The Effective Strength of Gravity and the Scale of Inflation

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Based on:

What is the strength of gravity?
The Effective Strength of Gravity

For any given momentum transfer, gravitational interactions have a strength set by a characteristic scale $M_\ast$. 

- Inferred from amplitudes calculated in an effective theory with a strong coupling scale $M_{\ast\ast}$.
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  - $M_* \neq M^{**} \neq M_{pl}$
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- Consider a particle w/ mass $M$.
- Scatter a test particle off a very heavy point mass; when $\Delta x \sim M^{-1}$, virtual pairs of these particles are created.
- Positive/negative energy solution attracted/repulsed from source, effectively anti-screening it. Gravity appears to have gotten ‘stronger’.
- N.B. There are many subtleties and caveat emptors when dealing with running quantities in EFT of gravity.
The Strong Coupling Scale

More concretely, every massive species contributes to lowering the scale at which strong gravitational effects become important.

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On a Minkowski background: Dvali, Redi, arXiv:0710.4344

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- In the limit \( p^2 \gg M^2 \), theory becomes conformal;

\[ \langle T(-p)T(p) \rangle \sim \frac{c}{16 \pi^2} p^4 \log \frac{p^2}{\mu^2} \]

- \( c := N = \frac{4}{3} N_\phi + 8 N_\psi + 16 N_\nu \) \( \text{Duff, Nucl. Phys. B 125, 334 (1977)} \)
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Comparison with the free propagator \( 1/(p^2 M^2_{pl}) \) implies that the perturbative expansion fails at \( p = M^{**} \) where \( M^{**} \sim \frac{M_{pl}}{\sqrt{N}} \).
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The Strong Coupling Scale

Generalized to curved backgrounds, places bounds on maximum allowed curvature.

- \( S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[ c_1 R^2 + c_2 R^\mu_\nu R^\mu_\nu \right] + \ldots \)

- \( c_1, c_2 \) is a weighted index counting spins and the numbers of species \( c_1, c_2 \sim N \)
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- e.g. During inflation, let's say we tried to calculate graviton 2-pt function; $h_{\mu\nu} = g_{\mu\nu} - g_{\mu\nu}^0$
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\[ \text{From leading term:} \]

\[ S = \frac{M_{pl}^2}{8} \int d^4x \sqrt{-g^0} \left[ \dot{h}_{ij} \dot{h}_{ij} - \frac{1}{a^2} \partial_k h_{ij} \partial_k h_{ij} \right] \]

\[ \text{... but also get contributions from higher curvature terms s.t.} \]

\[ M_{pl}^2 \rightarrow M_{pl}^2 \left( 1 + c \frac{H^2}{M_{pl}^2} + ... \right) \]
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- ... but also get contributions from higher curvature terms s.t.
  \[ M_{pl}^2 \rightarrow M_{pl}^2 \left( 1 + c \frac{H^2}{M_{pl}^2} + \ldots \right) \]
- Expansion breaks down when \( H^2 \sim M_{pl}^2/N \).
- Corollary: it is not possible to consistently infer a scale of inflation \( H \) greater than \( M_{pl}/\sqrt{N} \).
The strength of gravity $M_*$ is an independent quantity\textsuperscript{1}. Provided we are below $M_{**}$, any universally coupled species will also affect the effective strength of gravity depending on the process in question (equivalence principle, in general violated).

- KK gravitons do so universally for all conserved sources.

\[ M_{2*} = M_{pl}/N^*, \]

\[ N^* \] counts the number of species with masses below the momentum transfer of the process in question.

\textsuperscript{1}Can be $M_{pl}$ all the way up $M_{**}$ cf. Gasperini, arXiv:1508.06100
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For KK gravitons with mass $M$, we have tree level exchange:

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\frac{1}{M_{pl}^2 p^2} \rightarrow \frac{1}{M_{pl}^2 p^2} + \frac{n}{M_{pl}^2 (p^2 + M^2)}
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In the regime $M^2 \ll p^2 \ll M_{pl}^2 / N$, strength of gravity is modified immediately above $p = M$ as:

$$\frac{1}{M_{pl}^2 p^2} + \frac{n}{M_{pl}^2 (1 + M^2 / p^2)} \rightarrow \frac{n+1}{M_{pl}^2 p^2}$$
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- Universally coupled species, e.g. Higgs (w/ $D = 6$ interactions)
  \[ \Delta \mathcal{L}_{\text{eff}} \sim c_1 \frac{H^\dagger H}{M_{pl}^2} \partial_\mu \varphi \partial^\mu \varphi + c_2 \frac{H^\dagger H}{M_{pl}^2} \bar{\psi} i\gamma_\mu \psi \sim c_{\{1,2\}} \frac{H^\dagger H}{M_{pl}^2} T^\mu_\mu \]
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Non-minimal couplings do the same. $M_* = M_{pl}/N_*$, $N_*$ a (process dependent) weighted index.
It is widely understood that any detection of primordial tensor modes in the CMB determines the scale of inflation. Or does it?

► In foliation were inflaton fluctuations are gauged away (comoving/ unitary gauge):

\[ h_{ij}(t, x) = a^2(t) e^{2\mathcal{R}(t,x)} \hat{h}_{ij}; \quad \hat{h}_{ij} := \exp[\gamma_{ij}], \quad \partial_i \gamma_{ij} = \gamma_{ii} = 0 \]
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- Can compute two point correlators of \( \mathcal{R}, \gamma_{ij} \). Useful quantity is the so called dimensionless power spectrum
  \[ 2\pi^2 \delta^3(\vec{k} + \vec{q})\mathcal{P}_\mathcal{R}(k) := k^3 \langle 0|\hat{\mathcal{R}}_k \hat{\mathcal{R}}_{\vec{q}}|0\rangle_{\text{in in}} \]
  \[ \mathcal{P}_\mathcal{R} := \frac{H_*^2}{8\pi^2 M_*^2 \epsilon_*} = A \times 10^{-10}; \quad \epsilon_* := -\dot{H}_*/H_*^2 \]
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- But what is \( M_* s \)?
Consider a non-minimally coupled species with a mass between $10^{-2} \text{eV}$ and the putative scale of inflation:

$$\Delta \mathcal{L}_{\text{eff}} \sim \xi \eta^2 \frac{T_{\mu}^\mu}{M_{pl}^2} \equiv -\xi \eta^2 R$$
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Naively:

$$L_{\text{eff}} \supset \frac{M^2_{\text{pl}}}{2} \left( 1 - \frac{2\xi \eta^2}{M^2_{\text{pl}}} \right) R$$
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EM tensor scales as

$$T^\mu_\nu(\eta_*) = \frac{F^2(\eta_0)}{F^2(\eta_*)} T^\mu_\nu(\eta_0).$$

This is equivalent to keeping the EM tensor fixed, but scaling $M_*$ as $M_* \propto F(\eta)$. $M_*(\eta) = F(\eta)/F(\eta_0) \cdot M_{pl}$
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This is equivalent to keeping the EM tensor fixed, but scaling $M_*$ as

$$M_* \propto F(\eta) M_*(\eta) = F(\eta)/F(\eta_0) \cdot M_{pl}$$

$$M_*^2 = M_{pl}^2 \frac{F^2(\eta_*)}{F^2(\eta_0)} \approx \frac{M_{pl}^2}{1 + \tilde{N}_*}$$

where

$$\tilde{N}_* := \sum_i g_i, \text{ where for example, } g_i = 2\Delta(\xi_i \eta_i^2)/M_{pl}^2.$$
Similarly for tensor perturbations – all TT spin two species* lighter than $H$ contribute to the power spectrum:

$$P_{\gamma} := 2(1 + N_{KK}^T) \frac{H^2_*}{\pi^2 M^2_s}$$
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- Equivalent to the replacement $M_{*T}^2 = M_{*s}^2 / (1 + N_{KK}^T)$

$$P_\gamma := 2 \frac{H_*^2}{\pi^2 M_{*t}^2}; \quad r_* := \frac{P_\gamma}{P_R} = 16 \epsilon_* \left( \frac{M_{*s}^2}{M_{*T}^2} \right)$$

- Any positive detection (i.e. determination of $r_*$) implies

$$V_*^{1/4} = \frac{M_{pl}}{\sqrt{N_*}} \left( \frac{3 \pi^2 A r_*}{2 \cdot 10^{10}} \right)^{1/4}; \quad N_* \sim (1 + N_{KK}^T)(1 + N_s)$$

* N.B. Higuchi bound: massive spin 2 fields inconsistent on dS backgrounds s.t. $m^2 < 2 H_*^2$. However inflation is not dS, KK evidently gravitons are not described by the Fierz-Pauli Lagrangian in progress.
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$$\mathcal{P}_\gamma := 2(1 + N^T_{KK}) \frac{H^2_*}{\pi^2 M^2_* s}$$

Equivalent to the replacement $M^2_{*T} = M^2_{*s} / (1 + N^T_{KK})$

$$\mathcal{P}_\gamma := 2 \frac{H^2_*}{\pi^2 M^2_{*t}} ; \quad r_* := \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} = 16 \epsilon_* \left( \frac{M^2_{*s}}{M^2_{*T}} \right)$$

Any positive detection (i.e. determination of $r_*$ ) implies

$$V_*^{1/4} = \frac{M_{pl}}{\sqrt{N_*}} \left( \frac{3 \pi^2 A r_*}{2 \cdot 10^{10}} \right)^{1/4} ; \quad N_* \sim (1 + N^T_{KK})(1 + N_s)$$

e.g. $r_* = 0.1$ implies $V_*^{1/4} = 7.6 \times 10^{-3} M_{pl} / \sqrt{N_*}$. Uncertain up to unknown $N_*$.  

The Effective Strength of Gravity and the Scale of Inflation

Subodh P. Patil

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Equivalent to the replacement \(M_{*T}^2 = M_{*s}^2 / (1 + N_{KK}^T)\)

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P_{\gamma} := 2 \frac{H_*^2}{\pi^2 M_{*t}^2}; \quad r_* := \frac{P_{\gamma}}{P_R} = 16 \epsilon_* \left(\frac{M_{*s}^2}{M_{*T}^2}\right)
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- For putatively low scale inflationary models, extra species can bring down the scale of inflation even further. Collider constraints?
- Given $M_* \leq M_{pl}$ and that $M_{**} = M_{pl}/N$, $N$ being total number of species, can infer the absolute bound $N \leq \frac{9.15}{r_*} \times 10^7 \left( \frac{M^2_{pl}}{M^2_* T} \right)$.