First-order Fermi acceleration at pulsar wind termination shock

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Motivation

PSR B1259-63

Tam et al. 2015

Days since periastron

Tam et al. 2015
Motivation

PSR B1259-63

- superluminal waves for $\omega > \omega_{p0}$
- electromagnetically modified shock
- relevant in the context of $\gamma$-ray binaries

Amano & Kirk 2013
Motivation

**PSR B1259-63**

- superluminal waves for $\omega > \omega_{p0}$
- electromagnetically modified shock
- relevant in the context of $\gamma$-ray binaries
  - Amano & Kirk 2013

**Goal:** investigate the effects of the electromagnetically modified shock on particle acceleration

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Outline

1. Simulation of the pulsar wind
   - investigation of different $\omega$, $\sigma$ and $\Gamma$
   - precursor steady states

2. Test particle approach
   - reflection probability
   - energy spectrum

3. Simulation of Fermi-like process
   - pure scattering + normal deflection upstream, scattering downstream
   - pulsar wind upstream, scattering downstream
   - different regimes for $\lambda', r'_g, L'_\text{scatt}$
Simulation of the pulsar wind

Set-up
- two relativistic fluids ($e^-, e^+$)
- 1D simulation
- ultra-relativistic shock $\Gamma_s$
- magnetised flow
  \[ \sigma = \frac{\text{Poynting flux}}{\text{particle kinetic energy density}} \]

The wind
- fully transverse, circularly polarised magnetic shear wave
- null phase-averaged magnetic field
- $\omega \propto \omega_{p0}$ with $\omega_{p0} = \sqrt[\frac{8\pi ne^2}{m}}$
  (upstream proper plasma frequency)
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runs $\omega = 1.2\omega_{p0}$

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Run A: $\Gamma = 40, \; \sigma = 10$

snapshot at $\omega_p t = 1000$

$\omega = 0.4\omega_p, \; \omega = 1.2\omega_p, \; \omega = 2.5\omega_p$
Run A: $\Gamma = 40, \sigma = 10, \omega = 1.2\omega_{p0}$

looking for a steady state

$\omega_{p0}t[0:3000], x/c/\omega_{p0}[0:3000]$

$x_{sh}/c/\omega_{p0} = 2120$
Run B: $\Gamma = 100$, $\sigma = 25$

snapshot at $\omega_p t = 1000$

$\omega = 1.2\omega_p$, $\omega = 2.5\omega_p$, $\omega = 3.8\omega_p$
Run B: $\Gamma = 100$, $\sigma = 25$, $\omega = 1.2\omega_p$ looking for a steady state

\[ \omega_p t [0 : 1700], \ x/c/\omega_p [0 : 2000] \]

\[ x_{sh}/c/\omega_p = 1335 \]
Numerical integration of particle trajectories

\[
\frac{d\vec{x}}{dt} = \vec{\beta}
\]

\[
\frac{d\vec{n}}{dt} = \frac{q}{m\gamma\beta} [\vec{E}_\perp + \vec{\beta} \times \vec{B}]
\]

\[
\frac{d\gamma}{dt} = \frac{q}{mc} \beta \vec{n} \cdot \vec{E}
\]

using 4th order Runge-Kutta method in the test particle limit

Trajectories started off far upstream of the shock with isotropic distribution in the upstream fluid frame.
Typical trajectories

Particles are followed until they reach either one of the two spatial boundaries of the simulation box.
we set upstream and downstream absorbing boundaries to record particles
Injection/reflection probability

**RUN A**
\[ x_{sh} = 2120 \]
\[ x_{pre-sh} \approx 1800 \]

**RUN B**
\[ x_{sh} = 1335 \]
\[ x_{pre-sh} \approx 1150 \]
Injection/reflection probability

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\[ x_{sh} = 1335 \]
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\[ P_{refl} \sim 0.44 \]

\[ P_{refl} \sim 0.15 \]

\[ P_{refl} \sim 0.12 \] for ultrarelativistic, perpendicular shocks (Achterberg *et al.* 2001)
Energy spectrum

RUN A
upstream boundary $x_{up} = 1670$
downstream boundary $x_{down} = 2720$

RUN B
upstream boundary $x_{up} = 985$
downstream boundary $x_{down} = 1885$

$\gamma'_{inj} \approx \sigma \Gamma^2$ in the upstream fluid frame
First-order Fermi acceleration

Monte Carlo technique
- elastic scattering in the local fluid frame
- isotropic diffusion
- \[ f(p, \mu) = p^{-s} g(\mu) \]
First-order Fermi acceleration at pulsar wind termination shock

Monte Carlo technique

- elastic scattering in the local fluid frame
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- \( f(p, \mu) = p^{-s} g(\mu) \)

\[
d\vec{n} = \bar{a}(x, t) \ast dt + \bar{b}(x, t) \ast dW(t)
\]

\( d\vec{n} \) is small angle scattering 
\( \delta \psi \ll 1/\Gamma \implies D_\theta \)

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First-order Fermi acceleration at pulsar wind termination shock
Injection conditions in the upstream frame

- location: at the shock front
- direction: $\mu_{inj}' = -1$
- energy: $\gamma_{inj}' = \sigma \Gamma^2$

The trajectory integration is performed in the local fluid frame.

$$\mu = \cos \theta$$
Deflection: case of pure scattering

Comparison with the results of the eigenfunction method (Kirk et al. 2000) for different values of $\Gamma_u * \beta_u$. 

$\Gamma_u * \beta_u = 10.0$

![Graph showing comparison]
Deflection: regular deflection upstream, scattering downstream

\[ \Gamma_u \ast \beta_u = 100.0 \]

\begin{table}
\begin{tabular}{ ccc }
\hline
Achterberg et al. 2001 & this work \\
\hline
\( \Gamma_s \) & \( s \) & \( s \) \\
10 & 4.28 \( \pm 0.01 \) & 4.28 \( \pm 0.01 \) \\
100 & 4.30 \( \pm 0.01 \) & 4.30 \( \pm 0.01 \) \\
\hline
\end{tabular}
\end{table}
Deflection: pulsar wind upstream, regime I

Characteristic length-scales upstream

- \( \chi' = \frac{2\pi \beta_s \Gamma_s}{\omega} \) (\( \Gamma_u \beta_u = 10.0 \))
- \( L'_{\text{scatt}} = c t'_{\text{scatt}} = c/2D_\theta \) (\( \Delta \theta \sim 1/\Gamma \))
- \( r'_g = \gamma'/eB' \)

\[
L'_{\text{scatt}} \sim \chi' \gg r'_{g,\text{max}} \implies s = 4.28 \pm 0.01
\]
Deflection: pulsar wind upstream, regime II

Characteristic length-scales upstream

- \( \lambda' = \frac{2\pi \beta_s \Gamma_s}{\omega} \) \( (\Gamma_u \ast \beta_u = 10.0) \)
- \( L'_{\text{scatt}} = c t'_{\text{scatt}} = c/2D_\theta \) \( (\Delta\theta \sim 1/\Gamma) \)
- \( r'_{g} = \gamma'/eB' \)

\[ L'_{\text{scatt}} \gg r'_{g,\text{max}} \gg \lambda' \gg r'_{g,\text{min}} \implies s = 4.26 \pm 0.01 \]
Characteristics length-scales upstream

- \( \lambda' = \frac{2\pi\beta_s\Gamma_s}{\omega} \quad (\Gamma_u \ast \beta_u = 10.0) \)
- \( L'_{\text{scatt}} = ct'_{\text{scatt}} = c/2D_{\theta} \quad (\Delta\theta \sim 1/\Gamma) \)
- \( r'_g = \gamma'/eB' \)

- \( r'_g,_{\text{max}} \gg L'_{\text{scatt}} > r'_g,_{\text{min}} \sim \lambda' \quad \Rightarrow s = 4.23 \pm 0.01 \)
Realistic pulsar wind

\[
\sigma = \frac{B'^2}{4\pi \sum_s \Gamma_s w_s u_{s,x}} \quad B'_0 = \sqrt{8\pi \sigma}
\]

the specific enthalpy in the cold wind is

\[w_s = n_s m_s c^2\]

\[
\lambda' = \frac{2\pi \beta_s \Gamma_s}{\omega} \quad r'_g = \frac{\gamma}{\sqrt{\sigma}}
\]

\[\omega_{p0} = m = c = 1, \quad e = 1/\sqrt{8\pi}\]

we showed that \(\gamma'_{\text{inj}} \approx \sigma \Gamma^2_s\) for both RUN A and RUN B

\[\frac{\lambda'}{r'_g} = \frac{2\pi \beta_s}{\omega \sqrt{\sigma} \Gamma_s} \ll 1 \quad \Rightarrow \quad s = 4.23\]
1 Simulation of the pulsar wind
- investigation of parameter space \((\Gamma, \sigma, \omega)\)
- precursor steady state for \(\Gamma = 40, \sigma = 10\) and \(\Gamma = 100, \sigma = 25\) for \(\omega = 1.2\omega_p0\)

2 Test particle approach
- RUN A \(P_{inj} \sim 0.44\), RUN B \(P_{inj} \sim 0.15\)
- injection energy \(\gamma_{inj}' \approx \sigma \Gamma^2\)

3 Simulation of Fermi-like process
- results of our code in perfect agreement with semi-analytical (Kirk et al. 2000) and numerical (Achterberg et al. 2001) works
- study of the effects of the pulsar wind (magnetic shear) in different regimes
- for realistic pulsar wind \(\lambda'/r'_g \ll 1\) and \(s = 4.23\)
Summary

1. Simulation of the pulsar wind
   - investigation of parameter space \((\Gamma, \sigma, \omega)\)
   - precursor steady state for \(\Gamma = 40, \sigma = 10\) and \(\Gamma = 100, \sigma = 25\) for \(\omega = 1.2\omega_p\)

2. Test particle approach
   - RUN A \(P_{inj} \sim 0.44\), RUN B \(P_{inj} \sim 0.15\)
   - injection energy \(\gamma'_{inj} \approx \sigma\Gamma^2\)

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   - study of the effects of the pulsar wind (magnetic shear) in different regimes
   - for realistic pulsar wind \(\lambda'/r'_g \ll 1\) and \(s = 4.23\)

THANK YOU FOR YOUR ATTENTION
BACK UP SLIDES
Magnetic shear wave

**Lab. frame (Shock frame)**

\[
\begin{align*}
B_x &= 0 \\
B_y &= B_0 \cos(kx - \omega t) \\
B_z &= -B_0 \sin(kx - \omega t) \\
E_x &= 0 \\
E_y &= \beta B_z \\
E_z &= -\beta B_y \\
k &= \omega/\beta \\
\omega &= 1.2\omega_{p0} \\
\lambda &= 2\pi\beta/\omega
\end{align*}
\]

**Upstream fluid frame**

\[
\begin{align*}
B_x' &= 0 \\
B_y' &= B_y/\Gamma \\
B_z' &= B_z/\Gamma \\
E_x' &= 0 \\
E_y' &= 0 \\
E_z' &= 0 \\
k' &= k/\Gamma \\
\omega' &= 0 \\
\lambda' &= 2\pi\beta\Gamma/\omega
\end{align*}
\]

\[
\beta = \sqrt{1 - 1/\Gamma^2}, \quad \omega_{p0} = \sqrt{\frac{8\pi ne^2}{m}}, \quad \bar{k} = (k, 0, 0)
\]
Flux at the upstream boundary

RUN A
upstream boundary $x_{up} = 1670$
downstream boundary $x_{down} = 2720$

RUN B
upstream boundary $x_{up} = 985$
downstream boundary $x_{down} = 1885$

\[
\mu = \cos \theta
\]

$\mu_{inj}' \approx -1$ in the upstream fluid frame
Run A: $\Gamma = 40, \sigma = 10$

enlarged view of the precursor, snapshot at $\omega_p t = 1000$

$\omega = 0.4\omega_p, \omega = 1.2\omega_p, \omega = 2.5\omega_p$

$$\omega_{\text{eff}}^p = \sqrt{\frac{8\pi ne^2}{mh}}$$

$$h = 1 + \frac{\gamma_h}{\gamma_h - 1} \frac{T}{mc^2}, \gamma_h = 4/3 \text{ ratio of specific heat}$$
RUN A and B, $\sigma$ comparison

RUN A: $\Gamma = 40$, $\sigma = 10$
RUN B: $\Gamma = 100$, $\sigma = 25$
Time domain periodic boundaries

folding ensemble of snapshots $\Delta F$

We tested energy spectrum and angular distribution at both ends of the simulation box for different foldings, but same overall integration time.
Spectrum and angular distribution with periodic boundaries

overall integration time $\sim 8000$ for different values of $\Delta F$

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dependence of the spectral index $s$ on the size of the acceleration region compared to $L_{\text{scatt}}$