The m-z relation for type Ia supernovae, locally inhomogeneous cosmological models, and the nature of dark matter

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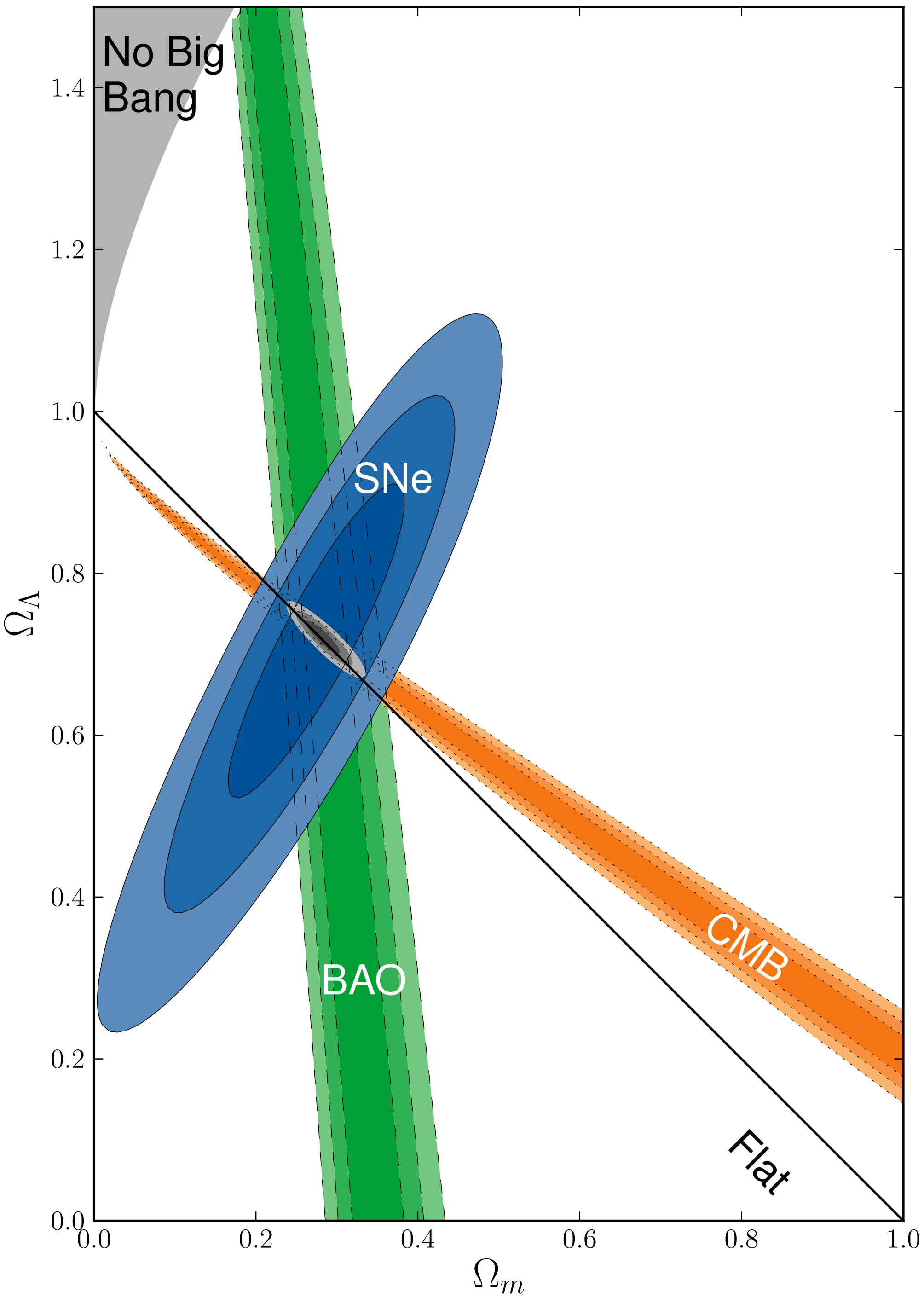
Overview

- Introduction
- Basic theory
- History
- This work
  - calculations
  - results and discussion—and surprises
  - averaging
- Conclusions

Introduction

- The m-z relation is a classic cosmological test.
- The 2011 Nobel Prize was awarded for work with type Ia supernovae.
- It is now one of many complementary cosmological tests.



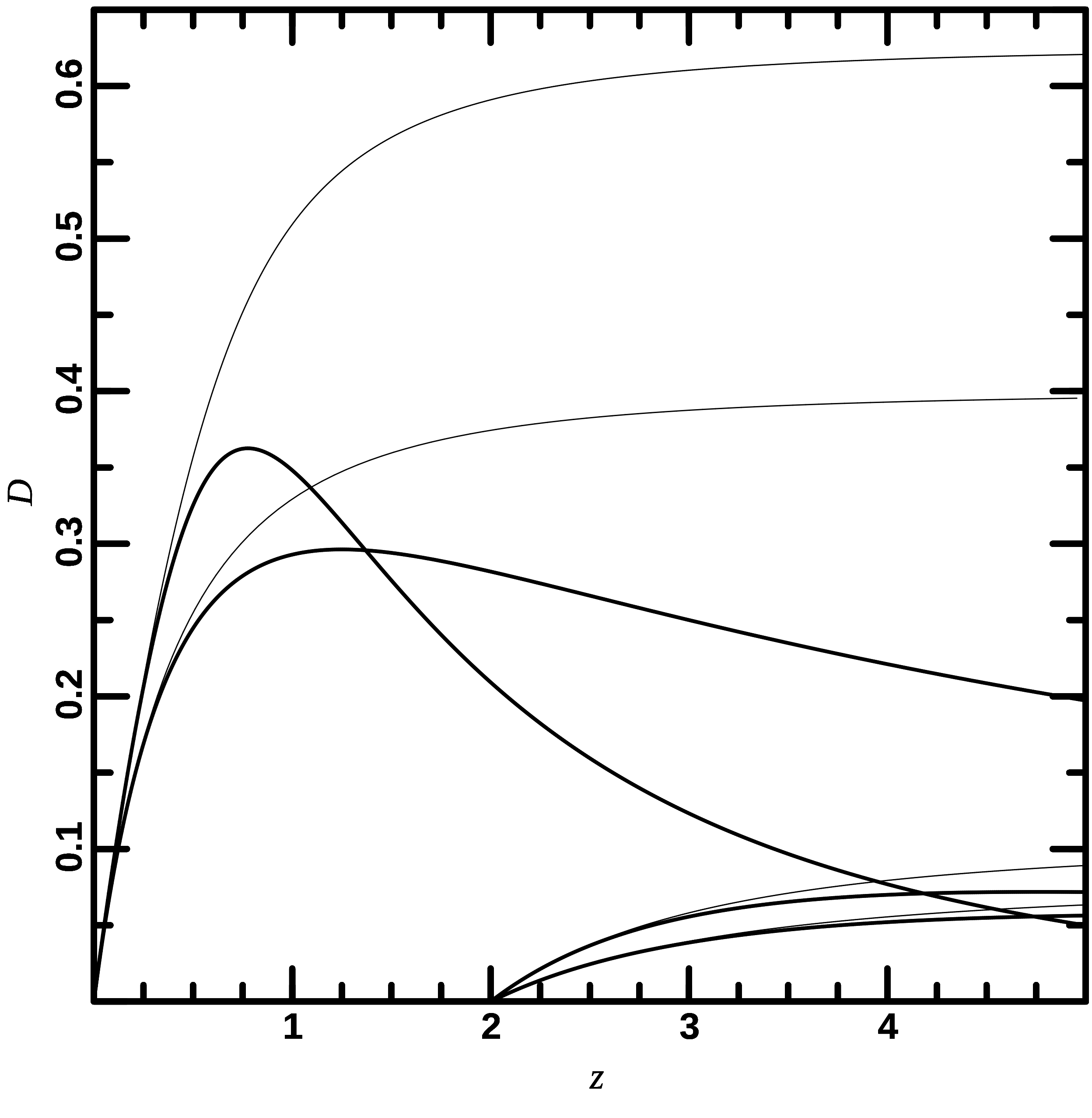
Suzuki *et al.* (2011)

Executive abstract

- The universe is obviously inhomogeneous.
- Does this matter for observational cosmology?
- It depends.
- The universe appears to be homogeneous on large scales but of course not on small scales.
- How does this affect one of the classical cosmological tests, the magnitude-redshift (m-z) relation, in particular for type Ia supernovae?

Basic theory (1/4)

- Fraction η is distributed homogeneously.
- Fraction $1 - \eta$ is distributed clumpily.
- η depends on the relevant scale.
- If η isn't mentioned, the authors assume $\eta = 1$.
- See Kayser, Helbig & Schramm (1997) for more details and Fortran code.



Thin: $\eta = 0$, thick: $\eta = 1$. The upper curves near $z = 0$ ($z = 2$ at lower right) are for $\lambda_0 = 2$, the lower for $\lambda_0 = 0$. $\Omega_0 = 1$ for all curves.

Basic theory (2/4)

- Small clumps are effectively homogeneous.

- Dark matter is transparent.

- $\eta \neq 1$ is the most obvious departure from the simplest model (Einstein-de Sitter with $\eta = 1$), though other departures are more often considered.

  – The universe is obviously not homogeneous at small scales.

  – $\Omega_0 = 1$ was believed by many people still alive today.

  – There is no evidence that 'dark energy' is anything other than the cosmological constant.

Basic theory (3/4)

- $\eta < 0$ is unphysical.

- $\eta > 1$ is possible, but would imply:
  - long, thin structures directed at us
  - gravitational-lens configuration

- η is an approximation, but if assumptions break down, treat as gravitational-lens effect with $\eta \approx 0$.

Basic theory (4/4)

- More complicated models are possible, but η is enough for now.

- η can be an arbitrary function of z, but here I consider only constant values.

- η can vary from one source to another:
  - additional source of uncertainty
  - in practice, not detectable due to worse fit
  - $\eta = 0$ or $\eta = 1$ indicates that there is no variation

Previous work

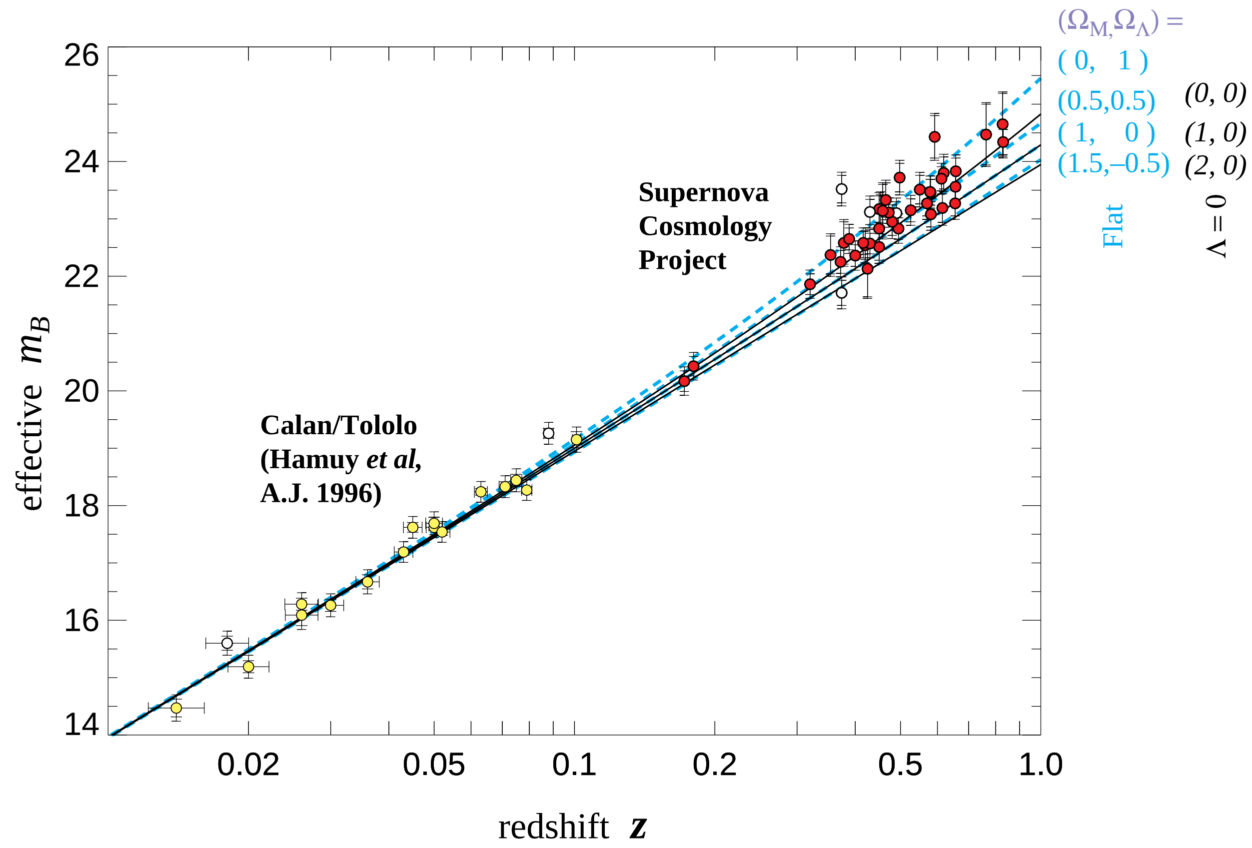
- Light propagation in locally inhomogeneous cosmological models was first investigated in detail by Zel'dovich (1964), Dashevskii & Zel'dovich (1965), Dashevskii & Slysh (1966)

- This topic entered 'mainstream cosmology' with papers by Dyer and Roeder (several papers, alone and together).

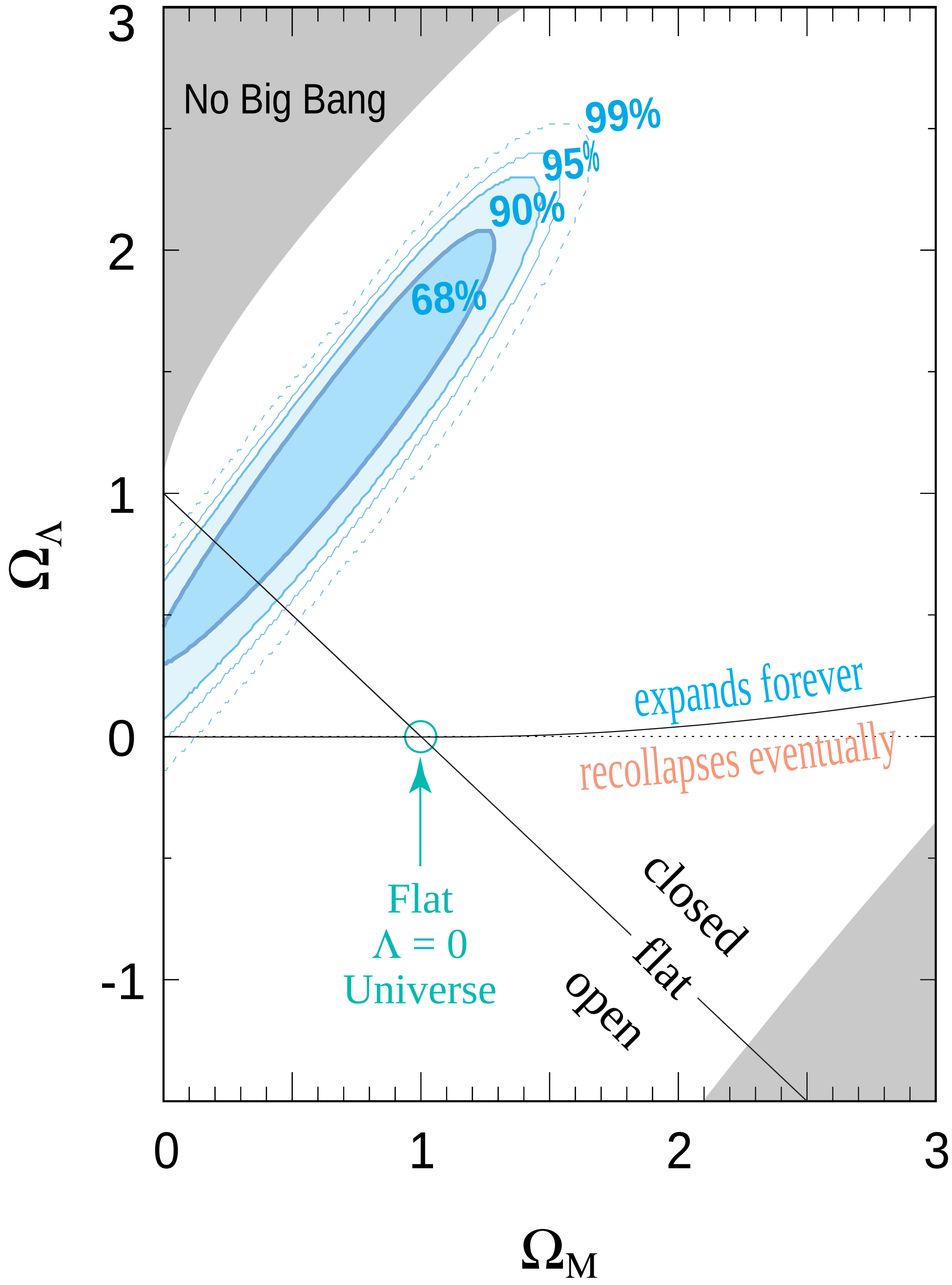
- The importance of η in the for the m-z relation has been emphasized by Kantowski.

Previous work

- Is it a good approximation?
  - Mörtsell (2002)
  - Jönsson, Kronborg, Mörtsell & Sollerman (2008)
  - Bergström, Goliath, Goobar & Mörtsell (2000)
- There is a simple analysis in the most important SCP paper.



Perlmutter *et al.* (1999)



Perlmutter *et al.* (1999)

What have I done?

- Calculations of χ^2 and the corresponding probability on two three-dimensional grids:
  - large, low-resolution grid
    - $-5 < \lambda_0 < 5$, $\Delta\lambda_0 = 0.02$ (500 points)
    - $0 < \Omega_0 < 10$, $\Delta\Omega_0 = 0.02$ (500 points)
    - $0 < \eta < 1$, $\Delta\eta = 0.01$ (100 points)
  - small, high-resolution grid
    - $0 < \lambda_0 < 1.5$, $\Delta\lambda_0 = 0.003125$ (480)
    - $0 < \Omega_0 < 1$, $\Delta\Omega_0 = 0.003125$ (320)
    - $0 < \eta < 1$, $\Delta\eta = 0.01$ (100)

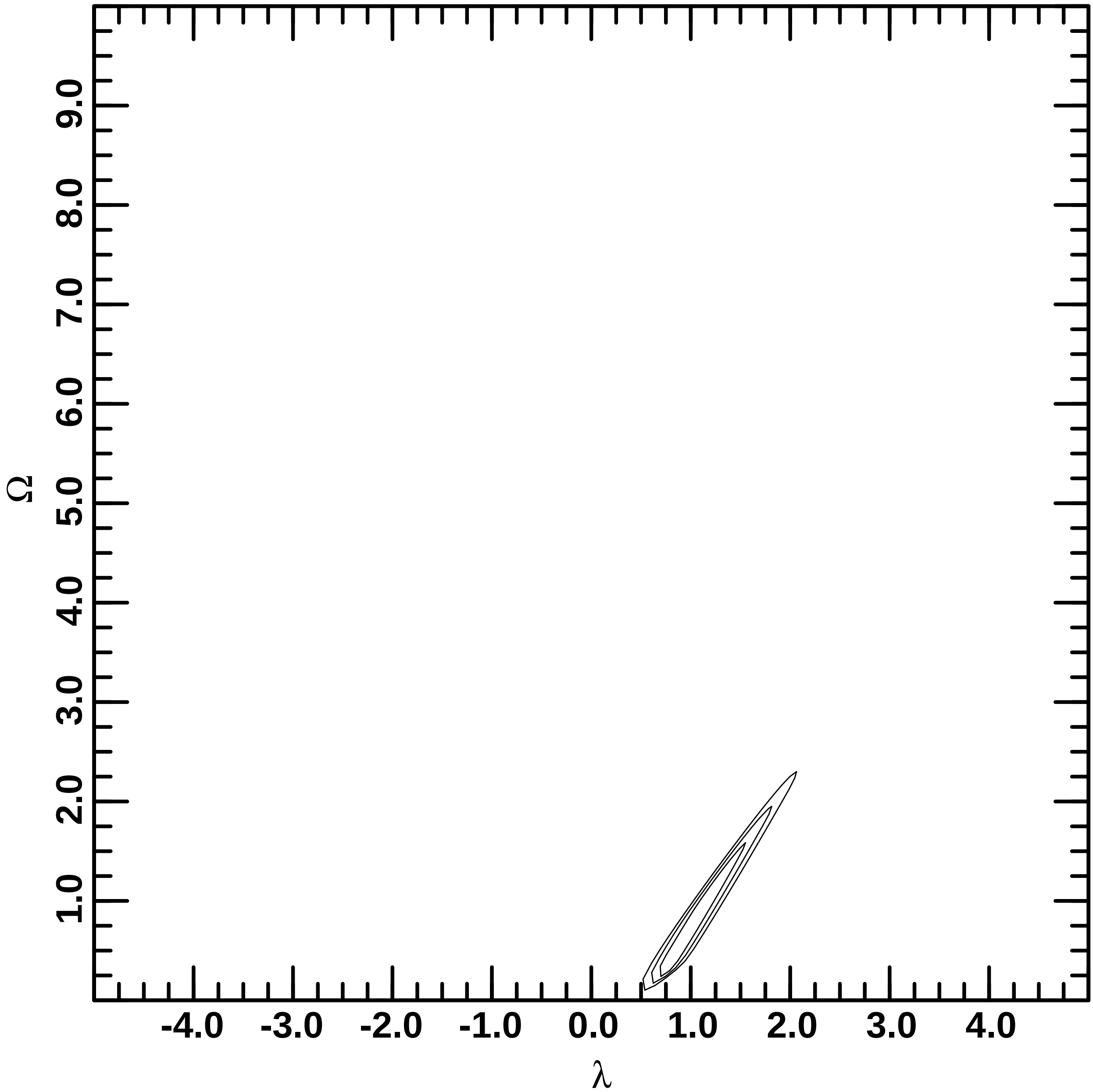
- Various two-dimensional visualizations

Calculate contours of

relative probability in

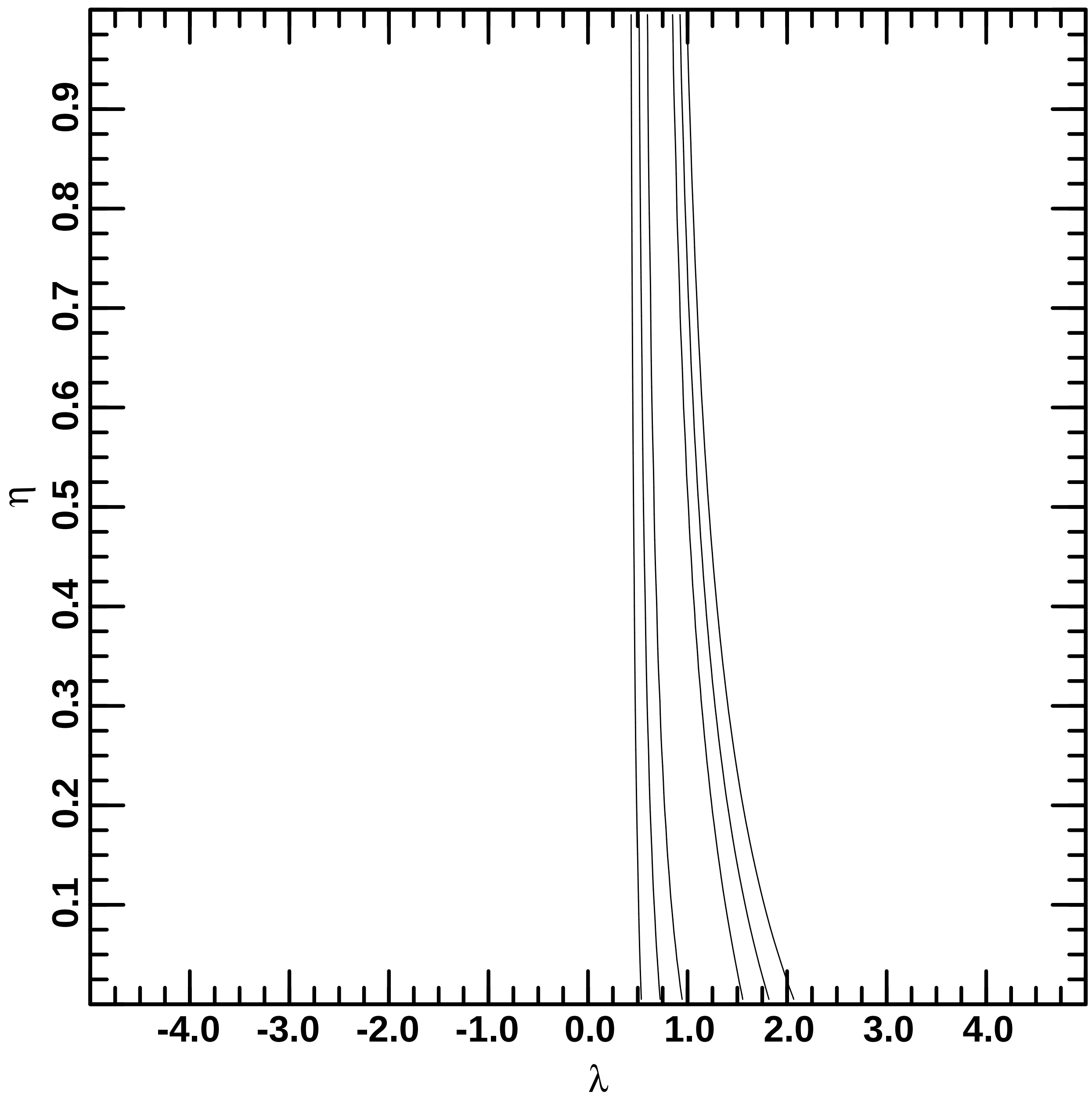
three-dimensional space;

project onto two dimensions



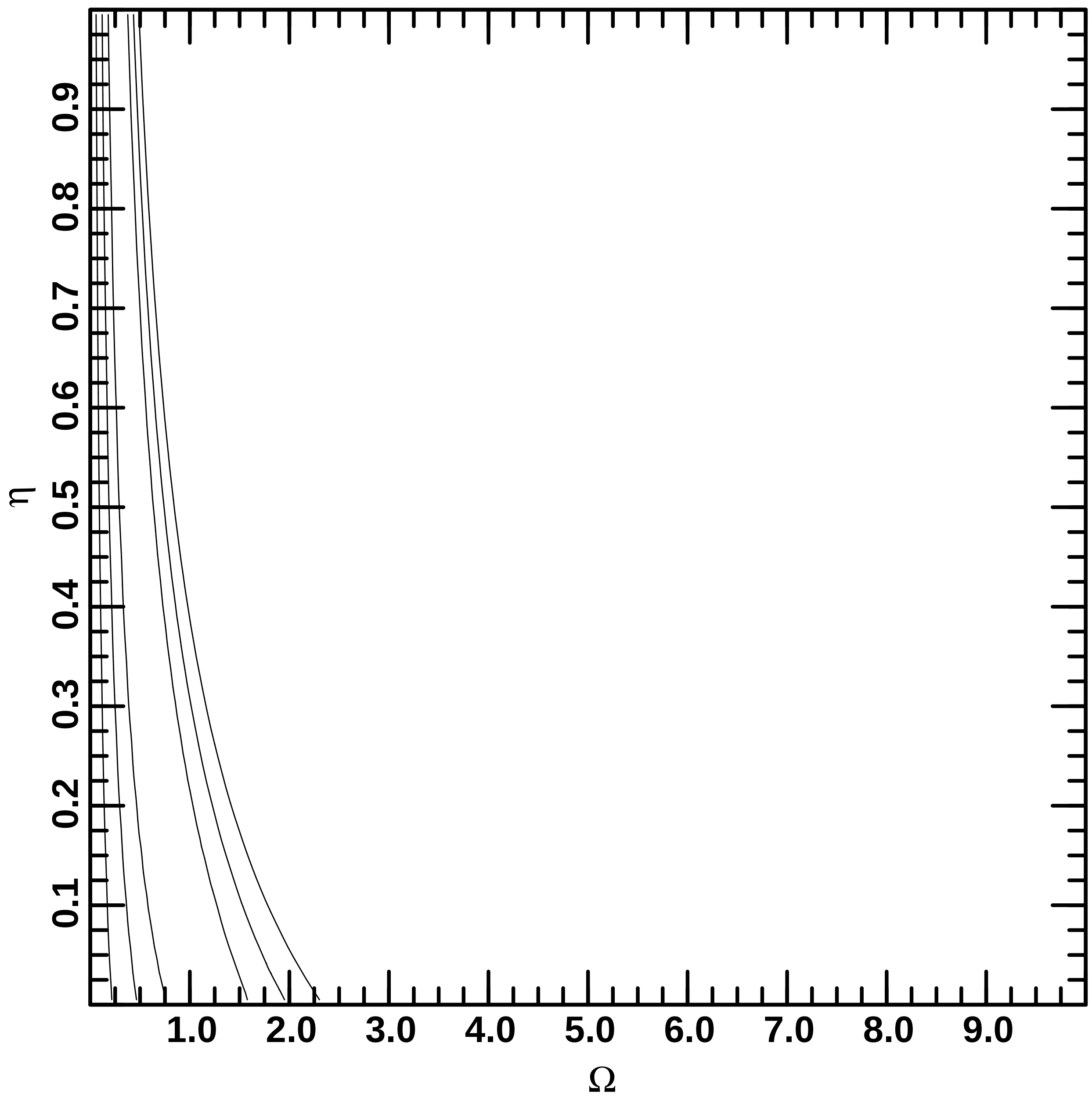
large, low-resolution grid

projected three-dimensional contours



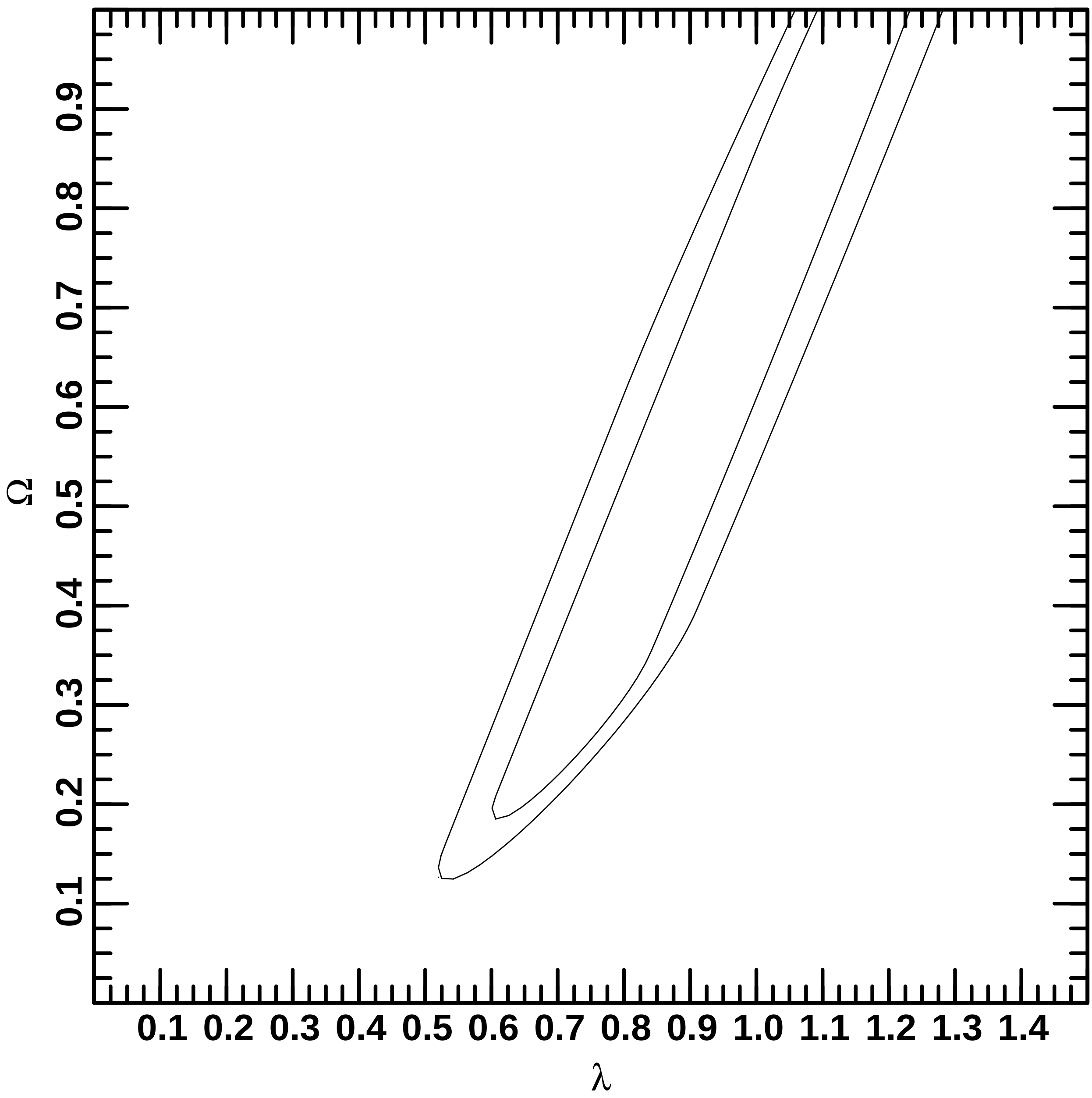
large, low-resolution grid

projected three-dimensional contours



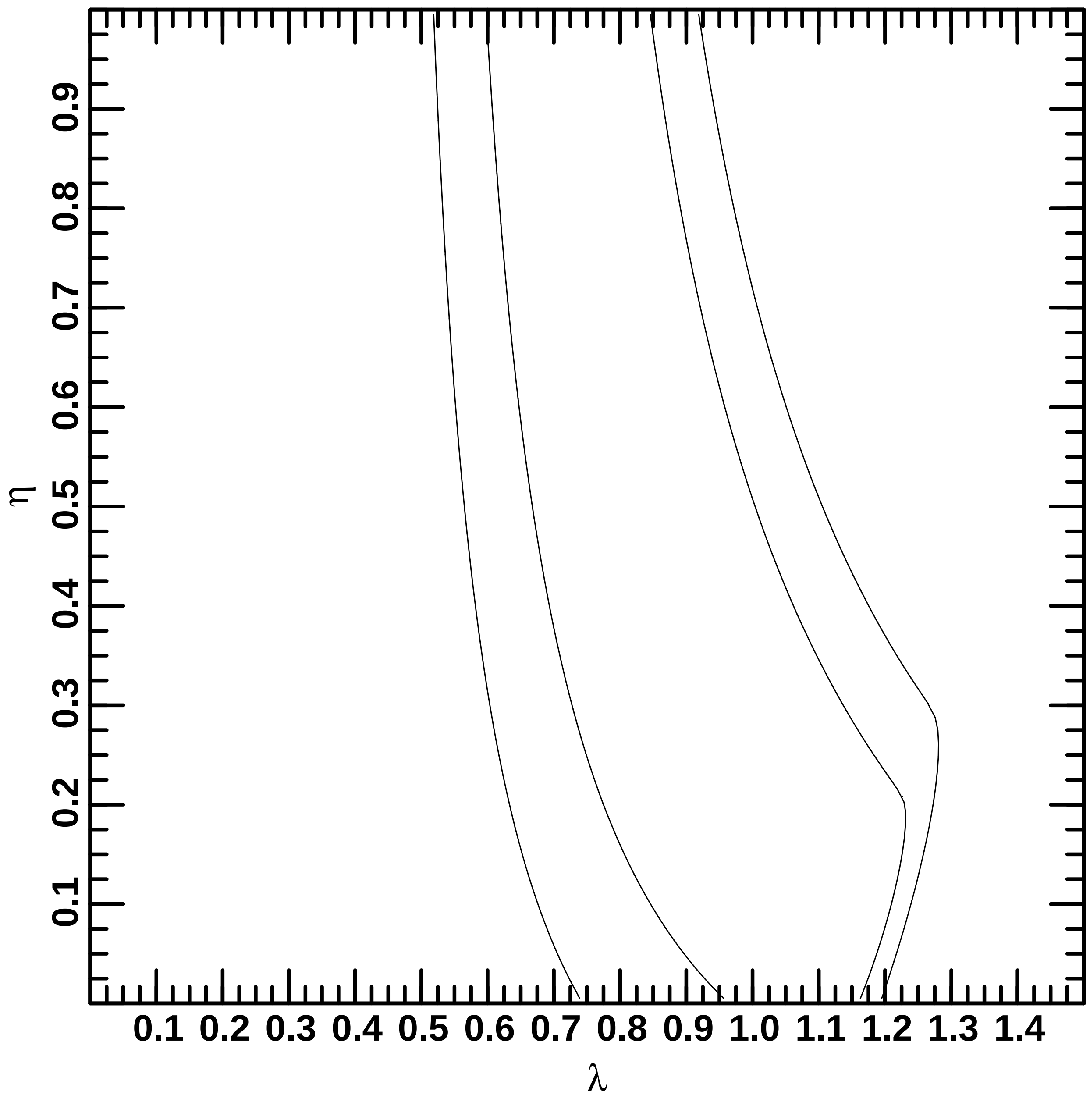
large, low-resolution grid

projected three-dimensional contours



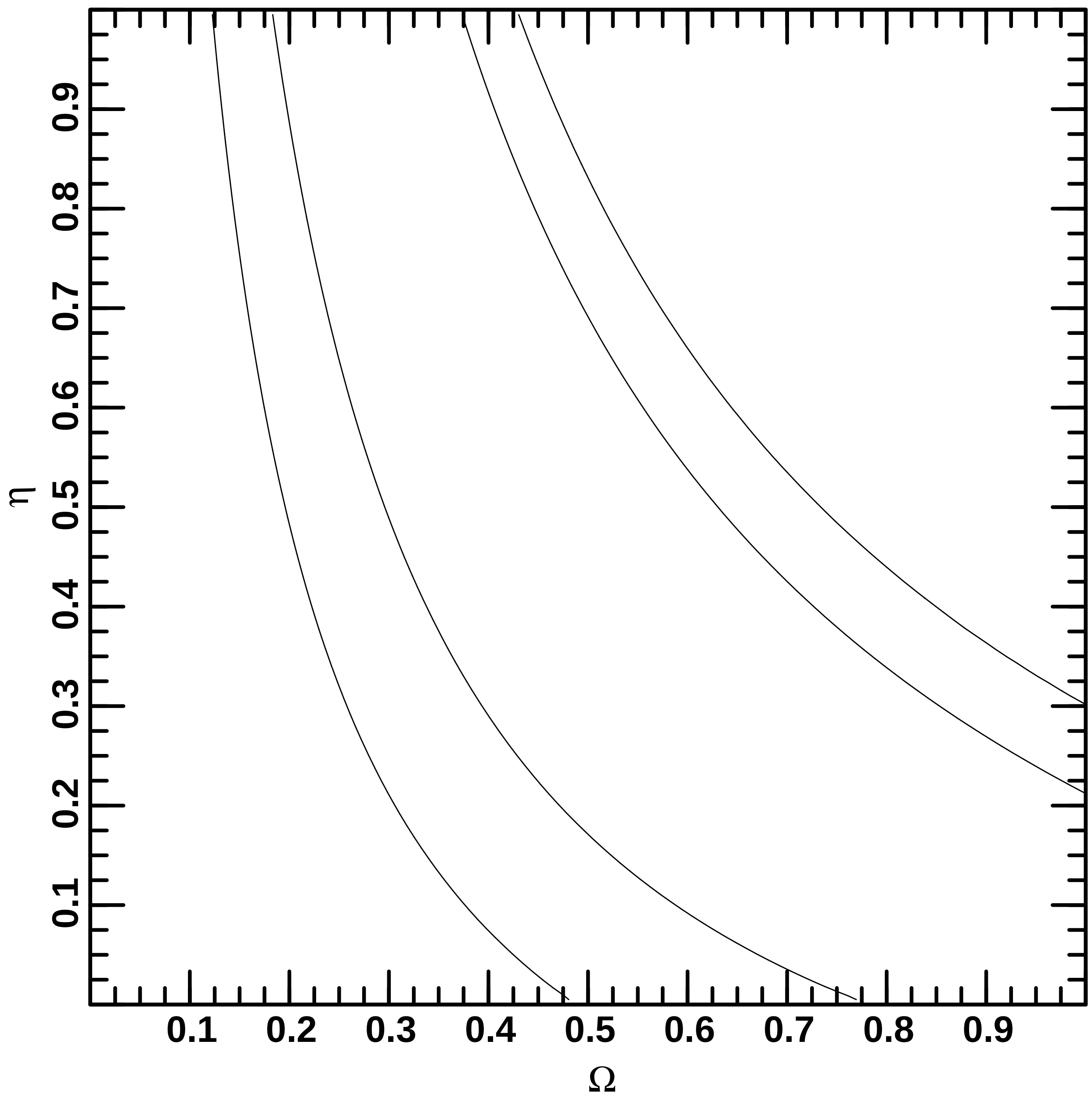
small, high-resolution grid

projected three-dimensional contours



small, high-resolution grid

projected three-dimensional contours



small, high-resolution grid

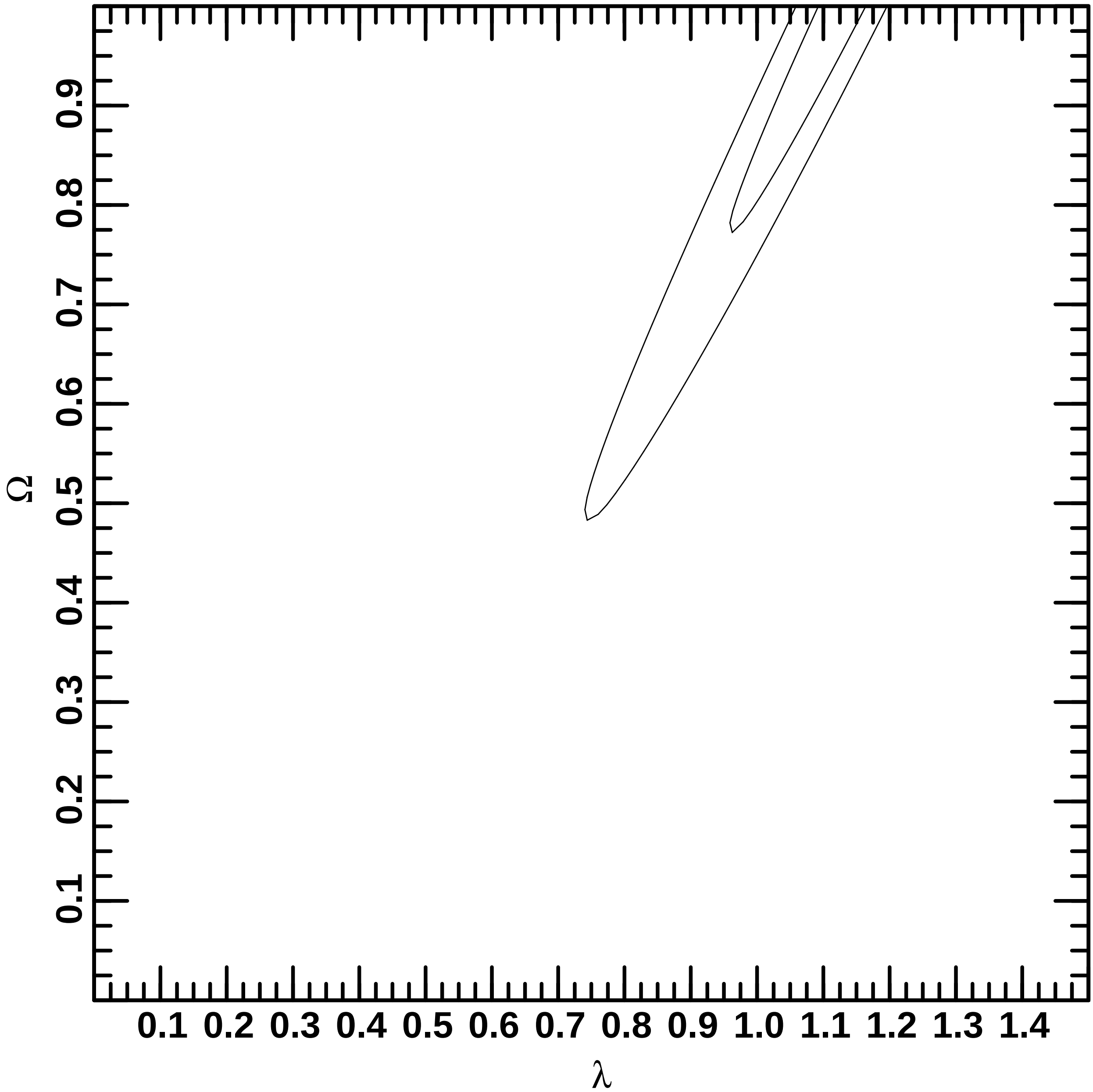
projected three-dimensional contours

Calculate contours of

relative probability in

three-dimensional space;

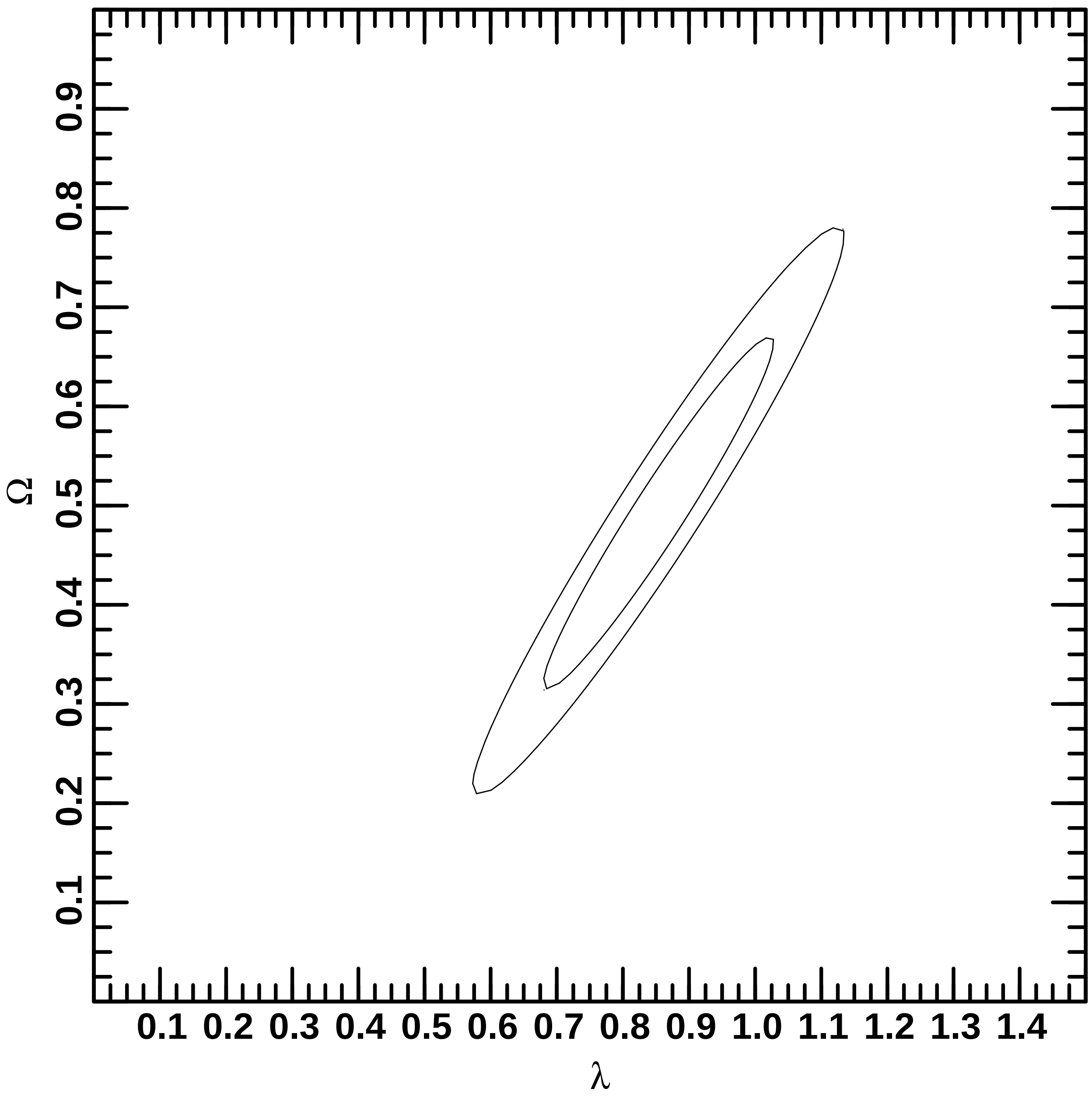
make **two-dimensional cuts**



small, high-resolution grid

cut through three-dimensional contours

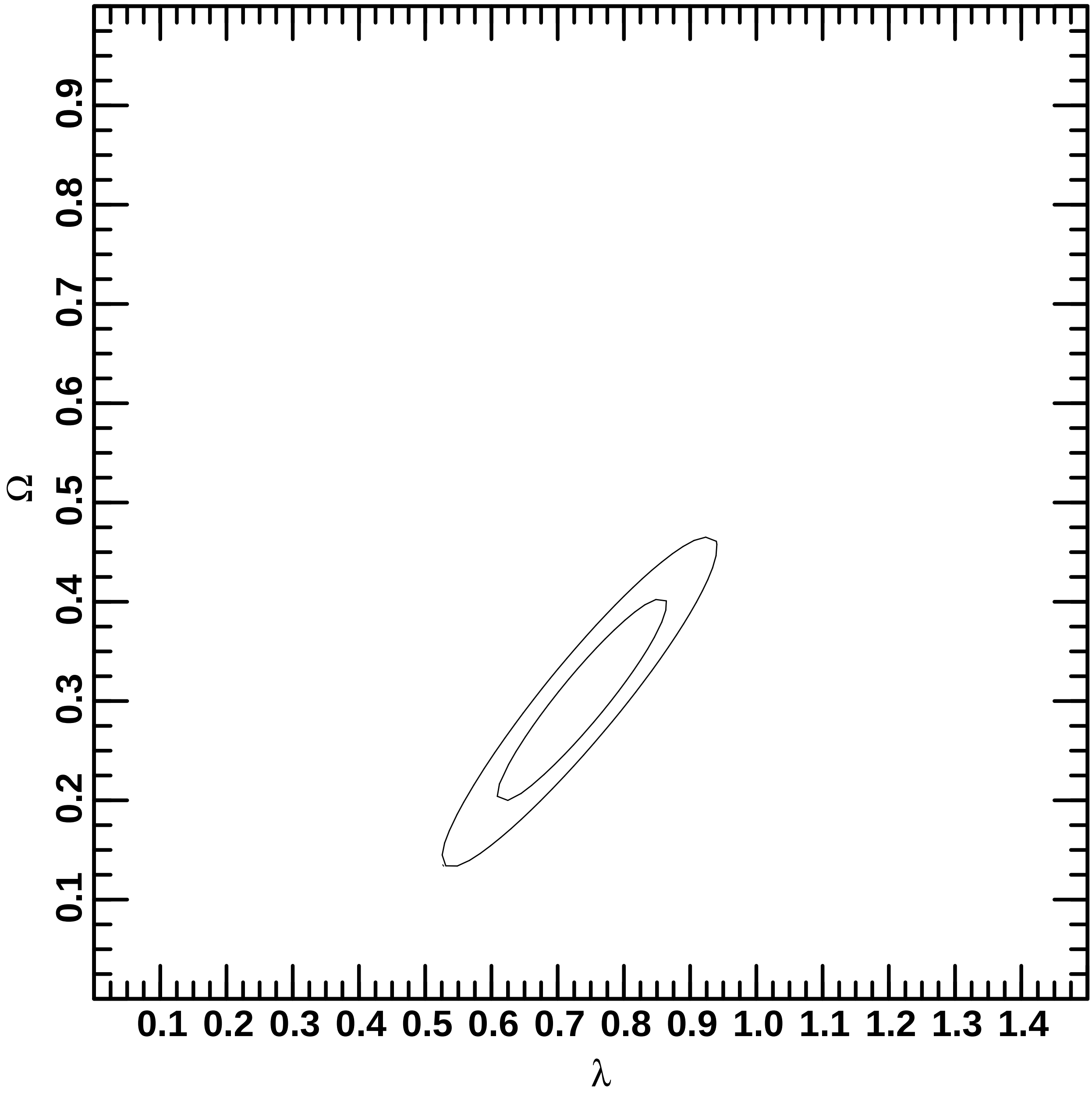
$\eta = 0.005$ (almost completely empty beam)



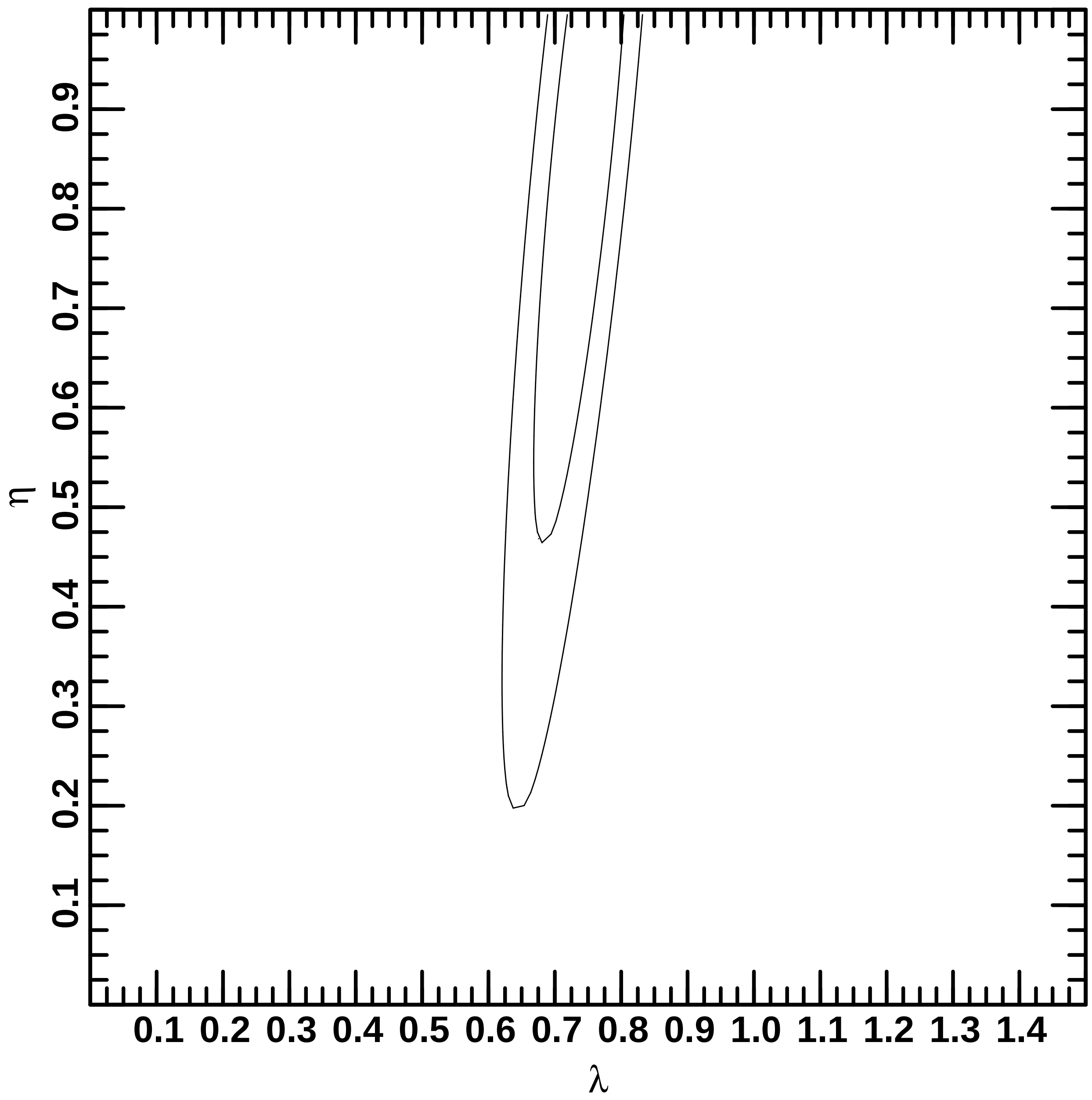
small, high-resolution grid

cut through three-dimensional contours

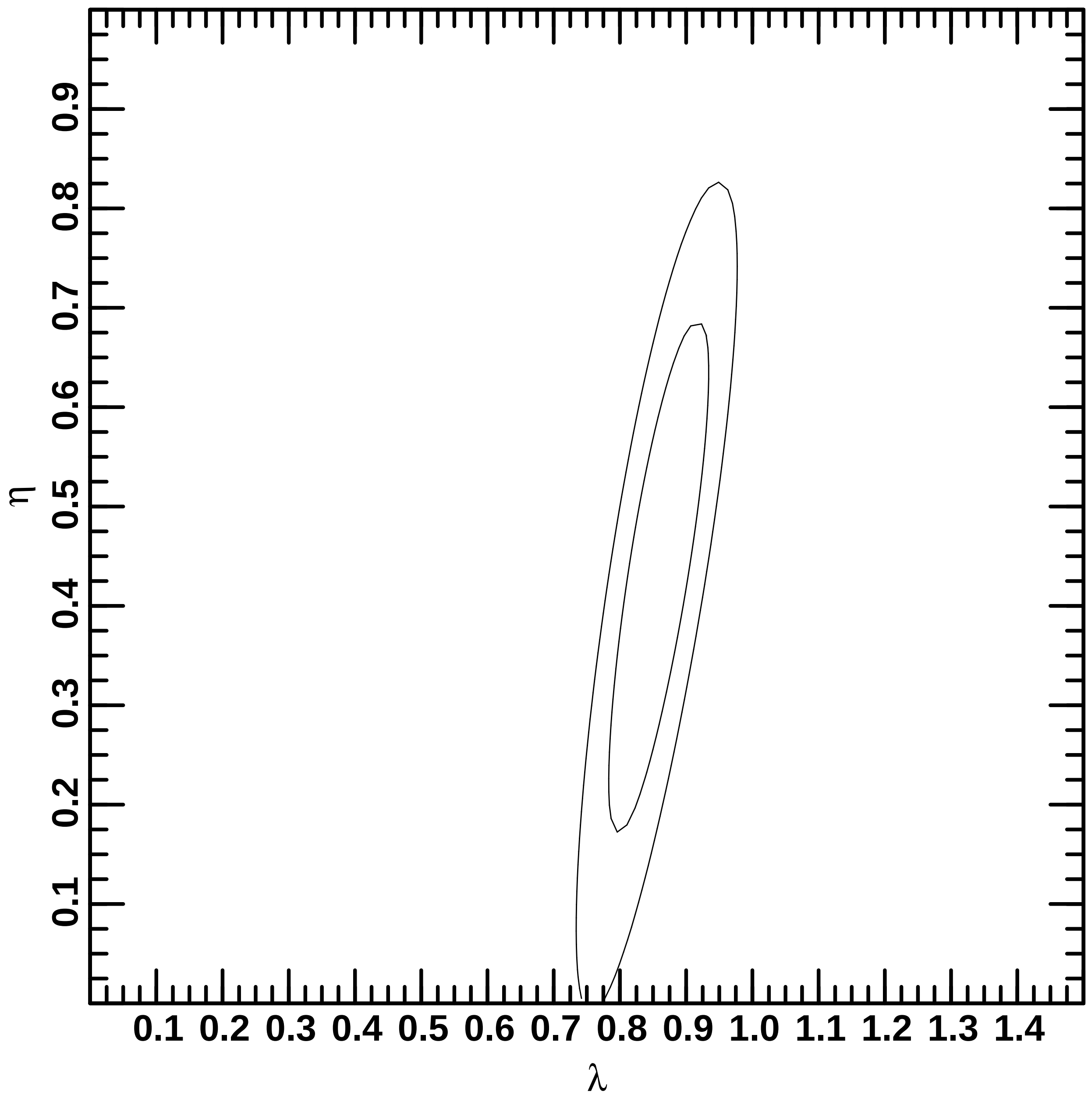
$\eta = 0.455$ (value of η at the overall maximum)



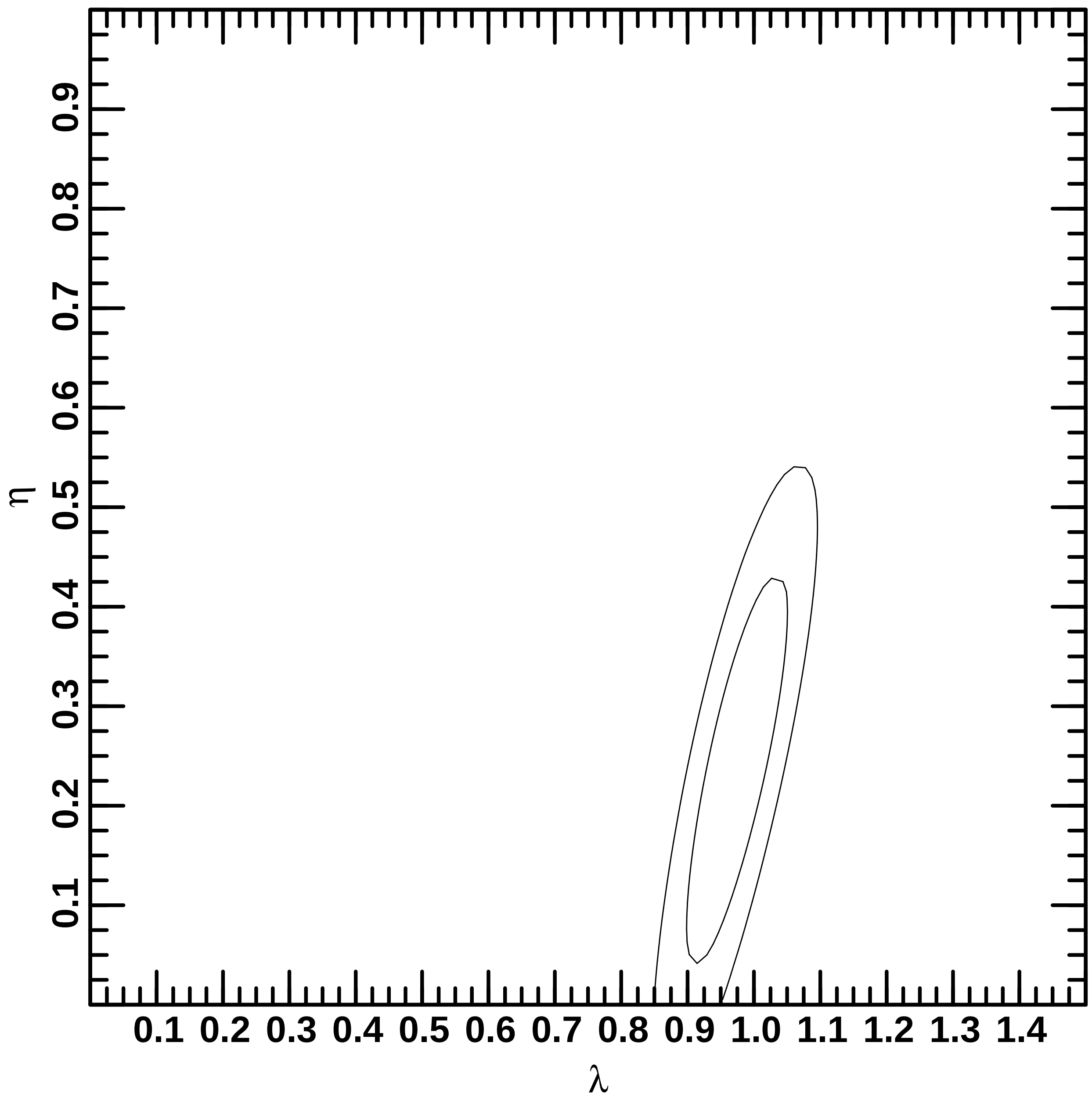
small, high-resolution grid
cut through three-dimensional contours
$\eta = 0.905$ (close to the value of η which is usually implicitly assumed)



small, high-resolution grid
cut through three-dimensional contours
$\Omega_0 = 0.3109375$ (close to the value of Ω_0 in the concordance model)



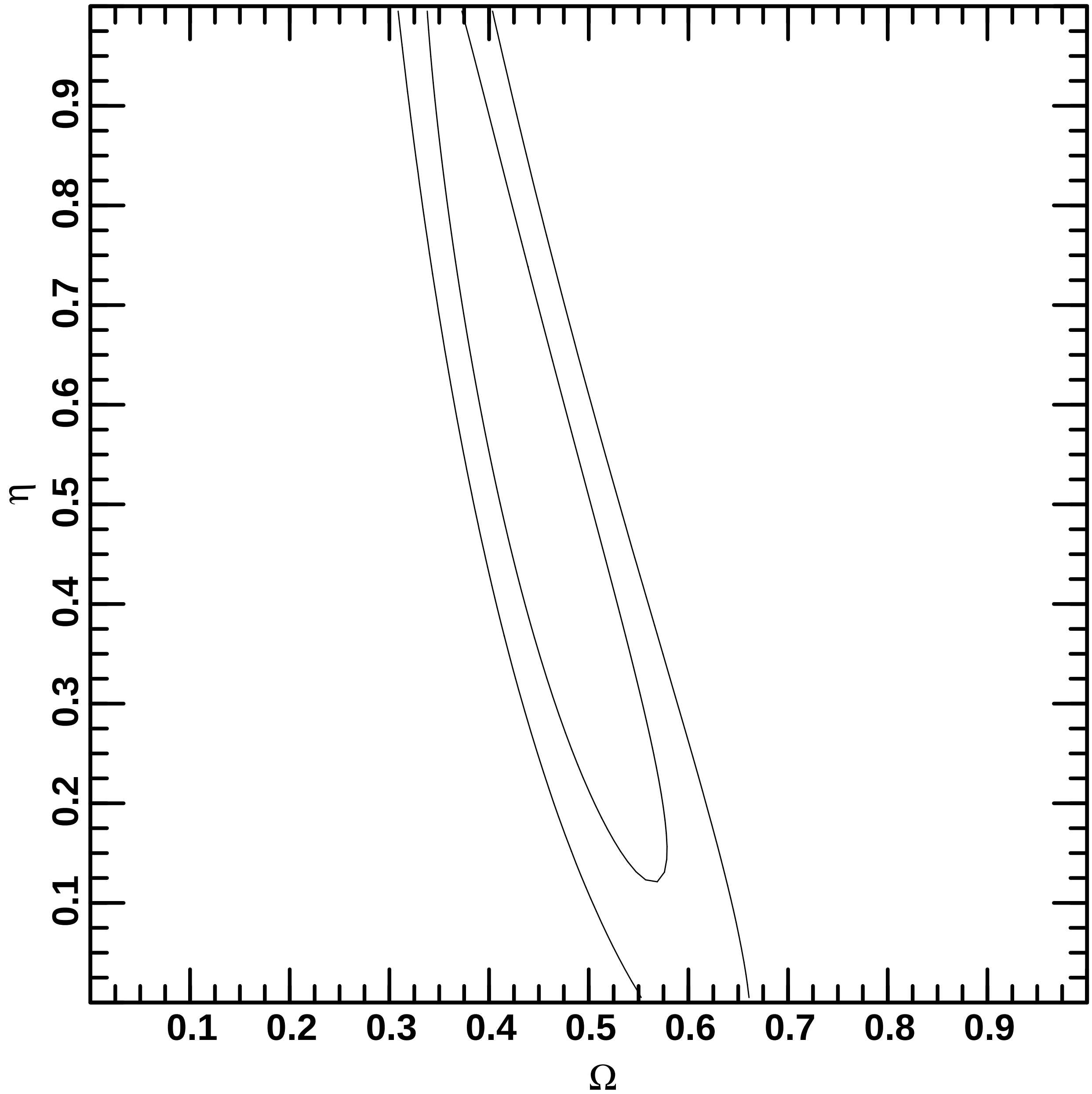
small, high-resolution grid
cut through three-dimensional contours
$\Omega_0 = 0.5015625$ (value of Ω_0 at the overall maximum)



small, high-resolution grid

cut through three-dimensional contours

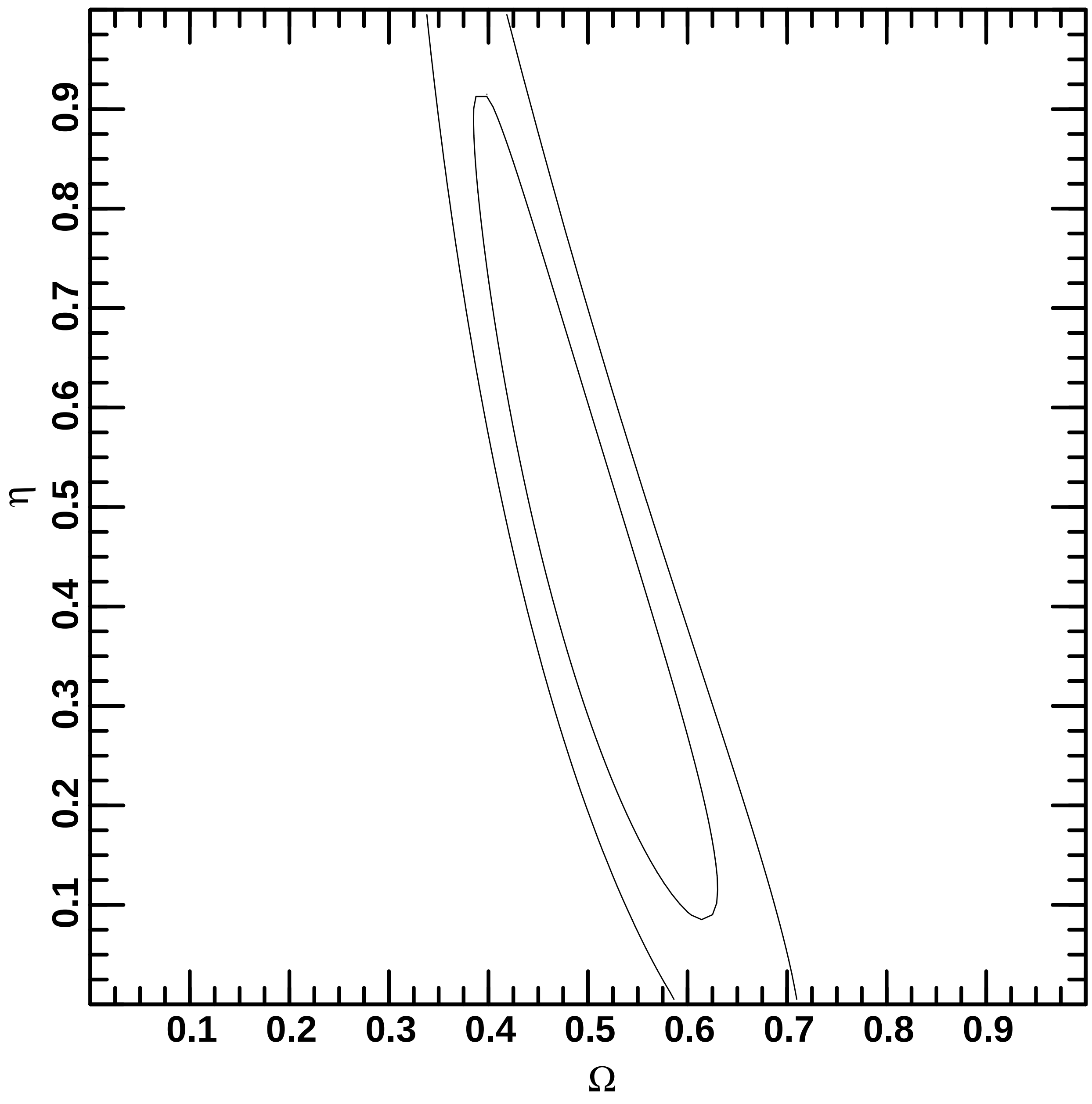
$\Omega_0 = 0.6921875$



small, high-resolution grid

cut through three-dimensional contours

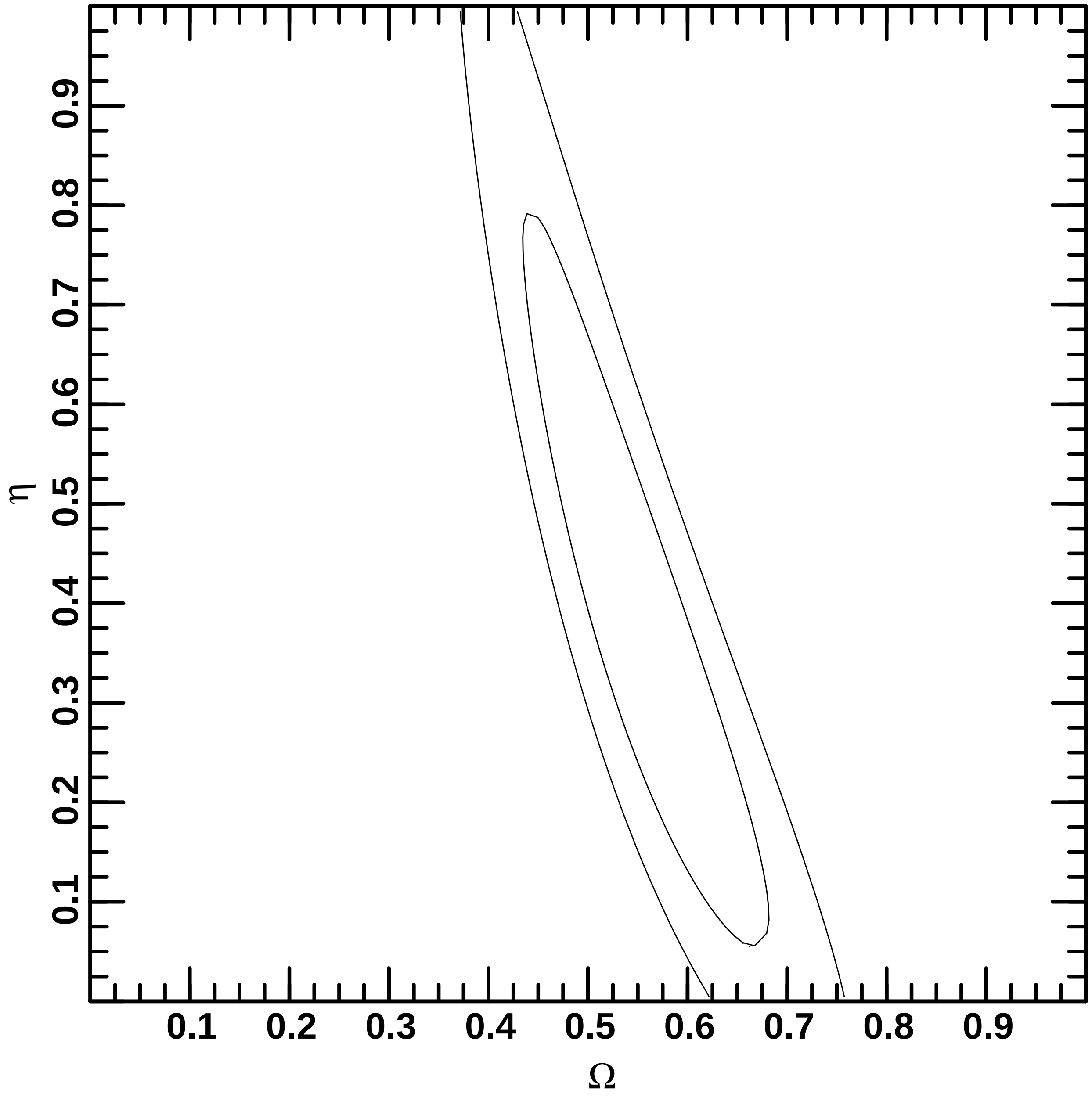
$\lambda_0 = 0.8296875$



small, high-resolution grid

cut through three-dimensional contours

$\lambda_0 = 0.8609375$ (value of λ_0 at the overall maximum)



small, high-resolution grid

cut through three-dimensional contours

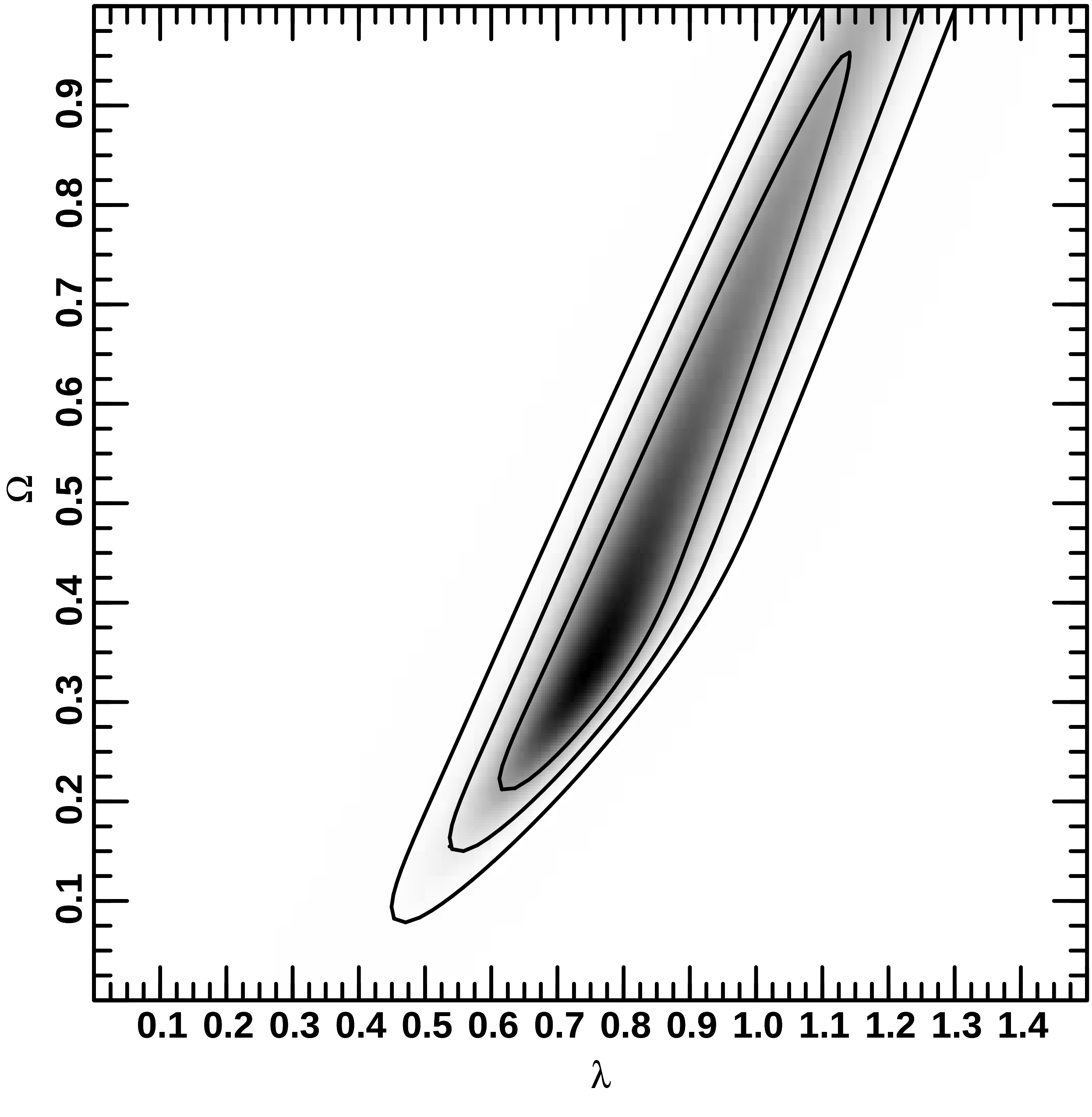
$\lambda_0 = 0.8921875$

Calculate contours of

relative probability in

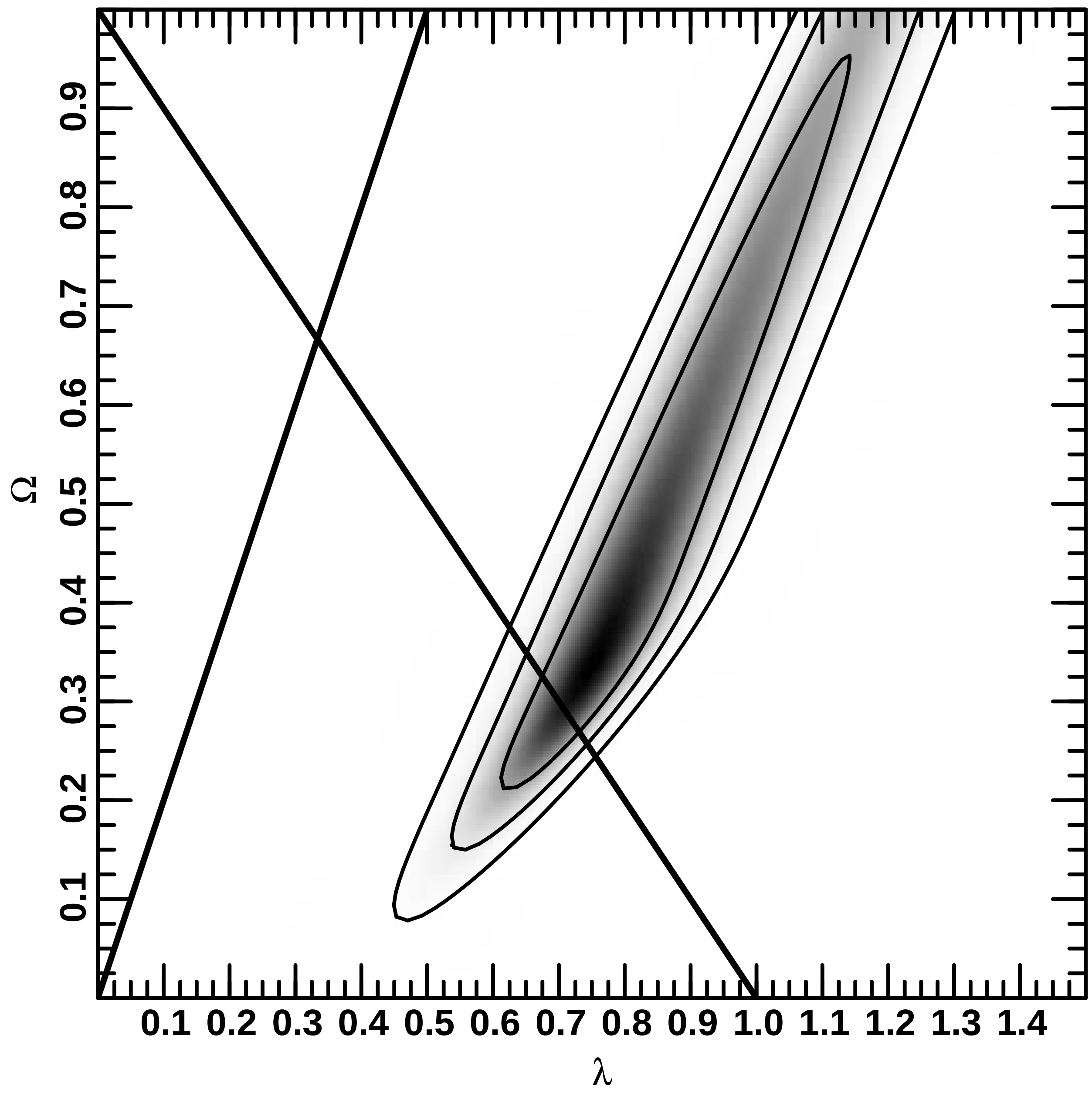
three-dimensional space;

marginalize over 1 dimension

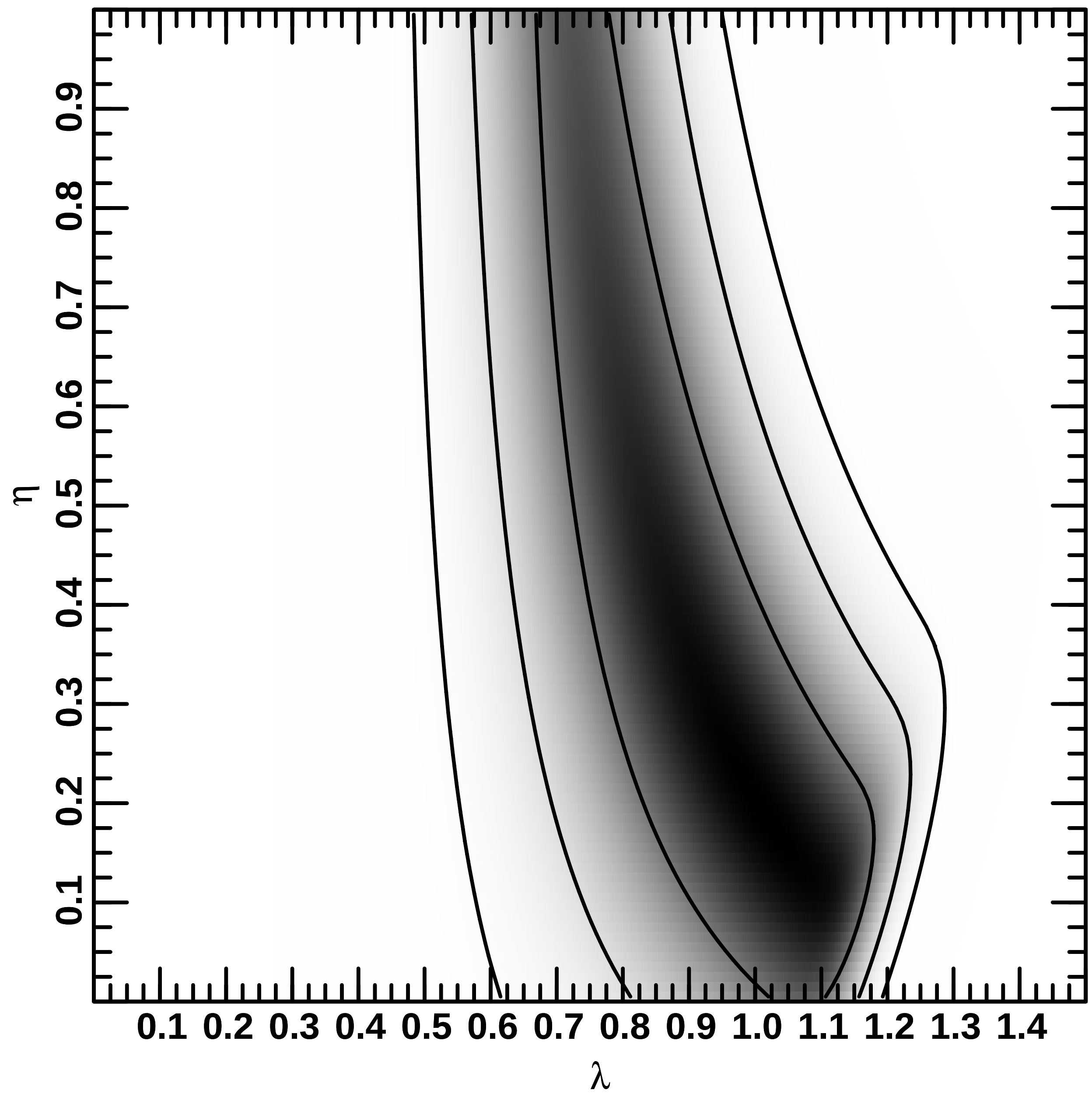


small, high-resolution grid

marginalized over η

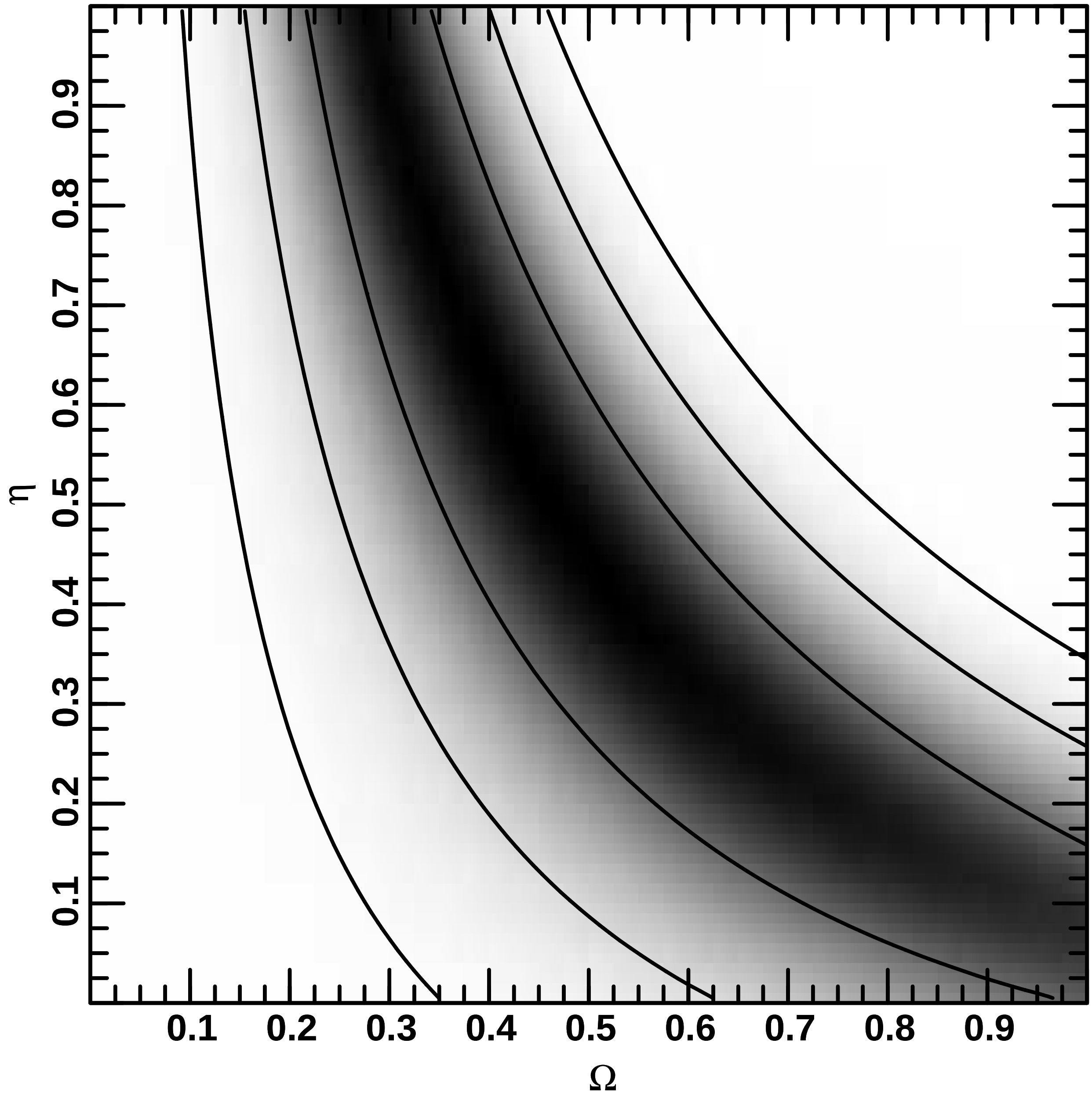


small, high-resolution grid
marginalized over η



small, high-resolution grid

marginalized over Ω_0



small, high-resolution grid

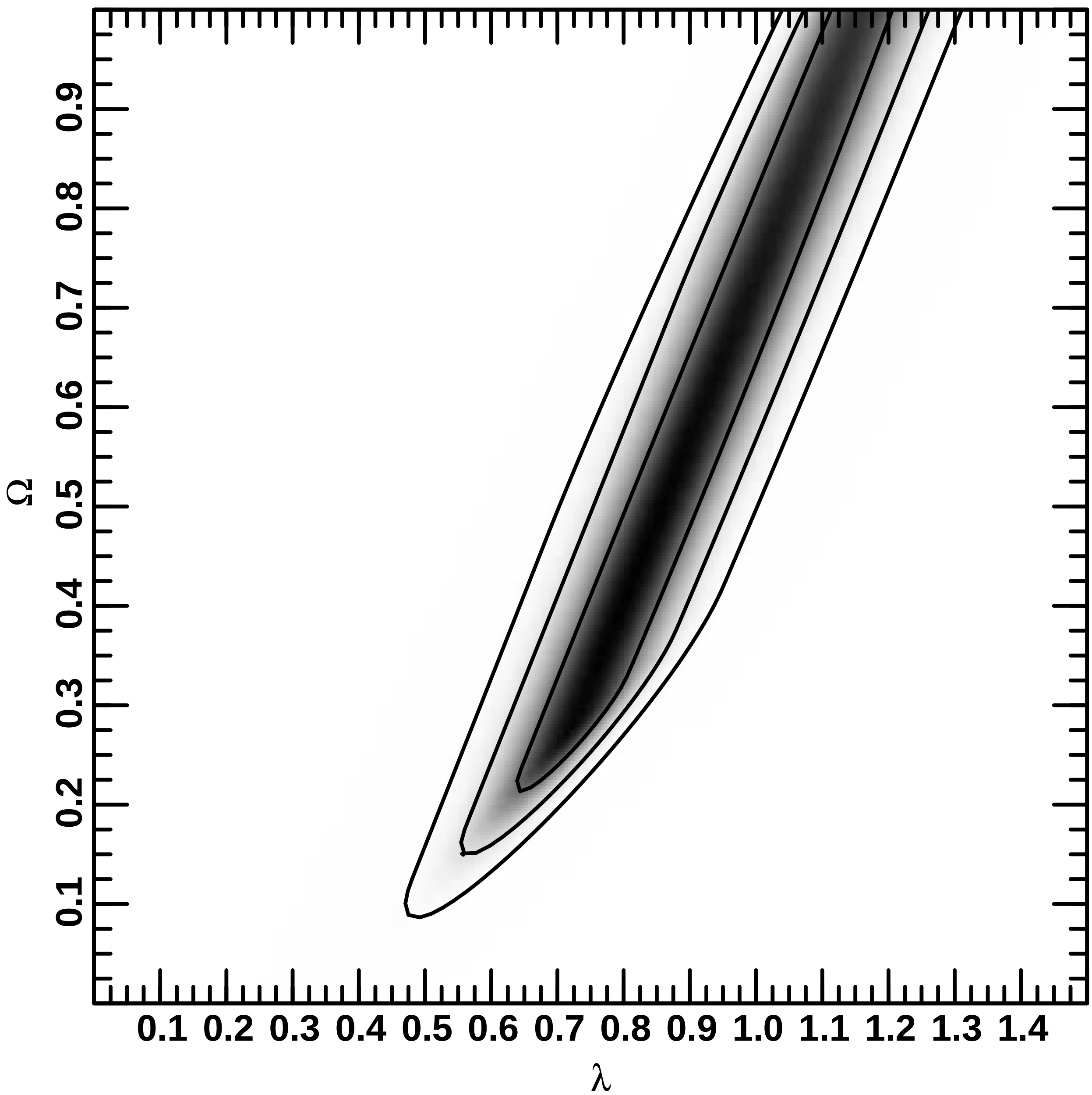
marginalized over λ_0

Calculate contours of

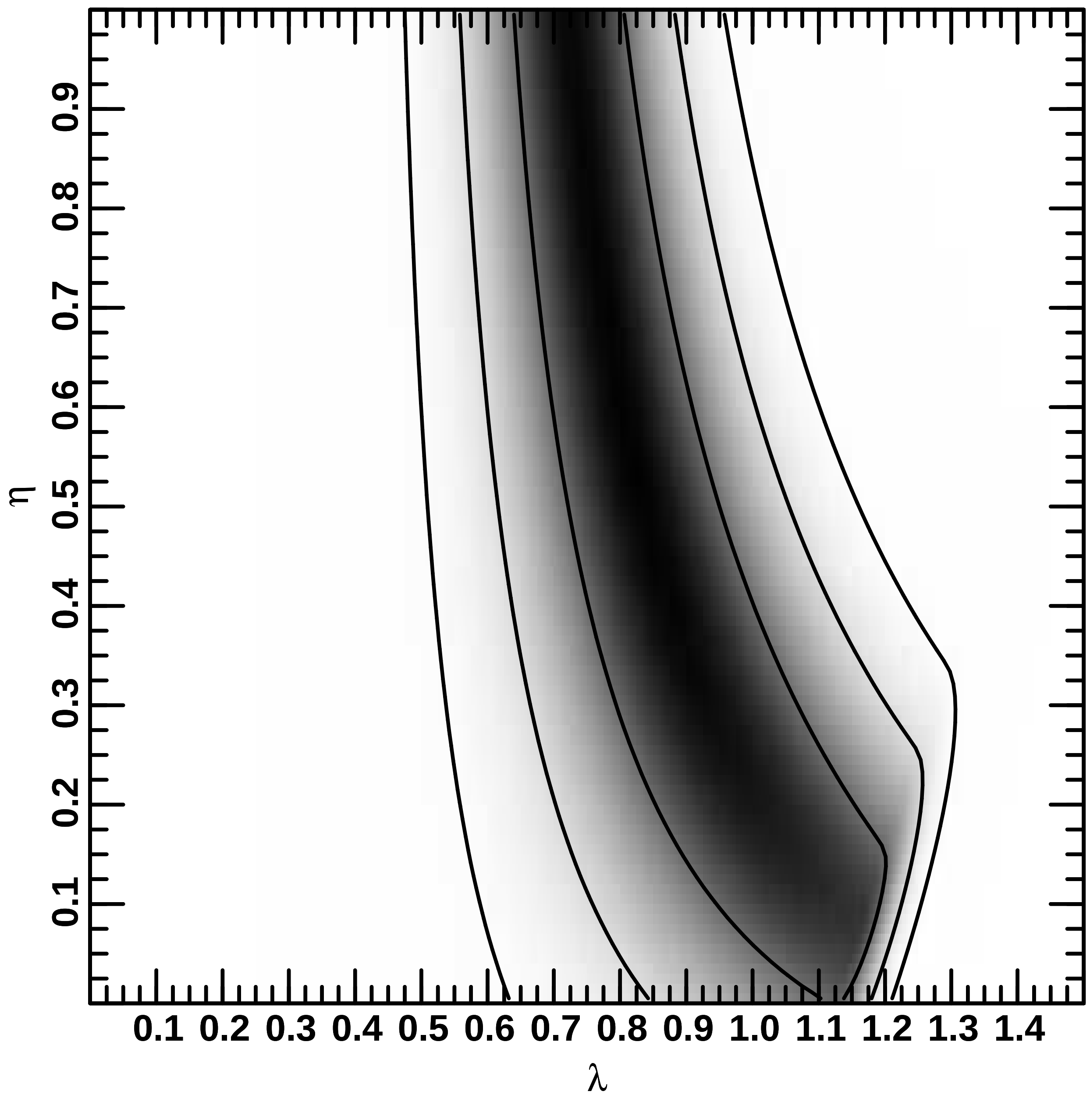
relative probability in

three-dimensional space;

maximize over 1 dimension

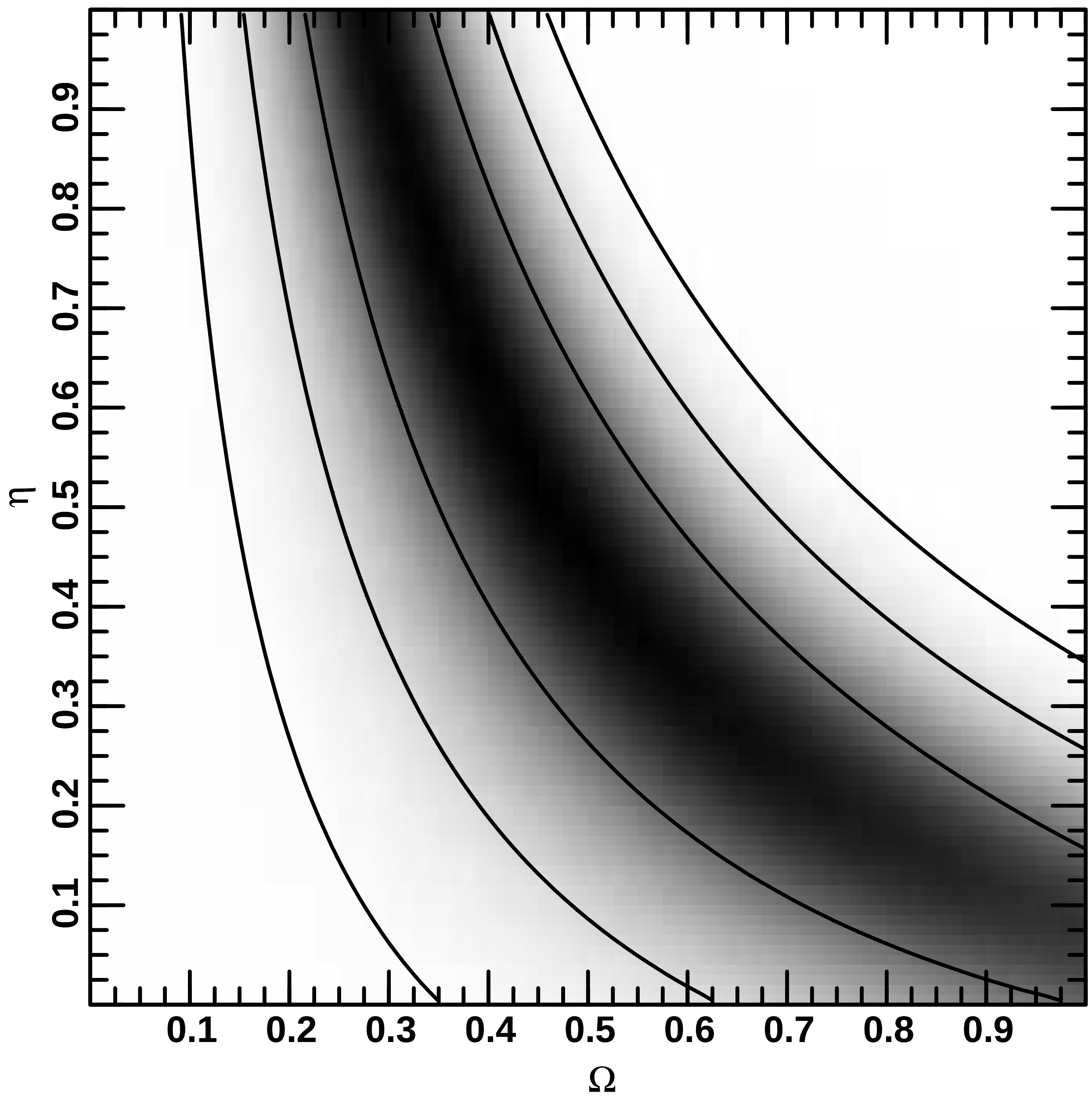


small, high-resolution grid
maximized over η



small, high-resolution grid

maximized over Ω_0



small, high-resolution grid

maximized over λ_0

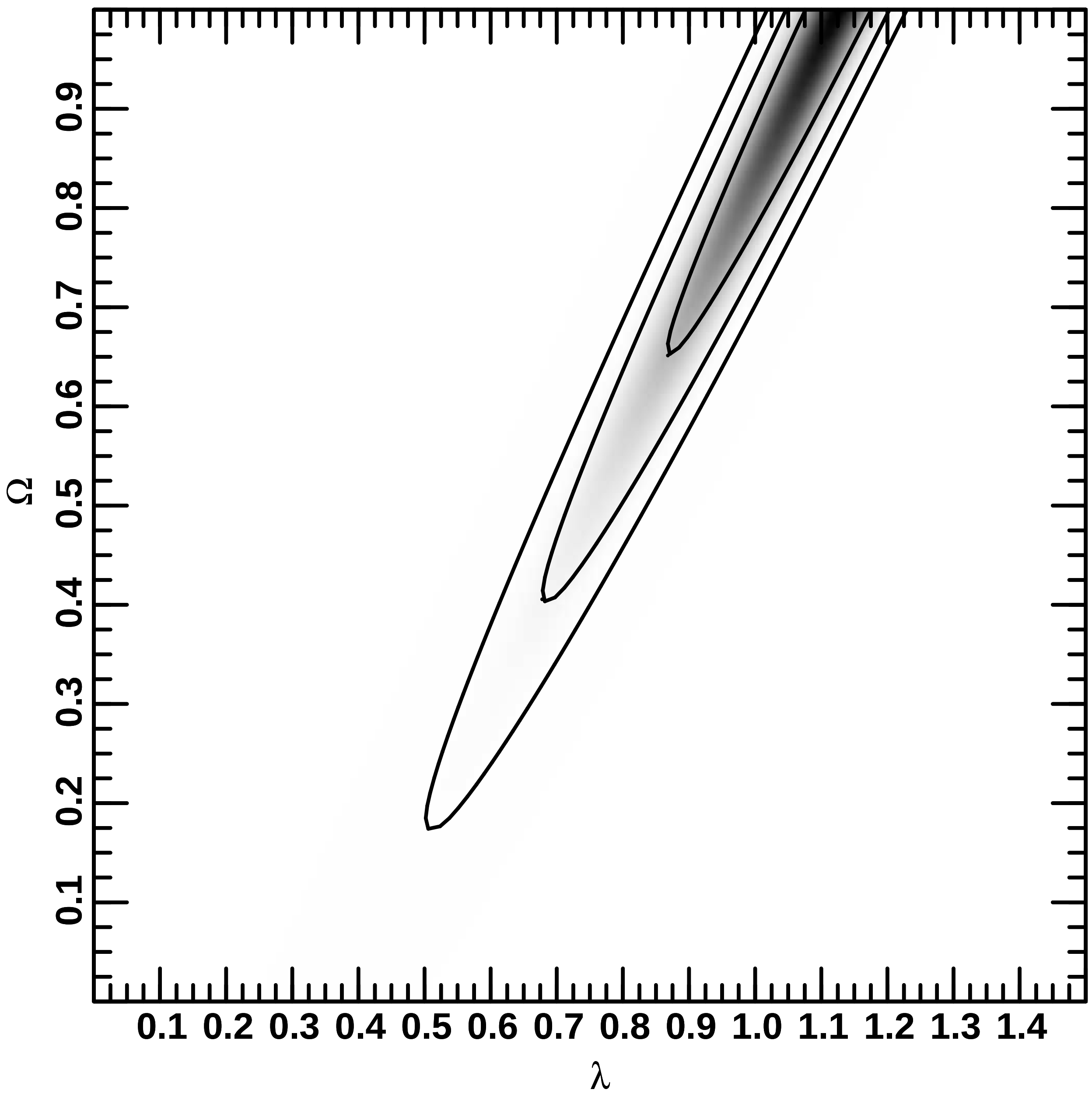
Calculate contours of

relative probability in

two-dimensional space;

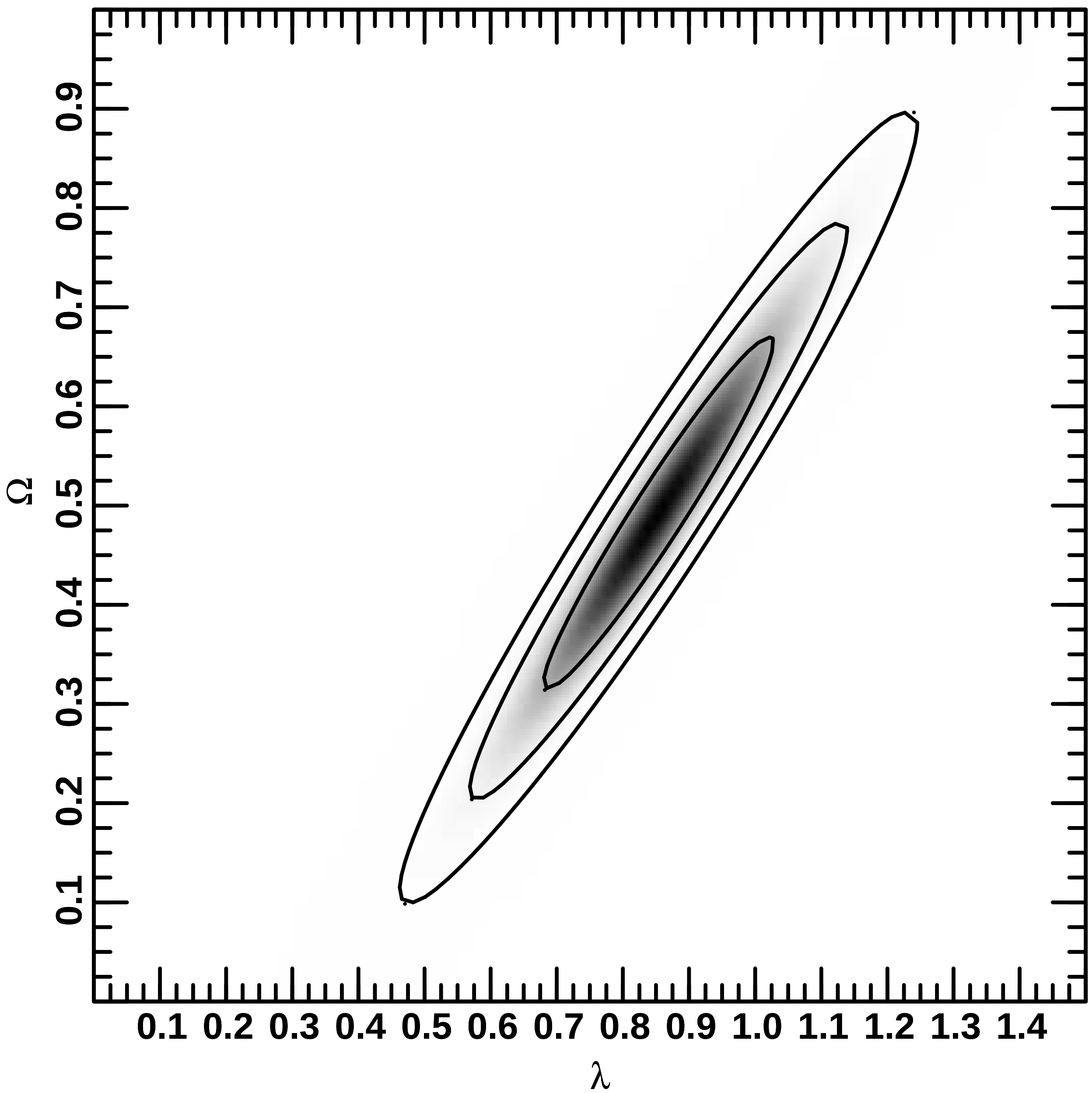
fix value of the 3rd dimension

(δ-function prior)



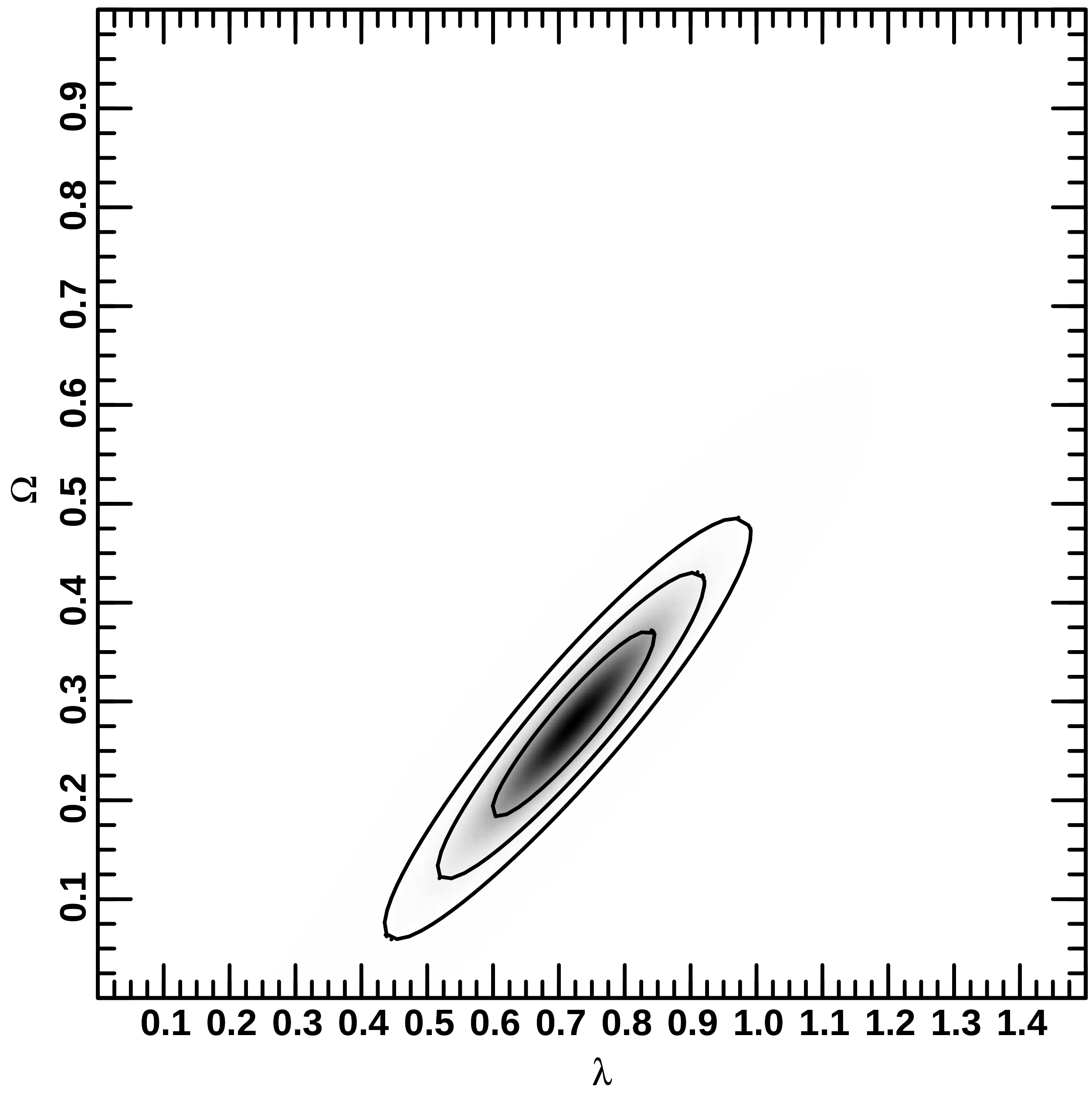
small, high-resolution grid

prior: $\eta = 0$ (completely empty beam)



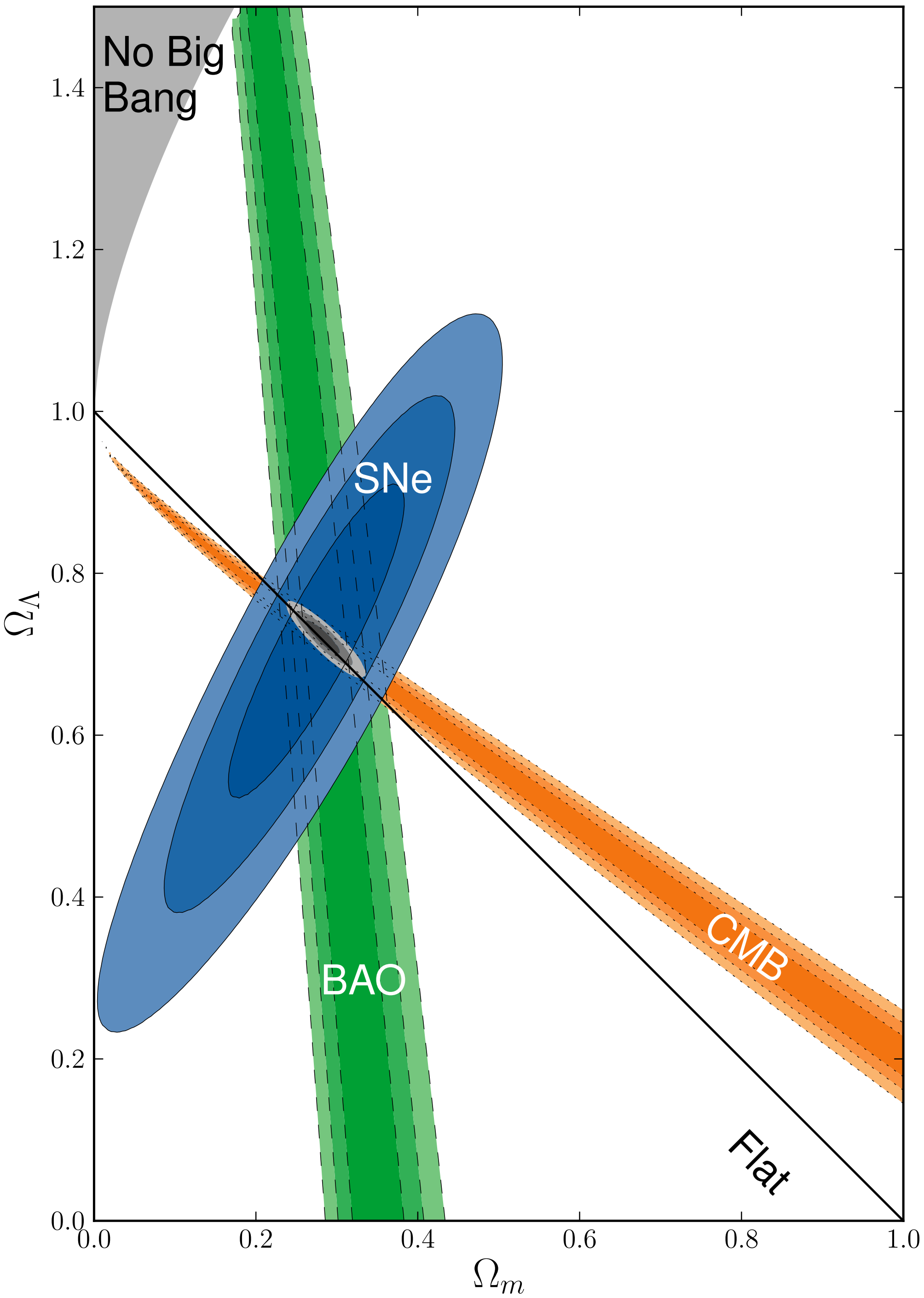
small, high-resolution grid

prior: $\eta = 0.455$ (value of η at the overall maximum)



small, high-resolution grid

prior: $\eta = 1$ (value which is usually implicitly assumed)



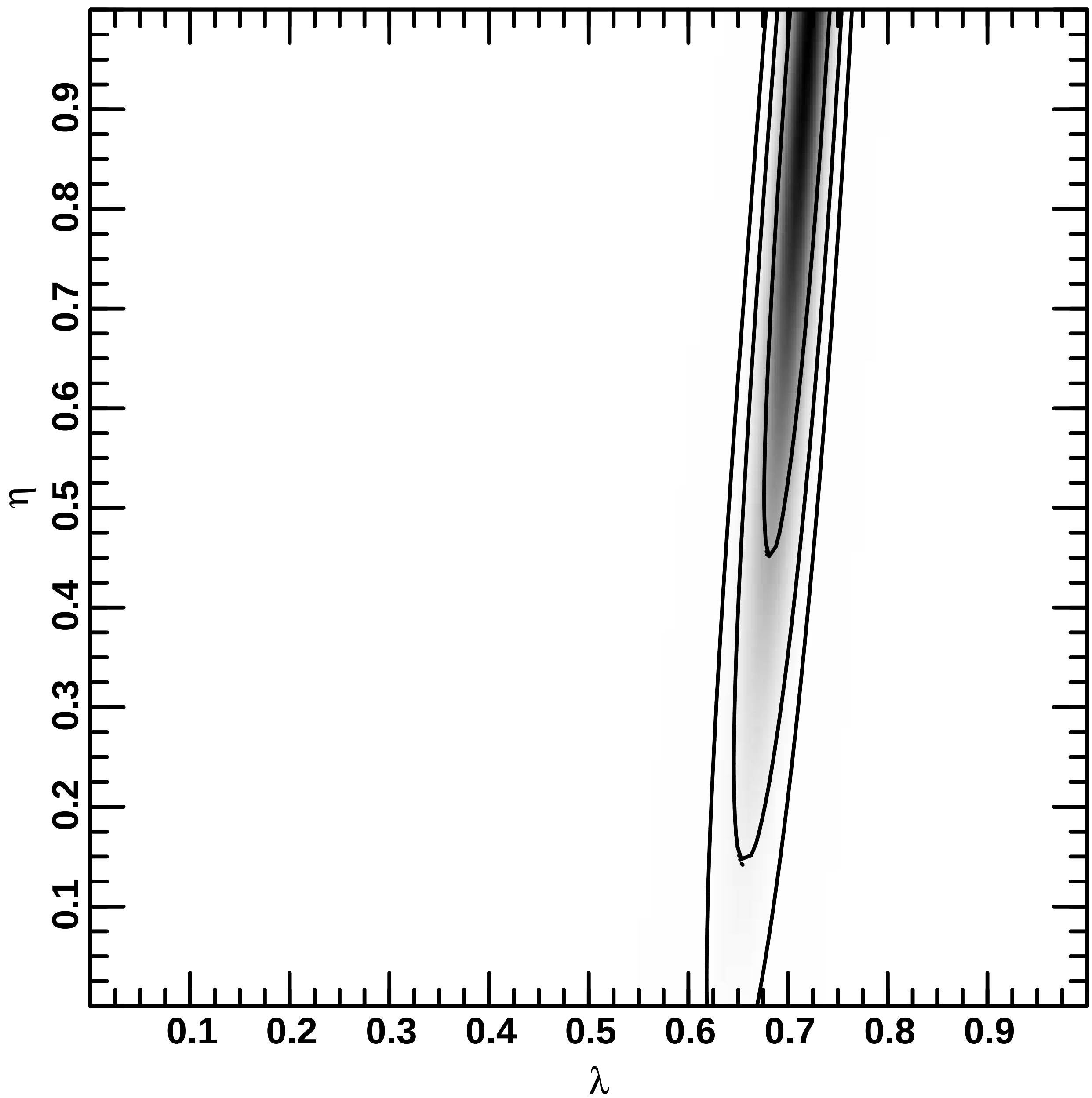
Suzuki *et al.* (2011)

Calculate contours of

relative probability in

two-dimensional space

assuming $k = 0$



small, high-resolution grid

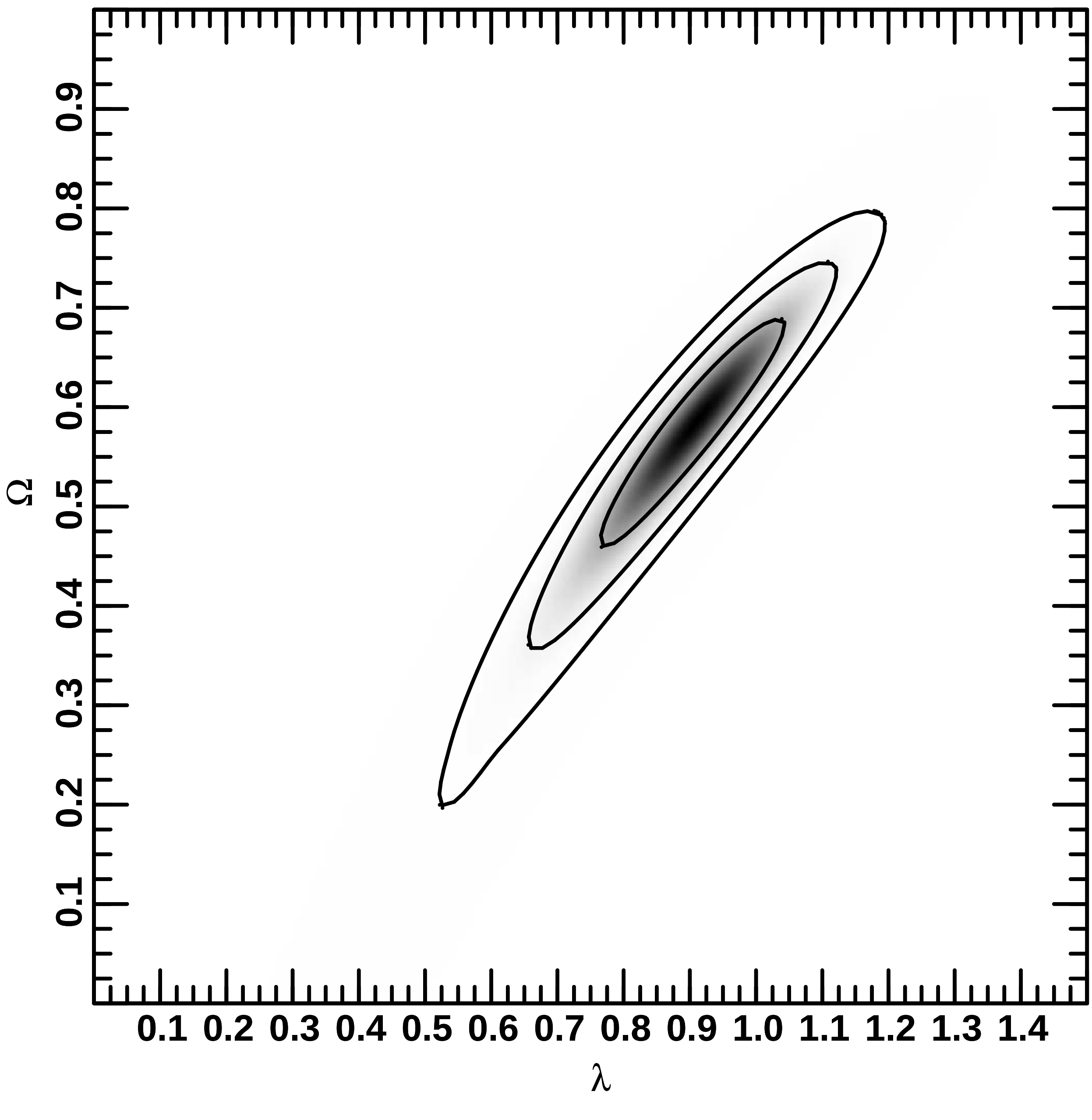
prior: $k = 0$

Calculate contours of

relative probability in

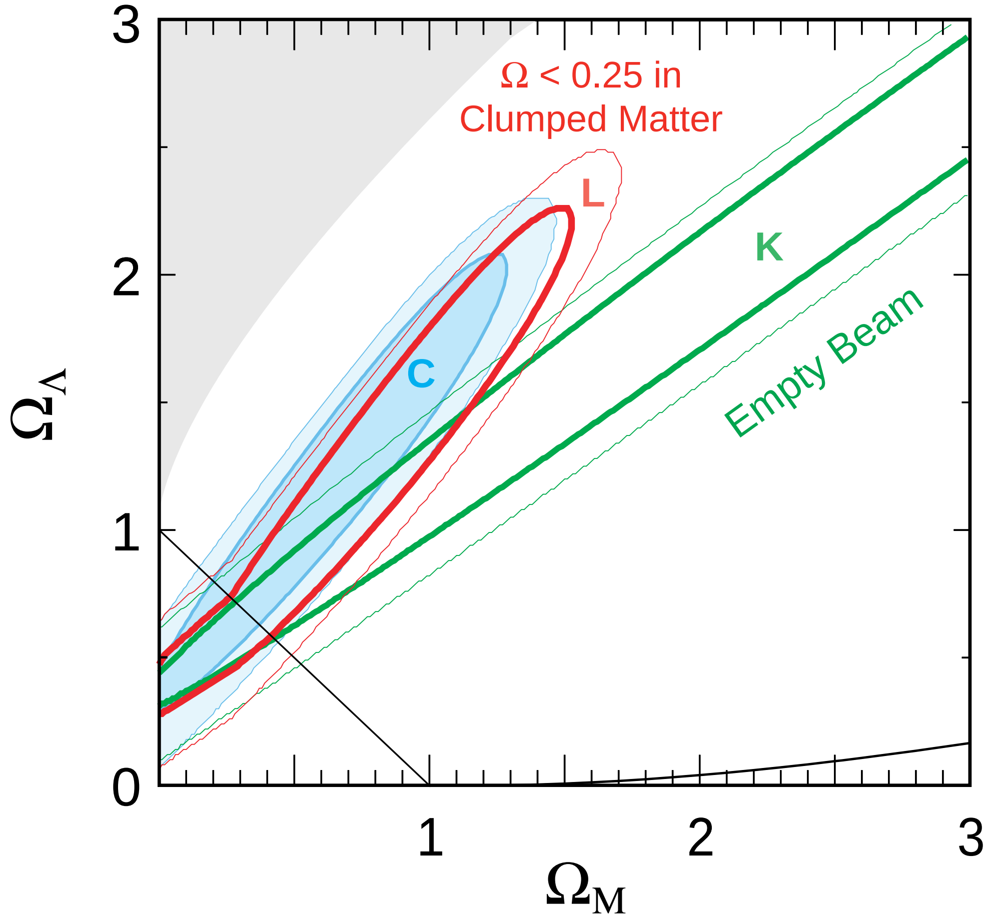
two-dimensional space

setting $\eta = f(\Omega_0)$



small, high-resolution grid

η **depends on** Ω_0

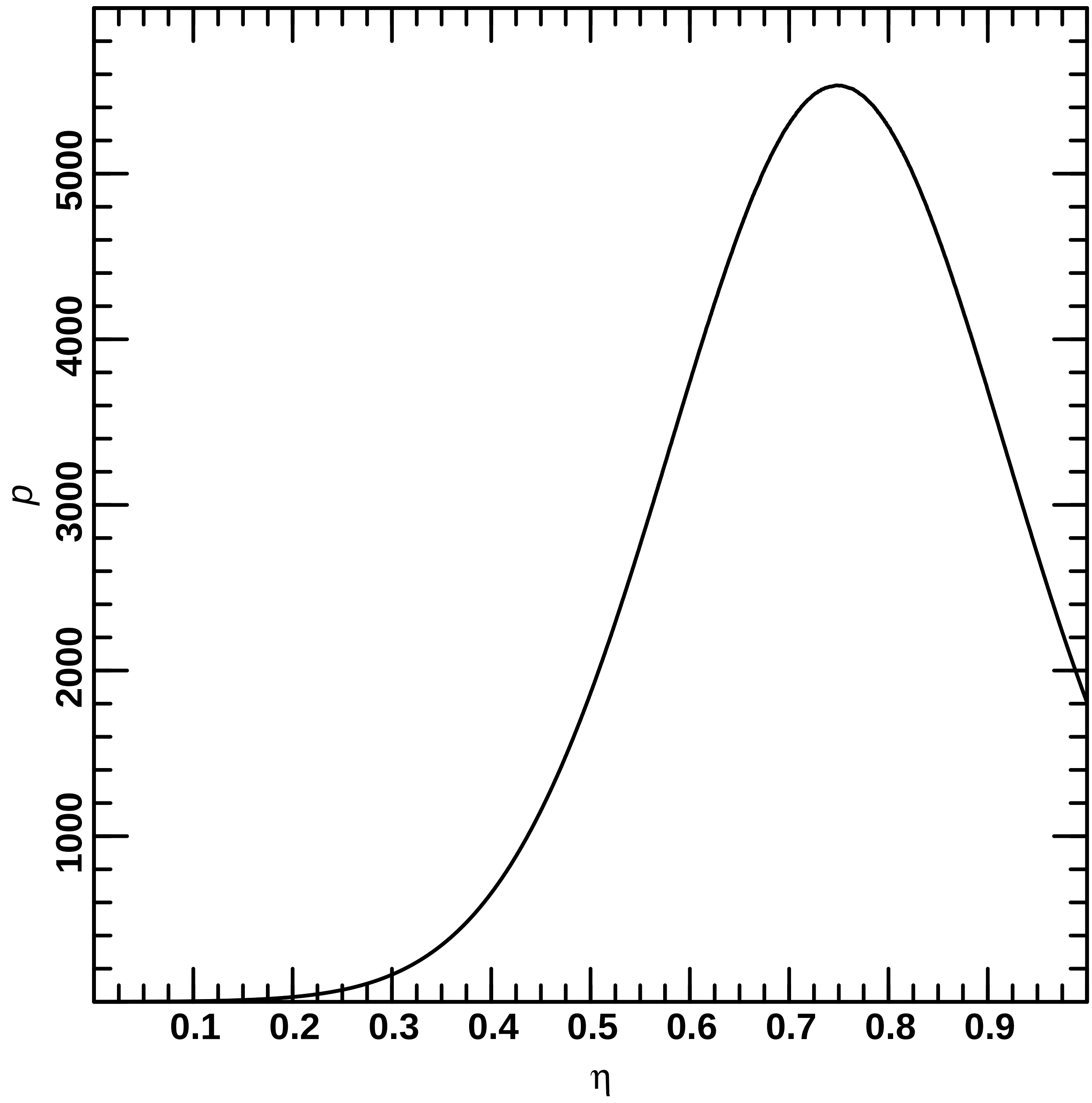


Perlmutter *et al.* (1999)

Calculate relative probability

in one-dimensional space

fix $\lambda_0 = 0.7$ and $\Omega_0 = 0.3$



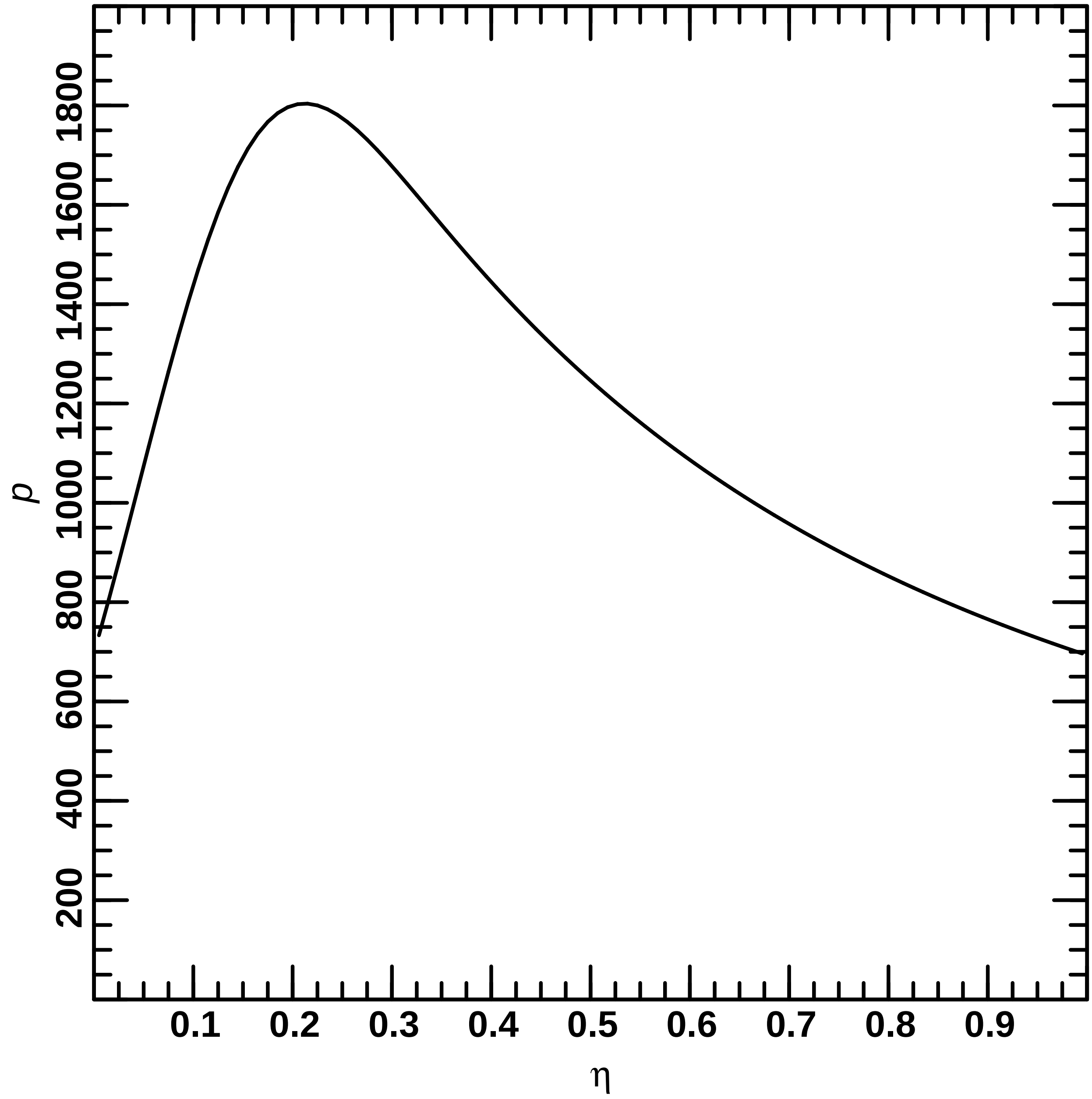
small, high-resolution grid

$p(\eta)$ for $\lambda_0 = 0.7$ and $\Omega_0 = 0.3$

Calculate relative probability

in one-dimensional space

marginalize λ_0 and Ω_0



small, high-resolution grid

$p(\eta)$ when marginalized over λ_0 and Ω_0

What do we find: expected

- Constraints on λ_0 and Ω_0 are weaker if η is not constrained.

- The concordance model is reasonably probable.

- There is a degeneracy between η and the amount of spatial curvature $(\lambda_0 + \Omega_0)$.

- λ_0 is constrained best, then Ω_0, then η.

What about dark matter?

- Assuming the concordance model, the best-fit value of η is approximately 0.75:
  - $0.60 < \eta < 0.90$ (68.3%)
  - $0.46 < \eta < 1.00$ (95.4%)
  - $0.28 < \eta < 1.00$ (99.7%)

- Question: can we take such constraints on η seriously, if we don't believe the overall best fit?

- Answer: The best fit is really not statistically significantly better than many other points, but some values of η can be ruled out at high confidence levels.

What do we find: surprising?

- Overall best fit is ruled out by other tests (overfitting?).

- If we *assume* $k = 0$ then the best fit is the concordance model *and* has $\eta = 1$.

- If we *assume* $\eta = 1$, then the best-fit is very close to the concordance model.

- If we assume the concordance model, then can probably rule out low values of η, even though the relevant scale is extremely small.

- We cannot rule out $\eta = 1$, and there is some tentative evidence for it.

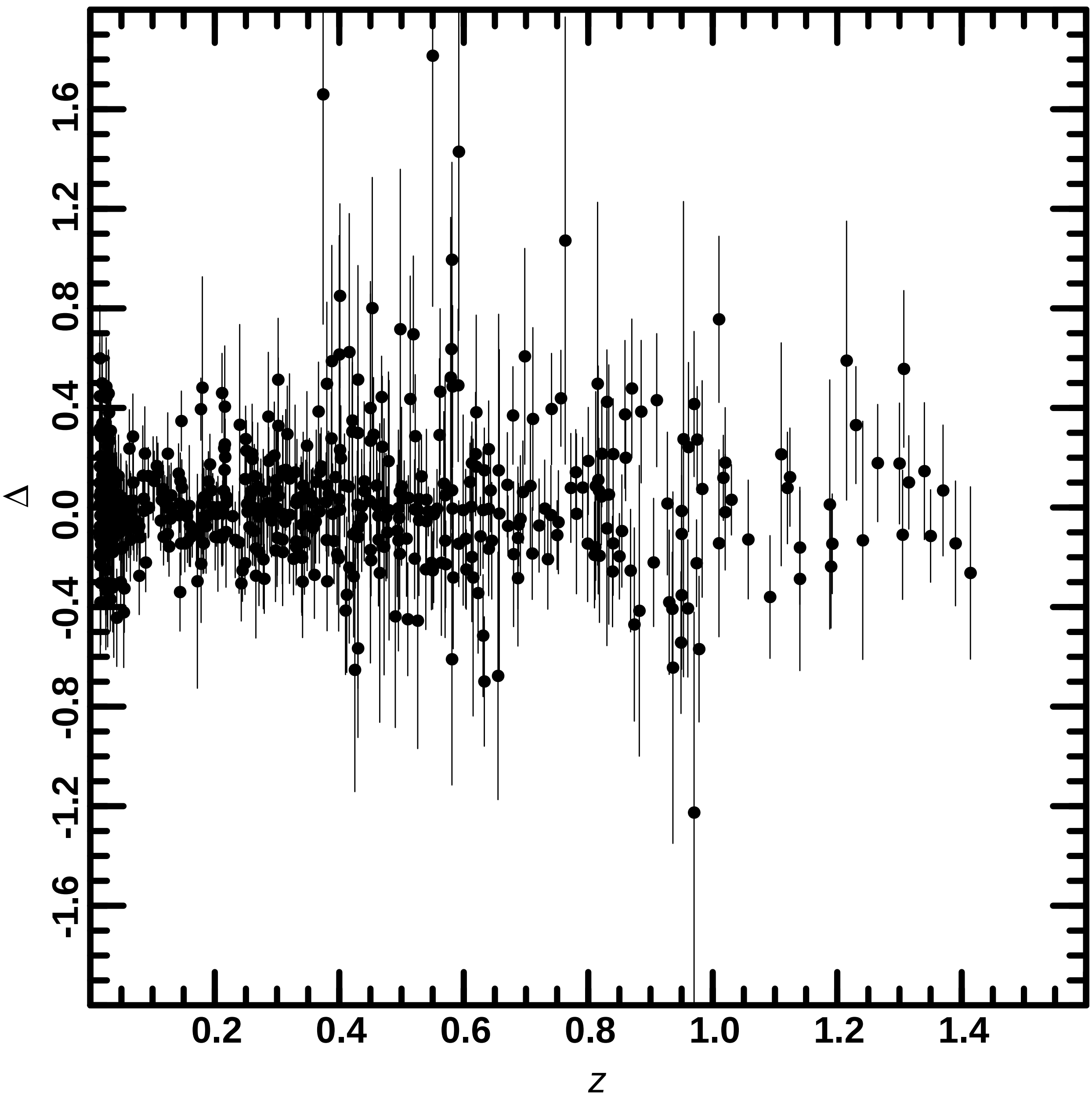
Averaging: Safety in numbers, or safely without worry?

- Two possibilities to have $\eta \approx 1$:

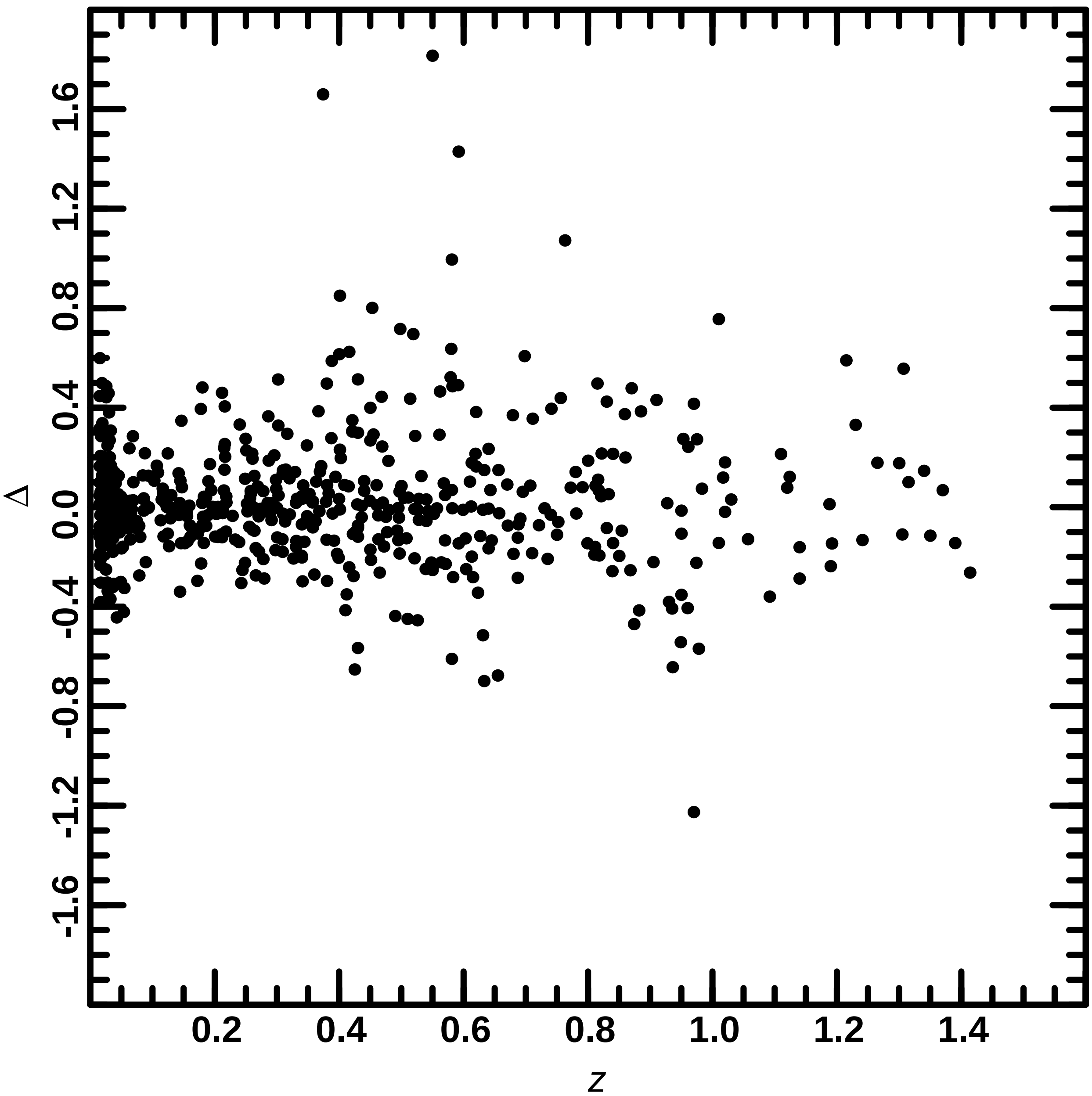
- Safety in numbers: Some lines of sight are overdense, some underdense (implies more general definition of η), and $\eta \approx 1$ results from appropriate 'averaging'.

- Safely without worry: Each line of sight has $\eta \approx 1$, so the classical distance formula (i.e. $\eta = 1$) can be used safely without worry.

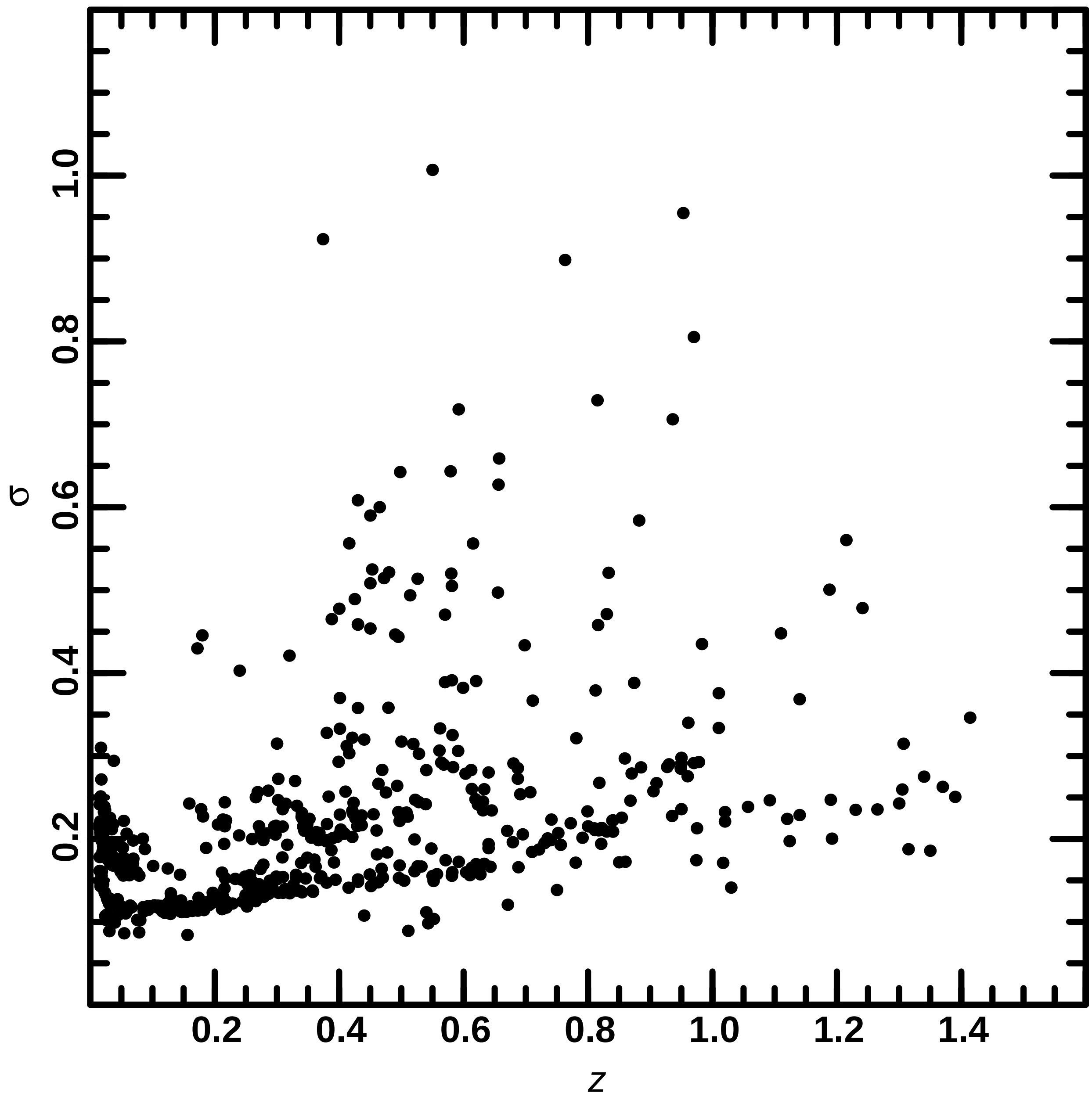
- Idea: compare residuals with observational uncertainties.



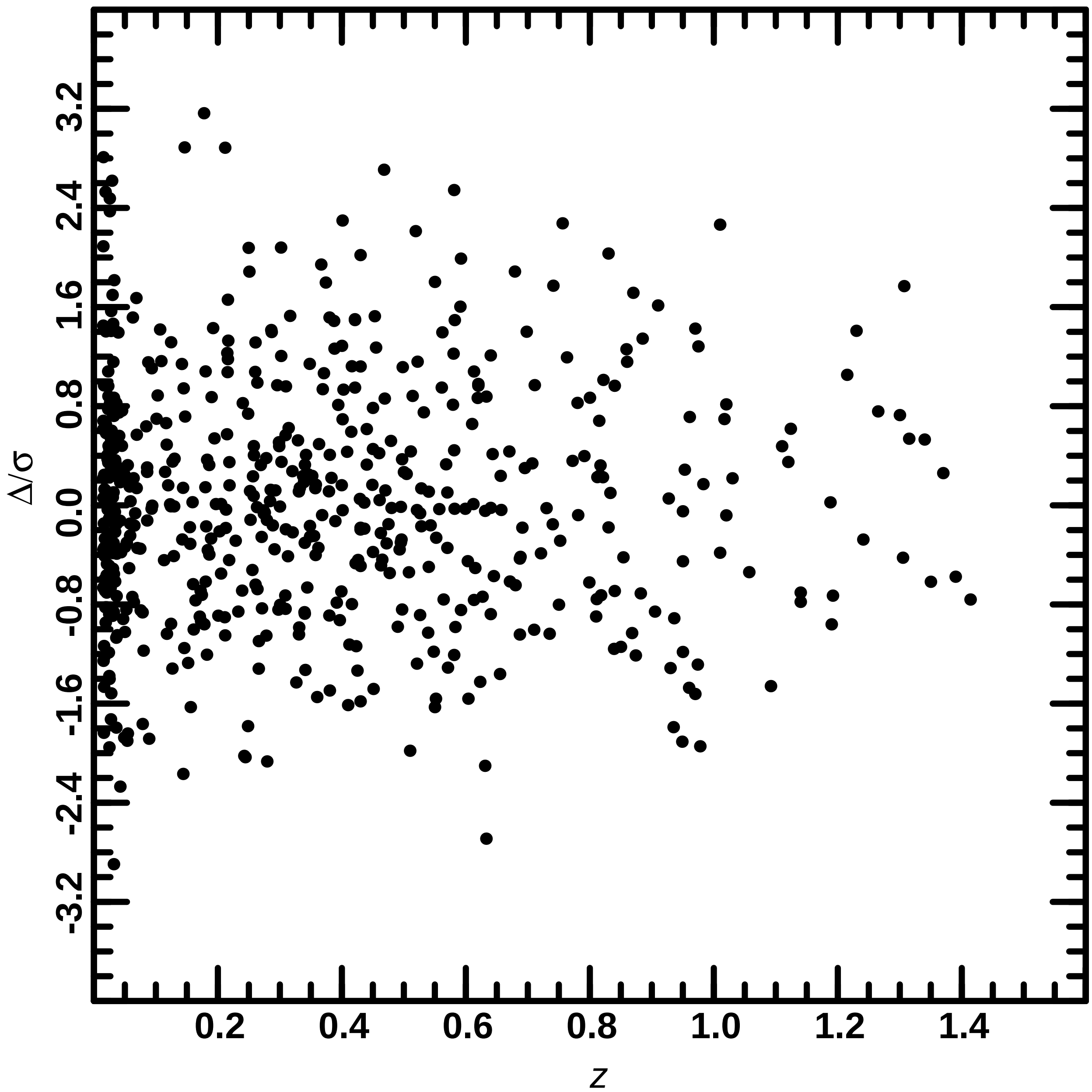
Residuals (points) with observational uncertainties (error bars)



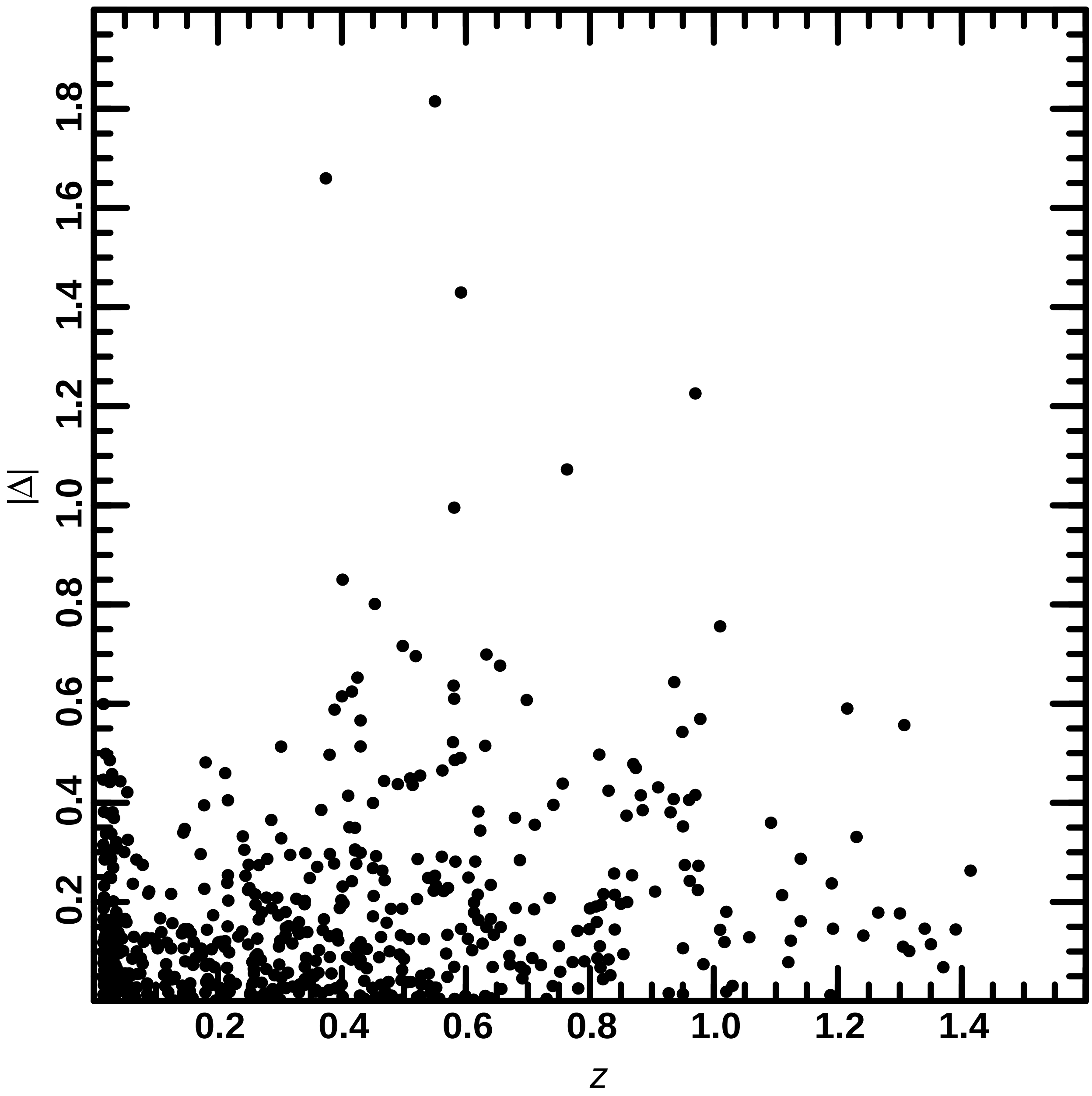
Residuals



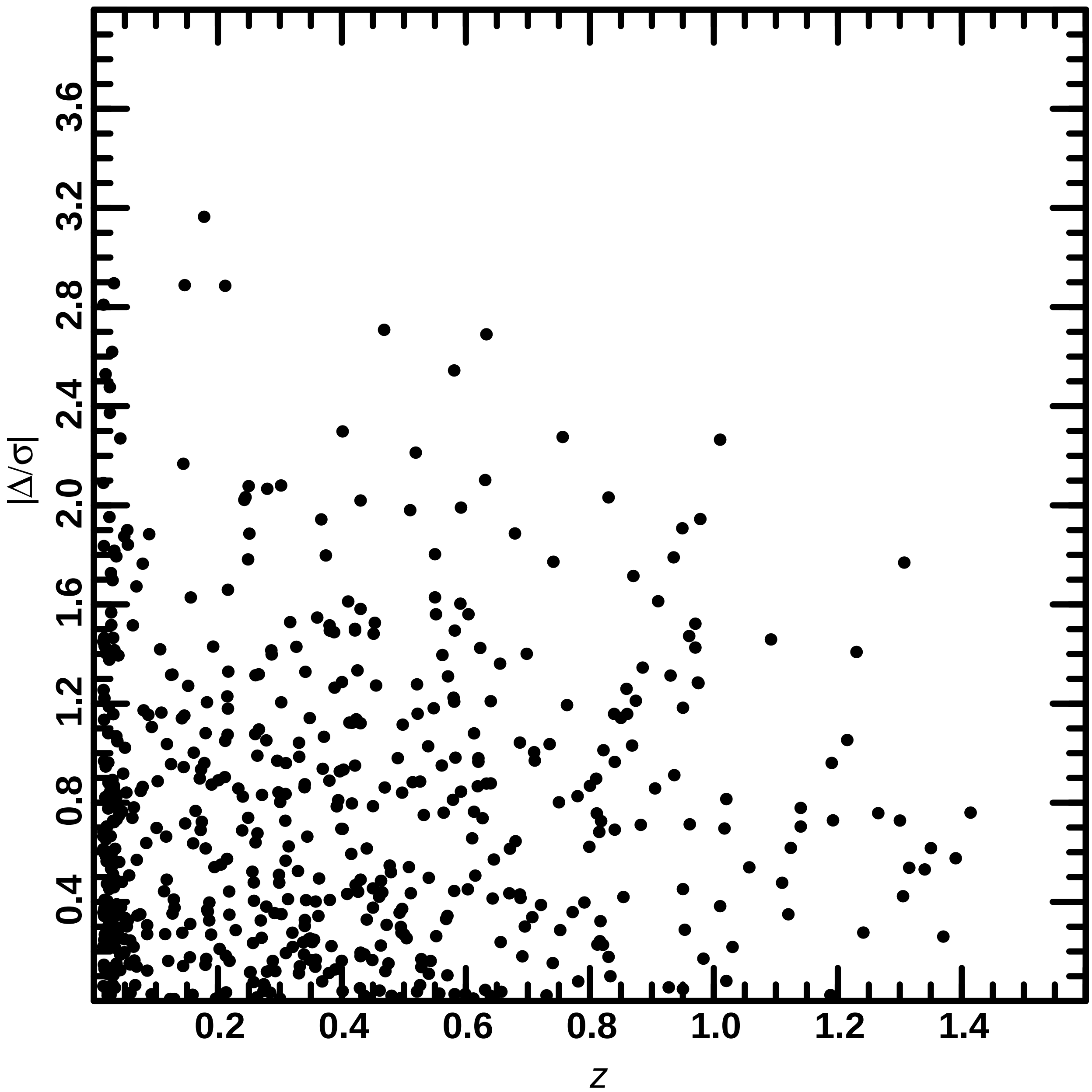
Observational uncertainties



Residuals scaled with observational uncertainties



Absolute value of residuals



Absolute value of residuals scaled with observational uncertainties

Conclusions

- Allowing η as a free parameter significantly alters both the best fit in the λ_0-Ω_0 plane and the allowed region of this plane.
- The concordance model is still allowed.
- There are non-significant hints that $\eta \approx 1$.
- Low values of η can probably be ruled out, which is not obvious considering the very small scales involved.
- Each line of sight is probably a fair sample of the universe; we don't have to average over many to justify $\eta \approx 1$.

Further reading

more general η concept:

Lima, Busti, & Santos: arXiv:1301.5360;
PRD, **89**, 6, 067301

η and the m-z relation:

P. Helbig: arXiv:1505.02917;
MNRAS, **451**, 2, 2097

residuals and uncertainties:

P. Helbig: arXiv:1508.05544;
MNRAS, **453**, 4, 3975

recent paper with all the gory details:

Kaiser & Peacock: arXiv:1503.08506
MNRAS (accepted)