onto the photon background, is needed to channel back part of their energy in the photon channel. The energy of these photons is significantly lower than the injected photon one: whenever they are re-injected below nuclear photo-dissociation thresholds they are actually lost for non-thermal nucleosynthesis, otherwise they would contribute to *strengthening* the bounds. Here, we will neglect these secondary photons, but see Ref. [15] for a quantitative assessment of their effect for the cases we illustrate. Within this approximation, the Boltzmann equation describing the non-thermal photon distribution function f_γ reads:

$$\frac{\partial f_\gamma(E_\gamma)}{\partial t} = -\Gamma_\gamma(E_\gamma, T(t)) f_\gamma(E_\gamma, T(t)) + \mathcal{S}(E_\gamma, t)\,, \tag{2}$$

where $\mathcal{S}(E_\gamma, t)$ is the source injection term, Γ_γ is the total interaction rate, and we neglected the Hubble expansion rate since interaction rates are much faster and rapidly drive f_γ to a quasi-static equilibrium, $\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t} = 0$. Thus, we simply have :

$$f_\gamma^{\rm S}(E_\gamma, t) = \frac{\mathcal{S}(E_\gamma, t)}{\Gamma_\gamma(E_\gamma, t)}\,, \tag{3}$$

where the term $\mathcal{S}$ for an exponentially decaying species with lifetime τ_X and density $n_X(t)$, whose total e.m. energy injected per particle is E_0, can be written as

$$\mathcal{S}(E_\gamma, t) = \frac{n_\gamma^0 \zeta_X (1+z(t))^3\, e^{-t/\tau_X}}{E_0 \tau_X}\, p_\gamma(E_\gamma, t)\,, \tag{4}$$

with $z(t)$ being the redshift at time t, and the energy parameter ζ_X (conventionally used in the literature) is simply defined in terms of the initial comoving density of the X particle n_X^0 and the actual one of the CMB, n_γ^0, via $n_X^0 = n_\gamma^0 \zeta_X / E_0$. We shall use as reference spectrum the one for a two body decay $X \to \gamma U$ leading to a single monochromatic line of energy E_0, corresponding to $p_\gamma(E_\gamma) = \delta(E_\gamma - E_0)$. If the not better specified particle U is (quasi)massless, like a neutrino, one has $E_0 = m_X/2$, where m_X is the mass of the decaying particle. Note that here, we will be interested in masses m_X between a few MeV and a few hundreds of MeV, and at temperatures of order few keV or lower, hence the thermal broadening is negligible and a Dirac delta spectrum as the one above is appropriate.

We calculate Γ_γ by summing the rates of processes that degrade the injection spectrum, namely scattering off CMB photons, Bethe-Heitler pair creation and Compton scattering over thermal electron. Since the critical energy for pair-production is a dynamical quantity, that increases at later times due to the cooling of the universe, it may happen that the primary photons energy E_0 is above threshold for pair-production at early times, and below it at late times (we do take into account that the decay is not instantaneous). In general, at each time we will compare E_0 with ϵ_c, and use the universal spectrum when $E_0 > \epsilon_c$, and the solution of Eq. (3) when $E_0 < \epsilon_c$. This gives always a qualitatively correct solution, albeit it is somewhat approximate when $E_0 \sim \epsilon_c$. Since this is realized only in a very narrow interval of time, however, the final result are also quantitatively robust, barring peculiar mass-lifetime fine-tuning.

3 Non-thermal nucleosynthesis

At temperatures of few keV or lower, the standard BBN is over, and the additional nucleosynthesis can be simply dealt with as a post-processing of the abundances computed in the standard scenario, for which we use the input values from Parthenope [16], with the updated value of Ω_b coming from [17].

The non-thermal nucleosynthesis due to electromagnetic cascades can be described by a system of coupled differential equations of the type

$$\begin{aligned} \frac{dY_A}{dt} &= \sum_T Y_T \int_0^\infty dE_\gamma f_\gamma(E_\gamma, t) \sigma_{\gamma+T\to A}(E_\gamma) \\ &- Y_A \sum_P \int_0^\infty dE_\gamma f_\gamma(E_\gamma, t) \sigma_{\gamma+A\to P}(E_\gamma) \end{aligned} \tag{5}$$

where: $Y_A \equiv n_A/n_b$ is the ratio of the number density of the nucleus A to the total baryon number density n_b (this factors out the trivial evolution due to the expansion of the universe); $\sigma_{\gamma+T\to A}$ is the photodissociation cross section of the nucleus T into the nucleus A, i.e. the production channel for A; $\sigma_{\gamma+A\to P}$ is the analogous destruction channel. Both cross sections are actually vanishing below the corresponding thresholds. In general one also needs to follow secondary reactions of the nuclear byproducts of the photodissociation, which can spallate on or fuse with background thermalized target nuclei but none of that is relevant for the problem at hand: According to [9], the only signification secondary production is that of ^{6}Li. Despite extensive work in the past, the current observational status of ^{6}Li as a reliable nuclide for cosmological constraints is doubtful, given than most claimed detections have not been robustly confirmed, and a handful of cases are insufficient to start talking of a "cosmological" detection, see [8]. We shall thus conservatively ignore this nuclide and the secondary reactions in the following.

With standard manipulations, namely by transforming Eq. (5) into redshift space, defining $H(z) = H_r^0(1+z)^2$ as appropriate for a Universe dominated by radiation (with $H_r^0 \equiv H_0\sqrt{\Omega_r^0}$, H_0 and Ω_r^0 being the present Hubble expansion rate and fractional radiation energy density, respectively) one arrives at

$$\begin{aligned}\frac{dY_A}{dz} &= \frac{-1}{H_r^0(z+1)^3} \\ &\times \left[\sum_T Y_T \int_0^\infty dE_\gamma f_\gamma(E_\gamma,z)\sigma_{\gamma+T\to A}(E_\gamma) - Y_A \sum_P \int_0^\infty dE_\gamma f_\gamma(E_\gamma,z)\sigma_{\gamma+A\to P}(E_\gamma)\right]. \quad (6)\end{aligned}$$

For the specific case of a decaying particle of lifetime τ_X and mass m_X into e.m. byproducts described before, one finally gets to the system of equations:

$$\begin{aligned}\frac{dY_A}{dz} &= -\frac{n_\gamma^0 \zeta_X}{H_r^0 E_0 \tau_X} \exp\left(\frac{-1}{2H_r(1+z)^2}\right) \\ &\times \left[\sum_T Y_T \int_0^\infty dE_\gamma \frac{p_\gamma(E_\gamma)\,\sigma_{\gamma+T\to A}(E_\gamma)}{\Gamma_\gamma(E_\gamma,z)} + Y_A \sum_P \int_0^\infty dE_\gamma \frac{p_\gamma(E_\gamma)\,\sigma_{\gamma+A\to P}(E_\gamma)}{\Gamma_\gamma(E_\gamma,z)}\right]. \quad (7)\end{aligned}$$

4 A simple solution to the lithium problem

If the injected energy is $1.59 < E_0/\mathrm{MeV} < 2.22$, the only open non-thermal BBN channel is $^7\mathrm{Be}(\gamma,{}^3\mathrm{He})^4\mathrm{He}$, whose cross-section we denote with $\sigma_\star$, there are no relevant source terms and only one evolving species (since $Y_7 \ll Y_{3,4}$). Furthermore, the relevant energy range beeing very small, almost all scattered photons are already below the ^{7}Be photodissociation threshold. In this special case, a monochromatic emission line would simply correspond to $p_\gamma(E_\gamma) = \delta(E_\gamma - E_0)$ thus yielding for the final (at z_f) to initial (at z_i) abundance ratio

$$\ln\left(\frac{Y_{^7\mathrm{Be}}(z_i)}{Y_{^7\mathrm{Be}}(z_f)}\right) = \int_{z_f}^{z_i} \frac{n_\gamma^0 \zeta_X\, \sigma_\star(E_0)\, c\, e^{\frac{-1}{2H_r^0\tau_X(z'+1)^2}}}{E_0 H_r^0 \tau_X \Gamma(E_0,z)} dz'\,. \quad (8)$$

H_0 and Ω_r^0 being the present Hubble expansion rate and fractional radiation energy density, respectively. By construction, equating the suppression factor given by the RHS of the Eq. (8) to $\sim 1/3$ provides a solution to the ^{7}Li problem which is in agreement with all other constraints from BBN. In Fig. 1 left panel, the lower band shows for each τ_X the range of ζ_X corresponding to a depletion from 40% to 70%, for the monochromatic photon injection case with $E_0 = 2\,\mathrm{MeV}$. Similar results would follow by varying E_0 by 10% about this value, i.e. provided one is not too close to the reaction threshold. The upper band represents the analogous region if we had distributed the same injected energy according to the spectrum of Eq. (1), up to $\min[\epsilon_c\,, E_0]$.
We now compare our region with constraints coming from the CMB. It is well known that a late injection of photons in the thermal bath can lead to additional measurable cosmological alterations. For instance, the injection of a significant amount of energy can lead to modification of the photon-baryon ratio η or equivalently, to the increase of the co-moving entropy. Since the inferred values of Ω_b at BBN and CMB epoch are compatible, no major entropy release could have taken place between nucleosynthesis and decoupling. We compute the small fractional change in entropy using Ref. [18]. The 2-σ limit around the measured value of $\Omega_b h^2$ by Planck translates into a constraint on the entropy variation of 0.022 [17]. Furthermore, as reviewed in detail in [19], the spectrum of the CMB itself can

also be affected through two types of deformation: a modification of the chemical potential μ and a modification of the Compton-y parameter, which is related to the energy gained by a photon after a Compton scattering. For the relatively early time we focus on, the constraints come essentially from μ-type distortions. We follow here the results of Ref. [19], which contains improvements with respect to the ones given in [20], notably for $z < 2 \times 10^6$, while [20] is accurate enough at late times (see Fig. 16 in [19]). We used the limit given by COBE on the chemical potential: $|\mu| \leq 9 \times 10^{-5}$ [21].

It is clear that in the correct treatment a large portion of this region survives other cosmological constraints, described below, while none survives in the incorrect treatment.

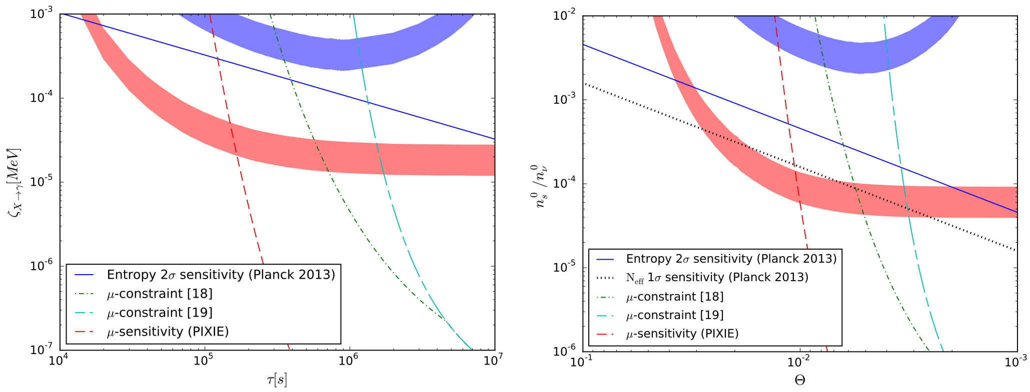

Figure 1: *Left Panel* - The lower band is the range of abundance parameter $\zeta_{X\to\gamma}$ vs. lifetime τ_X, for which the primordial lithium is depleted to 40% to 70% of its standard value, for a monochromatic photon injection with energy $E_0 = 2\,\text{MeV}$. The upper band represents the analogous region if we had distributed the same injected energy, up to $E_0 = 2\,\text{MeV}$, according to the erroneous spectrum of Eq. (1). Above the solid blue curve, a change in entropy (and Ω_b) between BBN and CMB time larger than the 2σ error inferred from CMB would be obtained. The region to the right of the dot-dashed green curve is excluded by current constraints from μ-distortions in the CMB spectrum [21] according to the computation of [19], while the dashed cyan curve illustrates the weaker bounds that would follow from the less accurate parameterization of [20]. The dotted red curve is the forecasted sensitivity of the future experiment PIXIE, corresponding to $|\mu| \sim 5 \times 10^{-8}$[22]. *Right Panel* - Constraints for the sterile neutrino model discussed in the text. Same legend as left panel.

One may wonder how realistic such a situation is in a concrete particle physics model. Although we refrain here from detailed model-building considerations, it is worth showing as a proof-of-principle that models realizing the mechanism described here while fulfilling the other cosmological constraints (as well as laboratory ones) can be actually constructed. Let us take the simplest case of a sterile Majorana neutrino with mass in the range $3.2 < M_s/\text{MeV} < 4.4$, mixing *mainly* with flavour α neutrinos via an angle θ_α. We also define $\Theta^2 \equiv \sum_\alpha \theta_\alpha^2$. The three main decay channels of this neutrino are (see e.g. [23] and refs. therein):

- $\nu_s \to 3\nu$, with rate $\Gamma_{\nu_s\to 3\nu} \simeq \frac{G_F^2 M_s^5 \Theta^2}{192\pi^3}$;
- $\nu_s \to \nu_\alpha e^+ e^-$, with a rate depending on single θ_α's;
- $\nu_s \to \nu\gamma$, with a rate $\Gamma_{\nu_s\to\nu\gamma} \simeq \frac{9 G_F^2 \alpha M_s^5}{256\pi^4}\Theta^2$.

The resulting branching ratios for the masses of interest and $\theta_e \ll \Theta$ are of the level of $0.9 : 0.1 : 0.01$, respectively. It is physically more instructive to normalize the abundance of the ν_s, n_s^0, in terms of one thermalized neutrino (plus antineutrino) flavour species, n_ν^0. In Fig. 1 right panel, we show the corresponding range of parameters in the $\Theta - n_s^0/n_\nu^0$ plane, for $M_s = 4.4\,\text{MeV}$, for which the ^{7}Li problem is solved, fulfills cosmological constraints and, provided that $\theta_e \ll \Theta$, also laboratory ones [24]. It is worth noting that: i) the entropy release bound is now close to the region of interest, since the decay mode $\nu_s \to \nu_\alpha e^+ e^-$, which is useless as far as the ^{7}Be dissociation is concerned, dominates the e.m. energy injection. ii) A non-negligible fraction of relativistic "dark energy" is now injected, mostly via the dominant decay mode $\nu_s \to 3\nu$; hence we added the current $1\,\sigma$ sensitivity of Planck to $N_{\rm eff}$ [17], with $\Delta N_{\rm eff}$ computed similarly to what done in [13]. The needed abundance could be obtained in scenarios with low reheating temperature [24].

5 Conclusions

We have argued that the universality of the photon spectral shape in electromagnetic cascades has often been used in cosmology even beyond its regime of applicability. When the energy of the injected photons falls below the pair-production threshold, i.e. approximately when $E_\gamma \leq m_e^2/(22\,T) \sim 10\,T_{\rm keV}^{-1}$ MeV, the universal form breaks down. The possibility to find new mechanisms to deplete the standard BBN prediction of lithium abundance in a consistent way is probably the most spectacular consequence of our investigation. In turn, this could stimulate more specific model-building activities. For instance, decays of relatively light new neutral fermionic particles X for which the $\nu+\gamma$ channel is the only two body SM channel opened—as it is the case for the light gravitinos in supergravity models—constitute a natural class of candidates. Alternatively, one may think of decaying scenarios involving a pair of quasi degenerate mass states X and Y, which are potentially much heavier than the MeV scale. Some of these scenarios may be motivated by other astroparticle or particle physics reasons and certainly deserve further investigation.

Let us quickly discussed the implication for the BBN constraints on early electromagnetically decaying particles. In Ref. [15], we have shown that, for illustrative cases of monochromatic energy injection at different epochs, these bounds are i) non universal, in contrary with what was commonly thought; ii) often much stronger than the ones presented in the literature (up to an order of magnitude), notably when the injected photon energy falls close to the peak of the photodisintegration cross-section of the relevant nucleus. In fact, the breaking of the non-universality is non-trivial and is essentially controlled by the energy behavior of the cross-sections: in the universal limit, most of the photons lie at relatively low-energies, so that the cross-section behaviour at the resonance just above threshold is what matters the most. In the actual treatment, the photons may be also sensitive to the high-energy tail of the process. Future studies aiming at assessing the nuclear physics uncertainties affecting these types of bounds would benefit from this insight. It cannot be excluded that in some cases constraints *weaken* a bit with respect to what considered in the literature.

References

[1] V. S. Berezinsky *et al.* Amsterdam, Netherlands: North-Holland (1990) 534 p.

[2] J. R. Ellis *et al.*, Nucl. Phys. B **373**, 399 (1992). doi:10.1016/0550-3213(92)90438-H.

[3] M. Kawasaki and T. Moroi, Astrophys. J. **452** (1995) 506.

[4] R. J. Protheroe *et al.*, Phys. Rev. D **51** (1995) 4134.

[5] F. Iocco *et al.*, Phys. Rept. **472**, 1 (2009).

[6] M. Pospelov and J. Pradler, Ann. Rev. Nucl. Part. Sci. **60**, 539 (2010).

[7] M. Spite *et al.*, Memorie della Società Astronomica Italiana Supplementi,2012 22, 9.

[8] F. Iocco, arXiv:1206.2396 [astro-ph.GA].

[9] R. H. Cyburt *et al.*, Phys. Rev. D **67** (2003) 103521.

[10] A. Fradette *et al.*, Phys. Rev. D **90**, 035022 (2014).

[11] M. Kusakabe *et al.*, Phys. Rev. D **90**, 045009 (2014).

[12] M. Kusakabe *et al.*, Phys. Rev. D **87**, no. 8, 085045 (2013).

[13] H. Ishida *et al.*, arXiv:1403.5995 [astro-ph.CO].

[14] V. Poulin and P. D. Serpico, Phys. Rev. Lett. **114**, no. 9, 091101 (2015).

[15] V. Poulin and P. D. Serpico, Phys. Rev. D **91**, no. 10, 103007 (2015).

[16] O. Pisanti *et al.*, Comput. Phys. Commun. **178**, 956 (2008).

[17] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5075 [astro-ph.CO].

[18] J. L. Feng *et al.*, Phys. Rev. D **68** (2003) 063504.

[19] J. Chluba and R. A. Sunyaev, Mon. Not. Roy. Astron. Soc. **419** (2012). 1294

[20] W. Hu and J. Silk, Phys. Rev. Lett. **70** (1993) 2661.

[21] D. J. Fixsen *et al.*, Astrophys. J. **473**, 576 (1996).

[22] A. Kogut *et al.*, JCAP **1107** (2011) 025.

[23] F. Bezrukov *et al.*, Phys. Rev. D **81** (2010) 085032.

[24] G. Gelmini *et al.*, JCAP **0810**, 029 (2008).